Department of Computer Science and Engineering (Data Science)
Image Processing and Computer Vision I (DJ19DSL603)

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AIM: To implement frequency domain filters on an image

THEORY:

1. Ideal Low Pass Filter

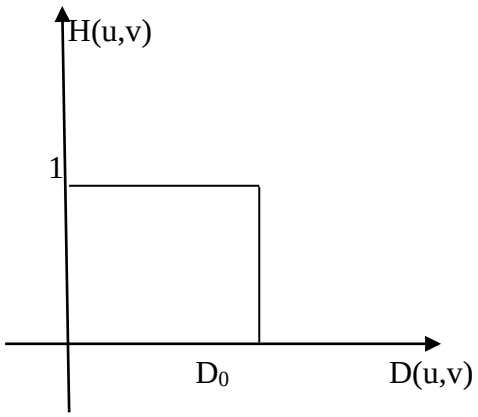
This filter cuts off all high frequency components of the Fourier transform that are at a distance greater than a specified distance D_0.

$$H(u,v) = 1; \text{ if } D(u,v) < D_0$$

$$= 0; \text{ if } D(u,v) > D_0$$

Where,

D_0 is the specified non negative distance.



Response of Ideal Low Pass Filter

D(u,v) is the distance from the point (u,v) to the origin of the frequency rectangle for an M X N image.

$$D(u,v)=[(u-M/2)^2 + (v-N/2)^2]^{\wedge} ½$$

Therefore ,

For an image, when u=M/2 , v=N/2

$$D(u,v)=0$$

This formula centers our H(u,v).

D(u,v) gives us concentric rings with each ring having a fixed value.

When an ideal low-pass filter is applied to an image, the high-frequency components (i.e., the high-frequency information, such as edges and details) are removed, and only the low-frequency components (i.e., the smooth areas and large details) are retained. This results in a blurring or smoothing effect on the image.

Observations:

1. The image appears smoother or less sharp, as high-frequency details are removed.
2. Edges and other high-contrast features may appear blurred or softened.
3. Noise and other high-frequency artifacts may be reduced, resulting in a cleaner appearance.
4. The overall contrast of the image may be reduced, especially in areas with fine details.
5. The filter may introduce ringing artifacts around edges or high-contrast areas, due to the ideal filter's inherent characteristics.

2. Ideal High Pass Filter

When an ideal high-pass filter is applied to an image, the low-frequency components (i.e., the smooth areas and large details) are removed, and only the high-frequency components (i.e., the

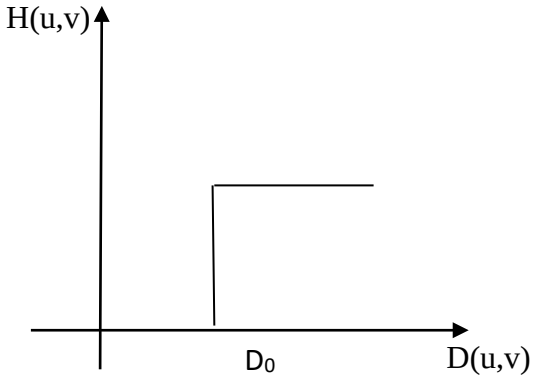
edges and fine details) are retained. This results in an image with enhanced edges and details, but with reduced low-frequency content.

Observations:

1. The image appears sharper, as high-frequency details are enhanced.
2. Edges and other high-contrast features appear more prominent and well-defined.
3. The overall contrast of the image may be increased, especially in areas with fine details.
4. Low-frequency content, such as smooth areas or large features, may appear blurred or reduced in prominence.

The filter may introduce ringing artifacts around edges or high-contrast areas, due to the ideal filter's inherent characteristics.

This filter cuts off all high frequency components of the Fourier transform that are at a distance greater than a specified distance D_0.



Where, H(u,v) = 0; if D(u,v)< D0

= 1; if D(u,v)> D0

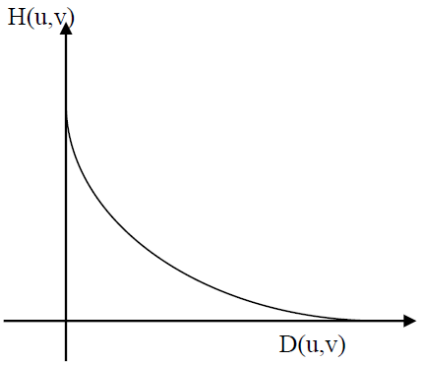
D0 is the specified non negative distance.

D(u,v) is the distance from the point (u,v) to the origin of the frequency rectangle for an M X N image.

3. Gaussian Low Pass Filter

Gaussian LPF is given by:

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$



Where, σ is the standard deviation and is a measure of spread of the Gaussian curve.
If we put σ =D0 we get, $H(u,v) = e^{-D^2(u,v)/2D0^2}$

The response of the Gaussian LPF is similar to that of BLPF but there are no ringing effects.

4. Gaussian High Pass Filter

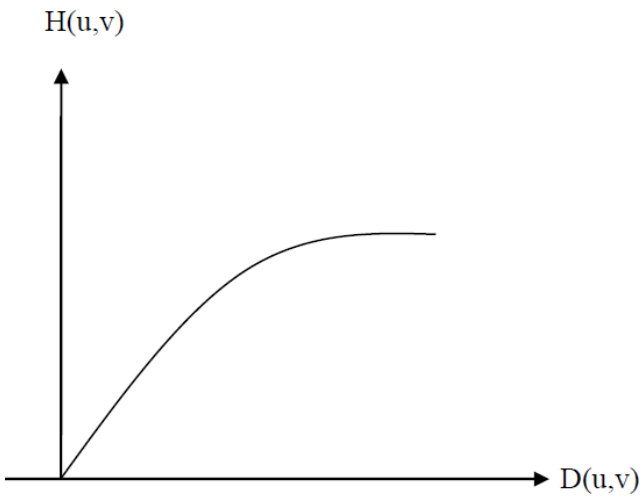
The basic formula is,

$H_{hp}(u,v) = 1 - H_{lp}(u,v)$

Therefore,

$H_{Gaussian\ hp}(u,v) = 1 - H_{Gaussian\ lp}(u,v)$

$H_{GHPF} = 1 - e^{-D^2(u,v)/2D0^2}$



The results of Gaussian high pass filter are smoother and cleaner

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Lab Assignments to complete in this session

Problem Statement: Develop a Python program utilizing the OpenCV library to manipulate images from the Fashion MNIST digits dataset. The program should address the following tasks:

1. Importing libraries
2. Read random image(s) from the MNIST fashion dataset.
3. **Dataset Link:** Fashion MNIST Github
4. Getting the Fourier Transform
5. Ideal Low Pass Filtering
6. Multiplication between the Fourier Transformed input image and the filtering mask
7. Taking Inverse Fourier Transform of the convoluted image
8. Ideal High Pass Filtering
9. Multiplication between the Fourier Transformed input image and the filtering mask
10. Taking Inverse Fourier Transform of the convoluted image
11. Gaussian Low Pass Filtering
12. Multiplication between the Fourier Transformed input image and the filtering mask
13. Taking Inverse Fourier Transform of the convoluted image
14. Gaussian High Pass Filtering
15. Multiplication between the Fourier Transformed input image and the filtering mask
16. Taking Inverse Fourier Transform of the convoluted image

The solution to the operations performed must be produced by scratch coding without the use of built in OpenCV methods.

```
import numpy as np
import matplotlib.pyplot as plt
import cv2
from scipy.fft import fft2, ifft2, fftshift
from keras.datasets import fashion_mnist


(x_train, y_train), (x_test, y_test) = fashion_mnist.load_data()


index = np.random.randint(0, len(x_test))
image = x_test[index]

plt.imshow(image, cmap='gray')
plt.title('Original Image')
plt.show()
```

```
Downloading data from https://storage.googleapis.com/tensorflow/tf-keras-datasets/train-labels-idx1-ubyte.gz
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Downloading data from https://storage.googleapis.com/tensorflow/tf-keras-datasets/train-images-idx3-ubyte.gz
26421880/26421880 ━━━━━━━━━━━━━━━━━━━━ 1s 0us/step
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```
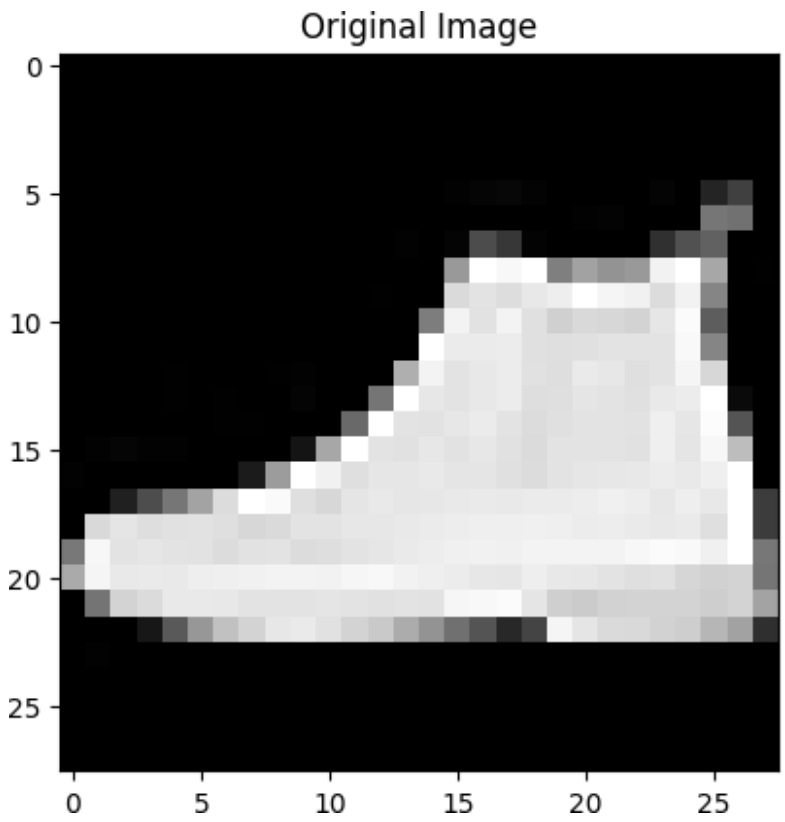
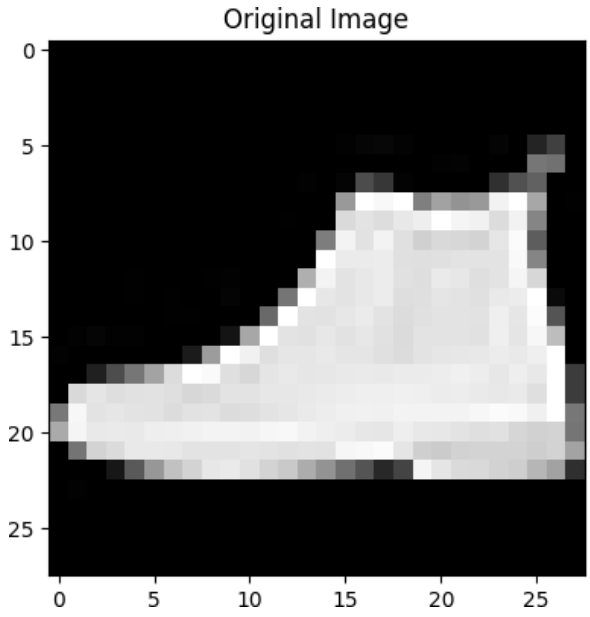




```
def fourier_transform(image):
    f_transform  = fft2(image)
    f_transform_shifted = fftshift(f_transform)
    return f_transform_shifted

def  inverse_fourier_transform(f_transform_shifted):

    f_ishift = np.fft.ifftshift(f_transform_shifted)
    img_back  =  ifft2(f_ishift)
    return np.abs(img_back)

def  ideal_low_pass_filter(shape,  D0):
    P,  Q  =  shape
    center = (P // 2, Q // 2)
    filter_mask  =  np.zeros((P,  Q))

    for  u  in  range(P):
        for  v  in  range(Q):
            D_uv  =  np.sqrt((u  -  center[0])**2  +  (v  -  center[1])**2)
            if  D_uv  <=  D0:
                filter_mask[u, v] = 1
    return  filter_mask

def  ideal_high_pass_filter(shape,   D0):
    P,  Q  =  shape
```

```
    center = (P // 2, Q // 2)
    filter_mask = np.ones((P, Q))

    for u in range(P):
        for v in range(Q):
            D_uv = np.sqrt((u - center[0])**2 + (v - center[1])**2)
            if D_uv <= D0:
                filter_mask[u, v] = 0
    return filter_mask

def gaussian_low_pass_filter(shape, D0):
    P, Q = shape
    center = (P // 2, Q // 2)
    filter_mask = np.zeros((P, Q))

    for u in range(P):
        for v in range(Q):
            D_uv = np.sqrt((u - center[0])**2 + (v - center[1])**2)
            filter_mask[u, v] = np.exp(-(D_uv**2) / (2 * (D0**2)))
    return filter_mask

def gaussian_high_pass_filter(shape, D0):
    return 1 - gaussian_low_pass_filter(shape, D0)

f_transform = fourier_transform(image)

D0 = 50
lpf_mask = ideal_low_pass_filter(image.shape, D0)
filtered_image_lpf = f_transform * lpf_mask
filtered_image_lpf_spatial =
inverse_fourier_transform(filtered_image_lpf)

plt.figure(figsize=(10, 5))
plt.subplot(1, 2, 1)
plt.imshow(image, cmap='gray')
plt.title('Original Image')
plt.subplot(1, 2, 2)
plt.imshow(filtered_image_lpf_spatial, cmap='gray')
plt.title('Low Pass Filtered Image')
plt.show()

hpf_mask = ideal_high_pass_filter(image.shape, D0)
filtered_image_hpf = f_transform * hpf_mask
filtered_image_hpf_spatial =
inverse_fourier_transform(filtered_image_hpf)


plt.figure(figsize=(10, 5))
plt.subplot(1, 2, 1)
plt.imshow(image, cmap='gray')
plt.title('Original Image')
```

```
plt.subplot(1, 2, 2)
plt.imshow(filtered_image_hpf_spatial, cmap='gray')
plt.title('High Pass Filtered Image')
plt.show()


gaussian_lpf_mask = gaussian_low_pass_filter(image.shape, D0)
filtered_image_gaussian_lpf = f_transform * gaussian_lpf_mask
filtered_image_gaussian_lpf_spatial =
inverse_fourier_transform(filtered_image_gaussian_lpf)

plt.figure(figsize=(10, 5))
plt.subplot(1, 2, 1)
plt.imshow(image, cmap='gray')
plt.title('Original Image')
plt.subplot(1, 2, 2)
plt.imshow(filtered_image_gaussian_lpf_spatial, cmap='gray')
plt.title('Gaussian Low Pass Filtered Image')
plt.show()


gaussian_hpf_mask = gaussian_high_pass_filter(image.shape, D0)
filtered_image_gaussian_hpf = f_transform * gaussian_hpf_mask
filtered_image_gaussian_hpf_spatial =
inverse_fourier_transform(filtered_image_gaussian_hpf)


plt.figure(figsize=(10, 5))
plt.subplot(1, 2, 1)
plt.imshow(image, cmap='gray')
plt.title('Original Image')
plt.subplot(1, 2, 2)
plt.imshow(filtered_image_gaussian_hpf_spatial, cmap='gray')
plt.title('Gaussian High Pass Filtered Image')
plt.show()
```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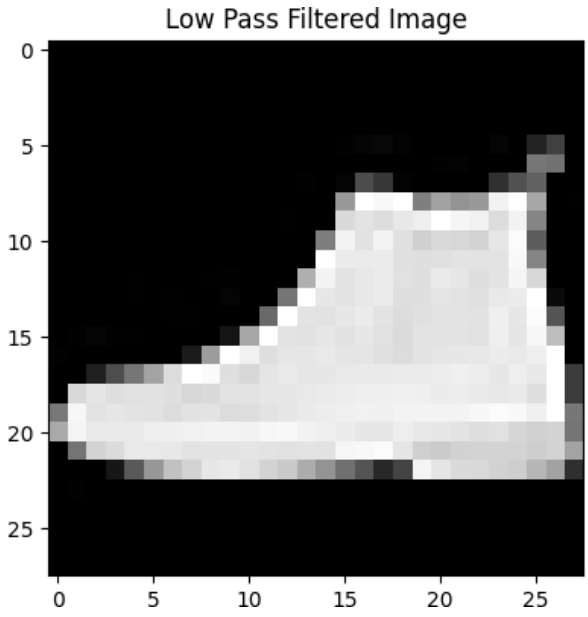
Low Pass Filtered Image
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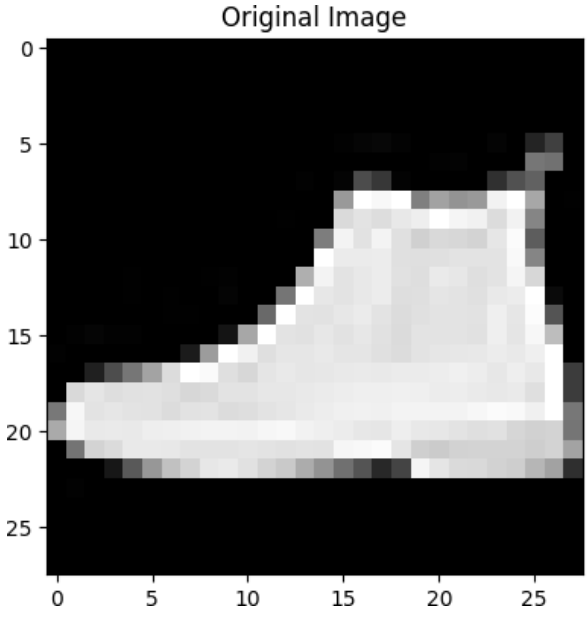
Original Image
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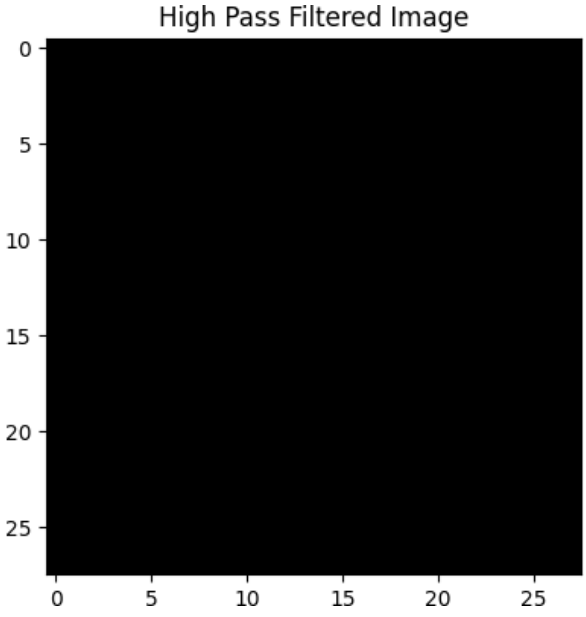
High Pass Filtered Image
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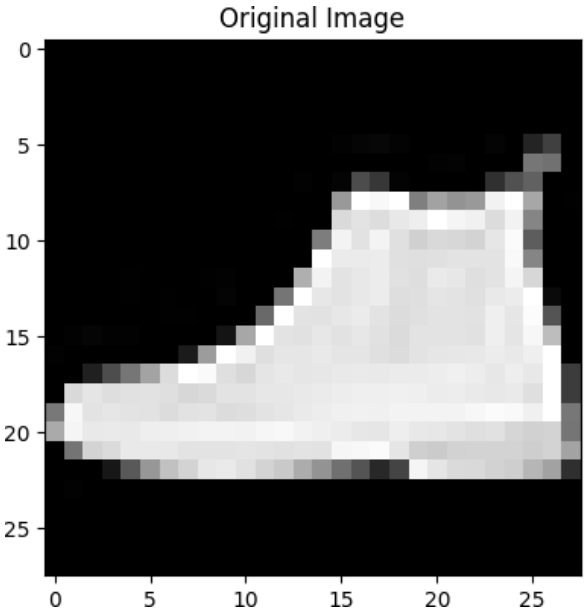
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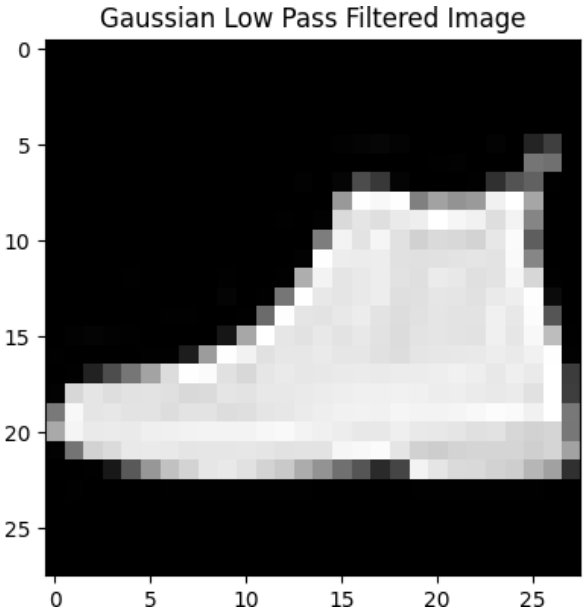
Gaussian Low Pass Filtered Image
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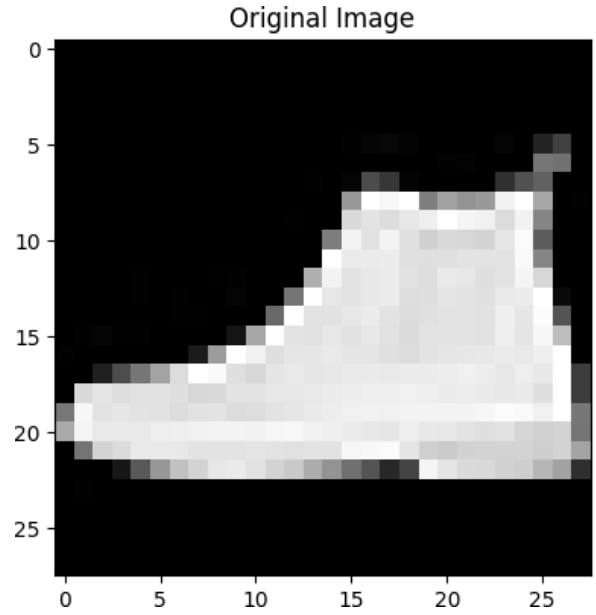
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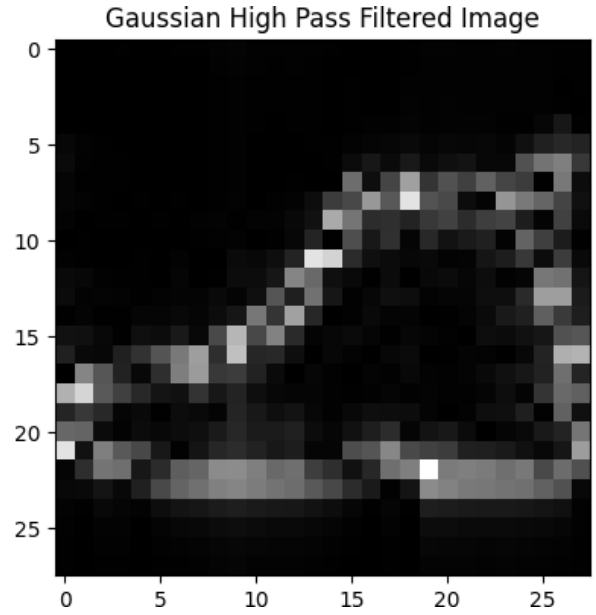
Gaussian High Pass Filtered Image
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Application
Frequency domain filtering is widely used in various image processing tasks, such as noise reduction, edge detection, and feature extraction. The ideal low pass filter can be used for tasks where we need to smooth the image or remove high-frequency noise, while the ideal high pass filter helps in edge detection by enhancing the high-frequency components. Gaussian filters, on the other hand, are more practical due to their smoother transitions, making them effective in real-world applications.

Conclusion
In this lab, we implemented several frequency domain filters—ideal and Gaussian, both low pass and high pass—on images from the Fashion MNIST dataset. We observed how each filter affects the image by either enhancing or suppressing different frequency components. Through this exercise, we gained a deeper understanding of how these filters work and their potential applications in image processing tasks.