

Challenge

1. What are the possible values of n given this set notation: $\{n \in \mathbb{Z} \mid n \text{ is a factor of } 8\}$?

$\{1, 2, 4, 8\}$

2. Identify each of the following as true or false:

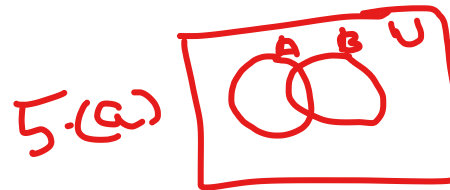
- a. $\{3\} \in \{1, 3, 5, 7\}$ **F**
- b. $\{3\} \subseteq \{1, 3, 5, 7\}$ **T**
- c. $\{3\} \in \{\{1\}, \{3\}, \{5\}, \{7\}\}$ **T**
- d. $\{3\} \subseteq \{\{1\}, \{3\}, \{5\}, \{7\}\}$ **F**

$$\begin{aligned} & A \cap (A \cap B) \\ \Rightarrow & A \cap (A \cup B) \\ \Rightarrow & (A \cap A) \cup (A \cap B) \\ \Rightarrow & A \cup (A \cap B) \\ \Rightarrow & A \end{aligned}$$

3. For all sets A , B , and C , prove: $A - (A \cap B) = A - B$. Identify the name of each law used.

4. Let set A be a set of all the NASA employees and B is the set of all astronauts. Describe the following sets:

- a. $A \cap B \rightarrow B$
- b. $A \cup B \rightarrow A$
- c. $A - B \rightarrow B'$
- d. $B - A \rightarrow \emptyset$



$$\begin{aligned} & A - B \\ \Rightarrow & A \cap B' \\ \Rightarrow & A \cap B \cap B' \\ \Rightarrow & \emptyset \end{aligned}$$

5. Illustrate the following using Venn Diagrams:

- a. $(A \cap B) - A = \emptyset$
- b. $(A - B) \cup (B - A)$



6. What is the power set of $\{a, b, c\}$?

$$P(S) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

7. Interest rates are an example of a geometric sequence. In this example, you deposit \$1,000 in a CD at your local bank, it earns 6% annual interest compounded monthly. What is the balance at the end of 12 months? (hint: don't forget to find the monthly interest rate)

$$\$1000 \cdot 2.441$$

8. Find the value for the given summation:

$$3 + 5 + 7 + 9 = 24$$

$$\sum_{j=2}^5 (2j - 1)$$

9. Rewrite the summation notation after separating out the last term:

$$\Rightarrow 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + \dots + 2^{n-1} + 2^n \quad \sum_{j=0}^n 2^j$$

✓ 10. Rewrite the summation by changing the summation index using $j = i+2$

$$\sum_{i=3}^{21} \frac{1}{i+3} = \sum_{j=5}^{23} \frac{1}{j+1} = \sum_{i=3}^{21} \frac{1}{i+3}$$

✓ 11. Given the recursive definition for a function g :

$$g(0) = 0$$

$$g(n) = g(n-1) + n^3$$

Find $g(3)$

$$\begin{aligned} g(1) &= 0 + 1 = 1 \\ g(2) &= 1 + 8 = 9 \\ g(3) &= 9 + 27 = 36 \end{aligned}$$