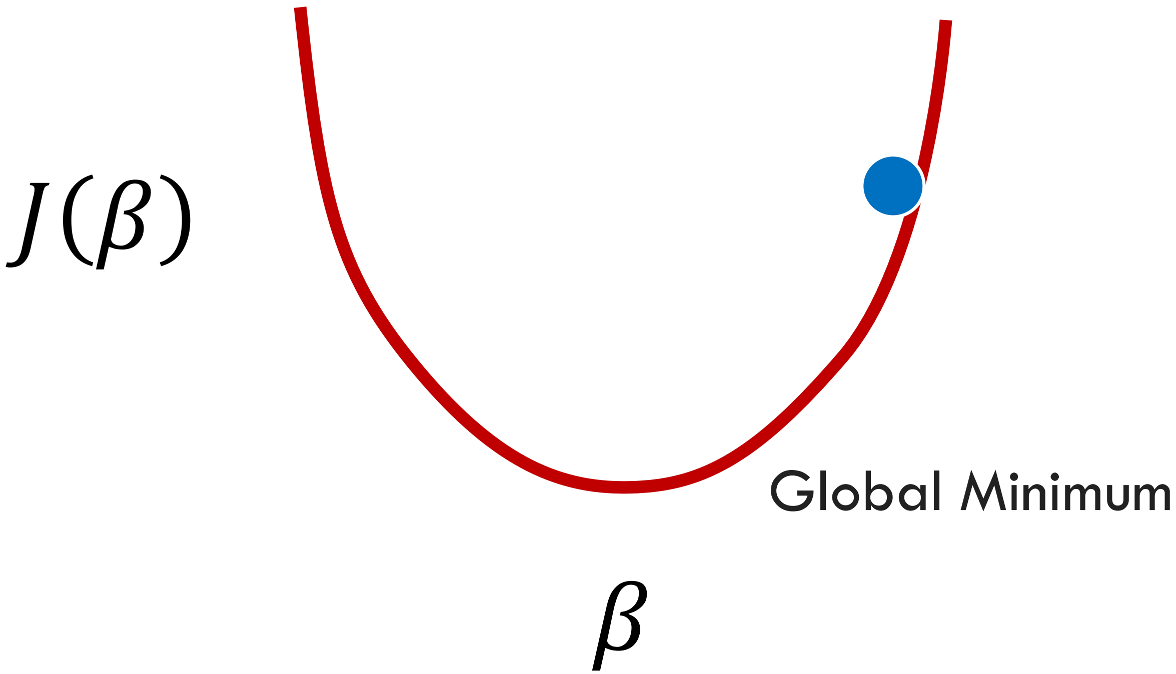
GRADIENT DESCENT

Gradient Descent

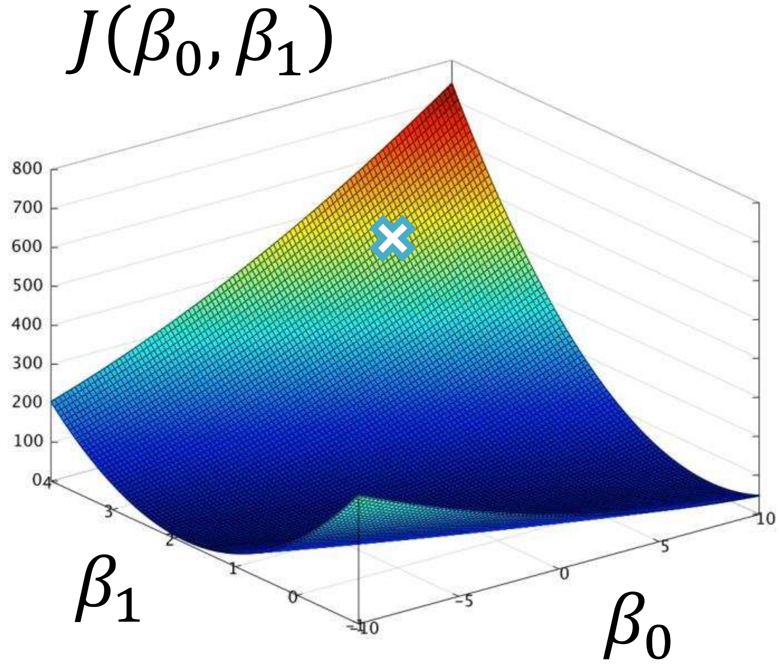
Start with a cost function J(β):



Then gradually move towards the minimum.

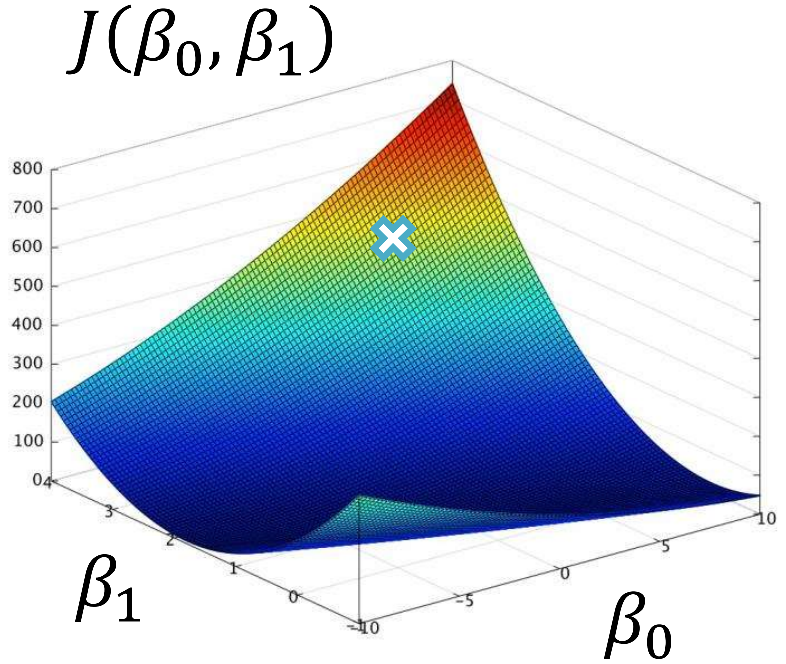
Gradient Descent with Linear Regression

- Now imagine there are two parameters (β_0, β_1)
- This is a more complicated surface on which the minimum must be found
- How can we do this without knowing what $J(\beta_0, \beta_1)$ looks like?



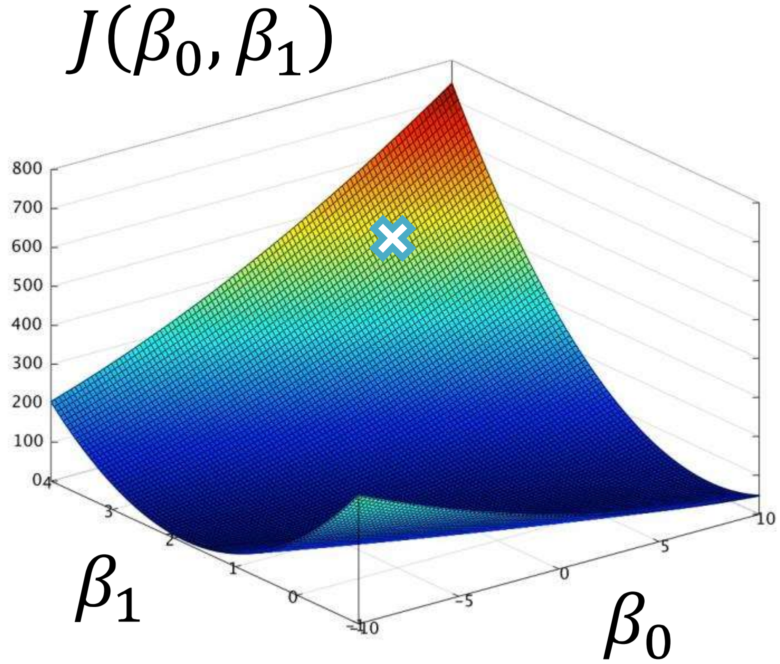
Gradient Descent with Linear Regression

- Compute the gradient, $\nabla J(\beta_0, \beta_1)$, which points in the direction of the biggest increase!
- -$\nabla J(\beta_0, \beta_1)$ (negative gradient) points to the biggest decrease at that point!



Gradient Descent with Linear Regression

- The gradient is the a vector whose coordinates consist of the partial derivatives of the parameters



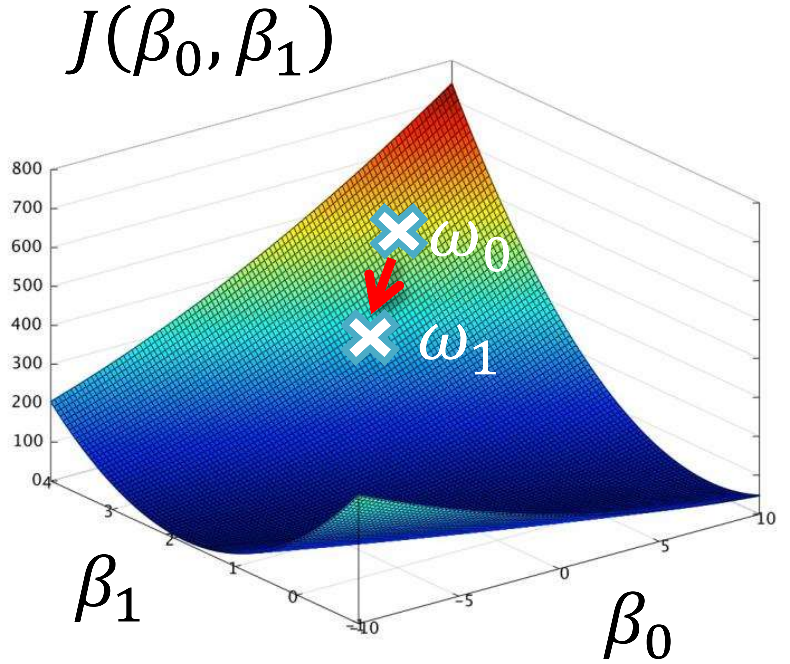
$$\nabla J(\beta_0, \dots, \beta_n) = < \frac{\partial J}{\partial \beta_0},\ \dots, \frac{\partial J}{\partial \beta_n} >$$

Gradient Descent with Linear Regression

- Then use the gradient (∇) and the cost function to calculate the next point (ω_1) from the current one (ω_0):

$$\omega_1 = \omega_0 - \alpha\nabla\frac{1}{2}\sum_{i=1}^{m}\left(\left(\beta_0 + \beta_1 x_{obs}^{(i)}\right) - y_{obs}^{(i)}\right)^2$$

- The learning rate (α) is a tunable parameter that determines step size

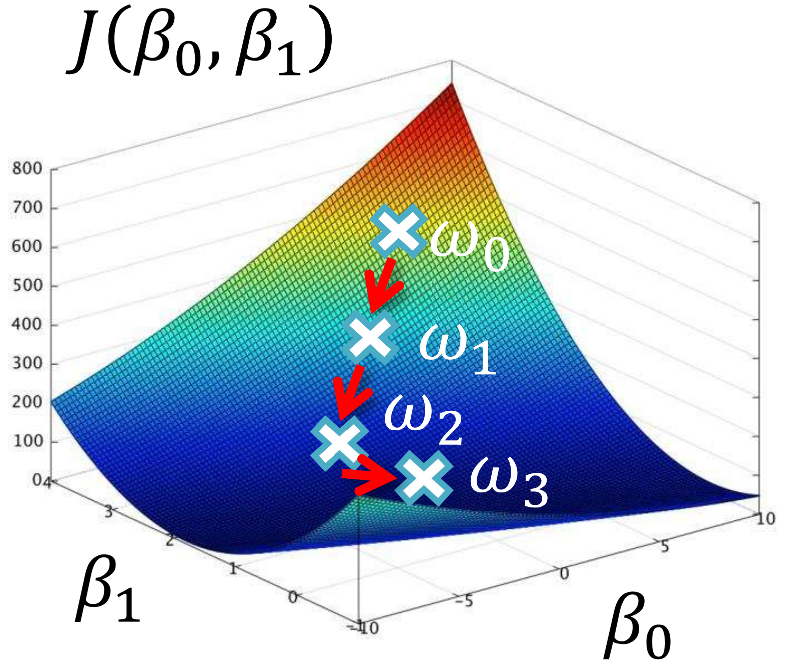


Gradient Descent with Linear Regression

- Each point can be iteratively calculated from the previous one

$$\omega_2 = \omega_1 - \alpha\nabla\frac{1}{2}\sum_{i=1}^{m}\left(\left(\beta_0 + \beta_1 x_{obs}^{(i)}\right) - y_{obs}^{(i)}\right)^2$$

$$\omega_3 = \omega_2 - \alpha\nabla\frac{1}{2}\sum_{i=1}^{m}\left(\left(\beta_0 + \beta_1 x_{obs}^{(i)}\right) - y_{obs}^{(i)}\right)^2$$

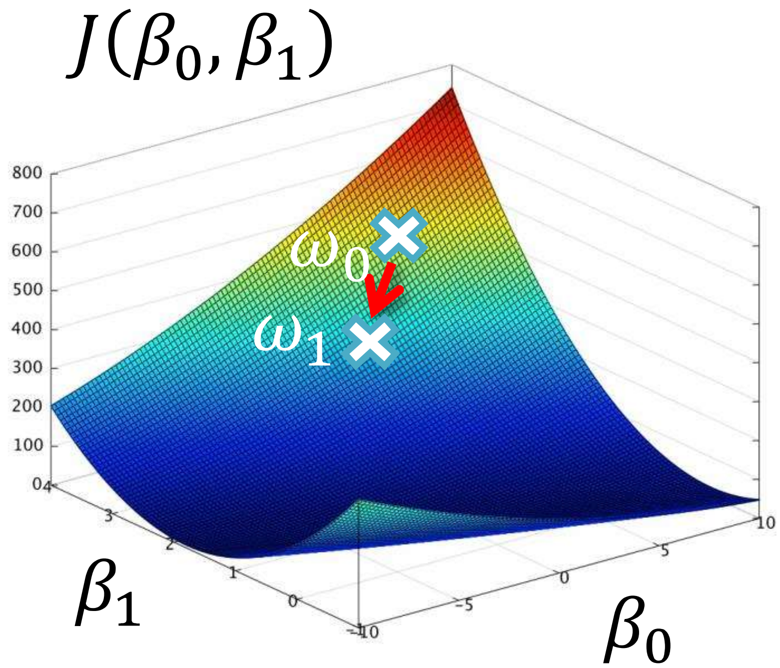


Stochastic Gradient Descent

- Use a single data point to determine the gradient and cost function instead of all the data

$$\omega_1 = \omega_0 - \alpha \nabla \frac{1}{2} \cancel{\sum_{i=1}^{m}} \left(\left(\beta_0 + \beta_1 x_{obs}^{(i)} \right) - y_{obs}^{(i)} \right)^2$$

$$\boxed{\omega_1 = \omega_0 - \alpha \nabla \frac{1}{2} \left(\left(\beta_0 + \beta_1 x_{obs}^{(0)} \right) - y_{obs}^{(0)} \right)^2}$$



Stochastic Gradient Descent

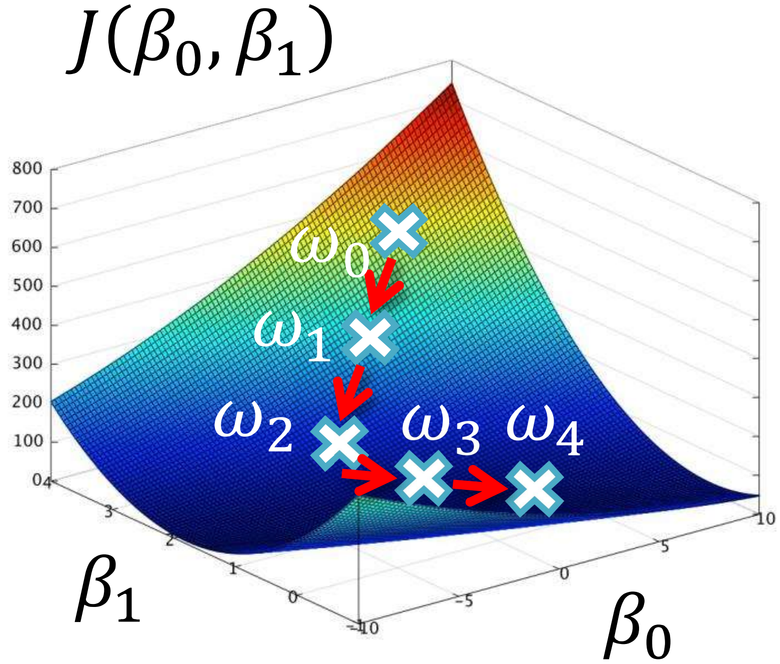
- Use a single data point to determine the gradient and cost function instead of all the data

$$\omega_1 = \omega_0 - \alpha\nabla\frac{1}{2}\left(\left(\beta_0 + \beta_1 x_{obs}^{(0)}\right) - y_{obs}^{(0)}\right)^2$$

$$\ldots$$

$$\omega_4 = \omega_3 - \alpha\nabla\frac{1}{2}\left(\left(\beta_0 + \beta_1 x_{obs}^{(3)}\right) - y_{obs}^{(3)}\right)^2$$

- Path is less direct due to noise in single data point—"stochastic"



Mini Batch Gradient Descent

- Perform an update for every n training examples

$$\omega_1 = \omega_0 - \alpha\nabla\frac{1}{2}\sum_{i=1}^{n}\left(\left(\beta_0 + \beta_1 x_{obs}^{(i)}\right) - y_{obs}^{(i)}\right)^2$$

Best of both worlds:

- Reduced memory relative to "vanilla" gradient descent
- Less noisy than stochastic gradient descent