UNIT I

OSCILLATION, ULTRASONICS AND DIELECTRIC MATERIALS

1.1 INTRODUCTION TO OSCILLATIONS

- A wave is a disturbance which travels through available space or medium. In a wave motion, energy is transferred from one place to other place due to the repeated periodic motion of the medium particles.
- When a wave passes the medium particles vibrate or oscillate about their mean position. The oscillating particles perform a periodic motion called **Harmonic Motion**.
- The three features associated with a **Low Energy Waves** are (i) energy is transmitted, (ii) medium is not transmitted and (iii) return to equilibrium is involved.
- Although the medium particles vibrate about their mean position, they transfer energy by transferring motion from one particle to the another at a regular interval of time.
- For propagation of a wave the medium must satisfy the following conditions :
  - The medium must be elastic, so that it returns to its original position after oscillation.
  - The medium particles must have inertia so that it swings to the other side of the oscillation.
  - The medium must be viscous so that the energy exchange takes place between the medium particles.
- The understanding of wave is an important aspect as it transfers the energy from one place to the other place. The most common types of waves are light, sound and heat etc. As wave propagation involves the oscillations of medium particles, we must understand the oscillations to understand the concept of wave propagation.

1.2 FREE OSCILLATION [Dec. 18]

- The motion of pendulum, piston of an engine, earth around the sun follows the same path and repeats it after equal intervals of time. Such type of motion is called as **Periodic Motion**. The periodic motion can be either circular or linear depending upon whether it moves along circular path or linear path.
- When we give a push to a pendulum or a string fixed at two ends is plucked and left free to oscillate, it oscillates with a frequency given by,

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{g}{l}\right)} \quad \text{... for pendulum ... (1.1)}$$

$$f = \frac{1}{2l}\sqrt{\left(\frac{T}{m}\right)} \quad \text{... for string ... (1.2)}$$

- This frequency is called **Natural Frequency** of the vibrating system. The frequency with which a body vibrates freely at its own is called its **Natural Frequency**.
- If no resistance is offered to the movement of the vibrating body, the body will keep on vibrating indefinitely. Such a vibration is called **Free Oscillation** or **Vibration**.
- In a free oscillation or vibration, whenever a body is disturbed, it vibrates with its own natural frequency for infinite time. But in practice, frictionless system is not possible and amplitude of vibrating body decreases slowly to zero. When the friction is very less, the system can be considered as **Free Oscillation**.
- Consider motion of a particle of mass m on which a restoring force is acting, such that the particle performs harmonic oscillation. For harmonic oscillations, the restoring force is linearly proportional to the displacement i.e. $-ky$ where k is the restoring force constant and negative sign indicates that it acts in opposite direction to displacement.

i.e. $$f = -ky \quad \text{... (1.3)}$$

According to Newton's law

$$f = ma \quad \text{... (1.4)}$$

i.e. $$f = m\frac{d^2y}{dt^2} \quad \text{... (1.5)}$$

Comparing equations (1.3) and (1.5):

$$\frac{md^2y}{dt^2} = -ky$$

i.e. $$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

Let, $$\frac{k}{m} = \omega^2 \quad \text{... (1.6)}$$

$$\therefore \quad \frac{d^2y}{dt^2} + \omega^2 y = 0 \quad \text{... (1.7)}$$

The solution of equation (1.7) is given in the form,

$$y = e^{\alpha t}$$

$$\therefore \quad \frac{dy}{dt} = \alpha e^{\alpha t}$$

and $$\frac{d^2y}{dt^2} = \alpha^2 e^{\alpha t} \quad \text{... (1.8)}$$

substituting in equation (1.7)

$$\alpha^2 e^{\alpha t} + \omega^2 e^{\alpha t} = 0$$

i.e. $$(\alpha^2 + \omega^2) e^{\alpha t} = 0$$

As, $$e^{\alpha t} \neq 0$$

$$\therefore \quad \alpha^2 + \omega^2 = 0$$

$$\therefore \quad \alpha^2 = -\omega^2$$

$$\alpha = \pm i\omega \quad \text{... (1.9)}$$

Thus the general solution of equation (1.7) is given by

$$y = Ae^{i\omega t} + Be^{-i\omega t} \quad \text{... (1.10)}$$

Where, A and B are constants to be determined.

$$\therefore \quad y = A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t)$$

$$y = (A + B)\cos\omega t + i(A - B)\sin\omega t$$

Let $$A + B = R\sin\phi$$

and $$i(A - B) = R\cos\phi$$

$$\therefore \quad y = R\sin\phi\cos\omega t + R\cos\phi\sin\omega t$$

$$\therefore \quad y = R\sin(\omega t + \phi) \quad \text{... (1.11)}$$

From equation (1.11) it is clear that R is the maximum value of y. Thus R is the amplitude of oscillation. The value of y repeats when t changes by $2\pi/\omega$, i. e.

$$y = R\sin[\omega(t + 2\pi/\omega) + \phi]$$

$$y = R\sin[(\omega t + \phi) + 2\pi]$$

$$y = R\sin(\omega t + \phi)$$

$$y = y$$

Thus after time interval of $2\pi/\omega$, the motion will repeat itself. The internal $2\pi/\omega$ is called the periodic time T.

$$\therefore \quad T = 2\pi/\omega \quad \text{... (1.12)}$$

The frequency,

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Using equation (1.6),

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m}\right)} \quad \text{... (1.13)}$$

From above discussions it is clear that the amplitude of oscillation is constant or independent of time.

1.3 DAMPED OSCILLATION [May 19]

- In ideal situation, the resistance offered to the oscillation is zero and therefore the oscillations will continue for indefinite time. But in practice the amplitude of oscillation keeps on decreasing due to resistive forces and hence oscillations will die out after some time. The time required to die out the oscillation will depend on the magnitude of the resistive force.
- A motion damped by resistive force results into **Damped Oscillation.**
- The resistive force is proportional to the velocity and in the direction opposite to direction of the motion. A damped system has following forces :

(i) Restoring force, $-ky$

(ii) Resistive force, $-r\frac{dy}{dt}$

where r is frictional force per unit velocity. The above forces are balanced by Newton's force,

i.e. $$ma = -ky - r\frac{dy}{dt} \quad \text{... (1.14)}$$

$$m\frac{d^2y}{dt^2} + r\frac{dy}{dt} + ky = 0$$

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$$

Let $$\frac{r}{m} = 2b \text{ and } \frac{k}{m} = \omega^2 \quad \text{... (1.15)}$$

$$\therefore \quad \frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = 0 \quad \text{... (1.16)}$$

The solution of equation (1.16) will be in the form,

$$y = Ae^{\alpha t} \quad \text{... (1.17)}$$

where, A and α are arbitrary constants. Differentiating equation (1.17),

$$\frac{dy}{dt} = A\alpha e^{\alpha t}$$

and $$\frac{d^2y}{dt^2} = A\alpha^2 e^{\alpha t}$$

$\therefore$ Equation (4) becomes,

$$A\alpha^2 e^{\alpha t} + 2b A\alpha e^{\alpha t} + \omega^2 Ae^{\alpha t} = 0$$

$\therefore\ Ae^{\alpha t}(\alpha^2 + 2b\alpha + \omega^2) = 0$

But, $Ae^{\alpha t} \neq 0$

$$\therefore\ \alpha^2 + 2b\alpha + \omega^2 = 0 \qquad \ldots (1.18)$$

$\therefore$ Roots of equation (1.18) gives

$$\alpha = \frac{-2b \pm \sqrt{(4b^2 - 4\omega^2)}}{2}$$

$$\alpha = -b \pm \sqrt{(b^2 - \omega^2)} \qquad \ldots (1.19)$$

The general solution of equation (1.16) is given by,

$$y = A\,e^{(-b + \sqrt{(b^2 - \omega^2)})\,t} + Be^{(-b - \sqrt{(b^2 - \omega^2)})\,t} \qquad \ldots (1.20)$$

where A and B are arbitrary constant.

Case I : Over Damped or Dead Beat :

- When $b^2 > \omega^2$, $\sqrt{(b^2 - \omega^2)}$ is real and less than b. In this case power of both the exponents is negative. Thus the displacement y consists of two terms both are decreasing exponentially. This type of motion is called **Over Damped** or **Dead Beat**.
- Example of such oscillation is pendulum in thick oil. Fig. 1.1 shows over damped oscillation.

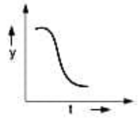


Fig. 1.1 : Over damped oscillation

Case II : Critically Damped :

- When $b^2 = \omega^2$, if we put $b^2 = \omega^2$ in the solution it will not satisfy the differential equation. Therefore, assume that $\sqrt{(b^2 - \omega^2)}$ is not zero but is tending to zero i.e. equal to a very small quantity h.

Therefore solution becomes, from equation (1.20)

$$y = Ae^{(-b+h)t} + Be^{(-b-h)t} \qquad \ldots (1.21)$$

$$y = e^{-bt}(Ae^{ht} + Be^{-ht})$$

$$y = e^{-bt}[A(1 + ht + \ldots) + B(1 - ht + \ldots)]$$

$$y = e^{-bt}[(A + B) + ht(A - B) + \ldots]$$

$$y = e^{-bt}[s + ut] \qquad \ldots (1.22)$$

where, $S = (A + B)$ and $u = h(A - B)$

- The equation (1.22) gives the solution of the differential equation. In equation (1.22) as t increases the factor e^{-bt} decreases and [S + ut] increases. Therefore the displacement approaches zero as t increases. Such a motion is called **Critically Damped**.
- The example is pointer of voltmeter and current meter which comes to rest without oscillation.

Case III : Under Damped :

When $b^2 < \omega^2$, the component $\sqrt{(b^2 - \omega^2)}$ is imaginary.

The solution will be given by,

$$y = Ae^{(-b + i\beta)t} + Be^{(-b - i\beta)t}$$

where, $\beta = \sqrt{(\omega^2 - b^2)}$

and $i = \sqrt{(-1)}$

$$y = e^{-bt}(Ae^{i\beta t} + Be^{-i\beta t})$$

$$y = e^{-bt}[A(\cos\beta t + i\sin\beta t) + B(\cos\beta t - i\sin\beta t)]$$

$$y = e^{-bt}[(A + B)\cos\beta t + i(A - B)\sin\beta t]$$

Let $A + B = a\sin\phi$

and $i(A - B) = a\cos\phi$

$$\therefore\ y = e^{-bt}(a\sin\phi\cos\beta t + a\cos\phi\sin\beta t)$$

$$y = e^{-bt}a\sin(\beta t + \phi) \qquad \ldots (1.23)$$

- The equation represents the simple harmonic motion with amplitude ae^{-bt}. The amplitude of motion will continuously decrease because of the factor e^{-bt}. The factor e^{-bt} is called the damping factor and b the damping coefficient.
- The decay in the amplitude is decided by the damping factor and the oscillation is called **Under Damped**.
- Example of under damped oscillation is pendulum in air, electric oscillator etc. Fig. 1.2 shows under damped oscillation.

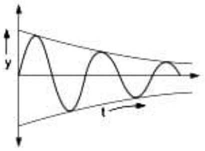


Fig. 1.2 : Under damped oscillation

1.4 FORCED OSCILLATION AND RESONANCE

1.4.1 Forced Oscillation [May 18]

- Till now we have discussed free vibrations in which the body vibrates at its own frequency without any external force. But the situation will be totally different when the body is subjected to an external force. Here the body oscillates because it is subjected to an external periodic force. Such oscillation is called **Forced Oscillation**.

- A forced oscillation can be defined as the oscillation in which a body vibrates with a frequency other than its natural frequency under the action of an external periodic force.

In a forced oscillation, the forces acting on the body are.

(i) Restoring force, $-kx$.

(ii) Resistive force, $-\frac{rdy}{dt}$

(iii) External periodic force, $f \sin pt$

The sum of above forces is balanced by Newton's force,

i.e. $$ma = -ky - \frac{rdy}{dt} + f \sin pt \quad \dots (1.24)$$

$$\therefore \quad m\frac{d^2y}{dt^2} = -ky - \frac{rdy}{dt} + f \sin pt$$

$$m\frac{d^2y}{dt^2} + \frac{rdy}{dt} + ky = f \sin pt$$

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{f}{m} \sin pt$$

Taking $\frac{r}{m} = 2b$, $\frac{k}{m} = \omega^2$ and $\frac{f}{m} = f$

$$\therefore \frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = f \sin pt \quad \dots (1.25)$$

The equation is differential equation of motion of particle.

- At a steady state the body oscillates with the frequency of applied force and not with its natural frequency. The solution of equation (1.25) will be of the form,

$$y = A \sin (pt - \theta) \quad \dots (1.26)$$

where, A and θ are arbitrary constants, Differentiating equation (1.26),

$$\frac{dy}{dt} = Ap \cos (pt - \theta)$$

and $$\frac{d^2y}{dt^2} = -Ap^2 \sin (pt - \theta)$$

Substituting in equation (1.25)

$$-Ap^2 \sin (pt - \theta) + 2b\, Ap \cos (pt - \theta) + \omega^2 A \sin (pt - \theta)$$
$$= f \sin pt = f \sin [(pt - \theta) + \theta]$$

$$A(\omega^2 - p^2) \sin (pt - \theta) + 2b\, Ap \cos (pt - \theta)$$
$$= f \sin (pt - \theta) \cos \theta + f \cos (pt - \theta) \sin \theta$$

comparing coefficients of $\sin (pt - \theta)$ and $\cos (pt - \theta)$ on both sides, we get,

$$A(\omega^2 - p^2) = f \cos \theta \quad \dots (1.27)$$

$$2b\, Ap = f \sin \theta \quad \dots (1.28)$$

Squaring and adding equations (1.27) and (1.28),

$$A^2 (\omega^2 - p^2)^2 + 4b^2 A^2 p^2 = f^2 (\cos^2 \theta + \sin^2 \theta)$$

$$A^2 [(\omega^2 - p^2)^2 + 4b^2 p^2] = f^2$$

$$\therefore \quad A = \frac{f}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}} \quad \dots (1.29)$$

Dividing equation (1.28) by (1.27), we get,

$$\tan \theta = \frac{2b\, Ap}{A(\omega^2 - p^2)}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right) \quad \dots (1.30)$$

Equation (1.29) gives the amplitude and equation (1.30) phase of oscillations.

Depending upon the relative values of p and ω we have following cases.

Case I : When driving frequency is low i.e. $p << \omega$.

In this case, the amplitude of oscillation is given by,

$$A = \frac{f}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}}$$

$$= \frac{f}{\sqrt{\omega^4}} = \frac{f}{\omega^2} = \text{constant}$$

and $$\theta = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right) = \tan^{-1}(0) = 0$$

The amplitude depends on the magnitude of applied force and force and displacement are in phase.

Case II : When $p = \omega$ i.e. frequency of the force is equal to the frequency of the body.

In this case the amplitude of oscillation is given by,

$$A = \frac{f}{2bp} = \frac{f}{r\omega}$$

and $$\theta = \tan^{-1}\left(\frac{bp}{0}\right) = \tan^{-1}(\infty) = \pi/2$$

Thus the amplitude of oscillation will depend on the damping force and the amplitude will be very large. The displacement and the force will have a phase difference of $\pi/2$.

Case III : When $p >> \omega$ i.e. the frequency of the force is greater than natural frequency ω of the body.

The amplitude in this case,

$$A = \frac{f}{\sqrt{p^2 + 4b^2 p^2}} = \frac{f}{p^2} = \frac{f}{mp^2}$$

and

$$\theta = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right) = \tan^{-1}\left(\frac{2bp}{-p^2}\right)$$

$$= \tan^{-1}(0) = \pi$$

Thus the amplitude A decreases and the phase difference tends to π.

1.4.2 Resonance

- In case of forced oscillation, a body vibrates with the frequency of the external force causing the oscillation rather than its natural frequency. The resultant amplitude under forced vibration is given by equation.

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2}} \qquad \dots (1.31)$$

- From above equation it is clear that the resultant amplitude of oscillation varies with the frequency value of force p. For a particular value of p the amplitude becomes maximum. This phenomenon is known as **Resonance.**
- Thus, phenomenon of making a body oscillate with its natural frequency under the influence of another oscillating body with the same frequency is called resonance. For amplitude to be maximum, $\sqrt{[(\omega^2 - p^2)^2 + 4b^2 p^2]}$ has to be minimum.

i.e. $$\frac{d}{dp}[(\omega^2 - p^2)^2 + 4b^2p^2] = 0$$

$$2(\omega^2 - p^2)(-2p) + 4b^2(2p) = 0$$

$$\therefore \quad \omega^2 - p^2 = 2b^2$$

or

$$p = \sqrt{(\omega^2 - 2b^2)} \qquad \dots (1.32)$$

If the damping is small i.e. b is negligible, then above equation reduces to

$$p = \omega \qquad \dots (1.33)$$

which is the condition for resonance.

Substituting this condition in the equation (1.31) we get,

$$A_{max} = \frac{f}{\sqrt{(p^2 - p^2)^2 + 4b^2p^2}}$$

$$= \frac{f}{2bp} \qquad \dots (1.34)$$

Thus A_{max} approaches to infinity when damping force approaches to zero.

Sharpness of Resonance :

- The amplitude of forced oscillation is maximum when the frequency of the applied force satisfies the condition of resonance i.e. $p = \sqrt{(\omega^2 - 2b^2)}$. If the frequency changes from this value the amplitude falls. The rate of fall in the amplitude with the change of forcing frequency on each side of the resonance frequency is called sharpness of the resonance.
- Fig. 1.3 shows the variation of amplitude with the forcing frequency at different damping values.

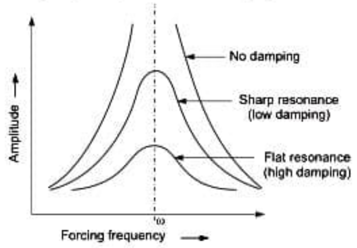


Fig. 1.3 : Resonance

1.5 DIFFERENTIAL WAVE EQUATION [Dec. 17]

- A wave motion is a disturbance which travels through available space or medium and the medium particle vibrates around their mean position when the wave approaches. The motion is handed over from one particle to the next after regular interval of time.
- Consider a progressive wave originating at the origin O and travelling along the positive x - axis with velocity v as shown in Fig. 1.4.

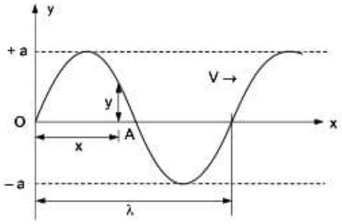


Fig. 1.4 : Progressive wave

- As the wave proceeds, each successive particle of the medium is set into simple harmonic motion. Let the time be measured from the instant when the particle at the origin O is passing through its equilibrium position. The displacement y of a particle at O from its mean position at any time t is given by,

$$y = a \sin \omega t \quad \ldots (1.35)$$

$$y = a \sin \frac{2\pi}{T} t \quad \ldots (1.36)$$

where, $\omega = \frac{2\pi}{T}$

- Now consider a particle at point A at a distance x from O, the wave starting from O would reach the point in (x/v) seconds later than the particle at O. Therefore, there is a phase lag of (x/v) sec between the particle at points A and O. Therefore, the displacement of the particle at A at a time t will be same as that of particle at O at a time (x/v) sec earlier i.e. at time (t – x/v). Thus equation (1.36) becomes,

$$y = a \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right) \quad \ldots (1.37)$$

- This represents the equation of a plane progressive wave.

The other forms of the equation are,

$$y = a \sin \frac{2\pi}{Tv} (vt - x)$$

$$\therefore \quad y = a \sin \frac{2\pi}{\lambda} (vt - x) \quad \ldots (1.38)$$

($\because$ Tv = v/f = fλ/f = λ)

Also,

$$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{vT}\right)$$

$$\therefore \quad y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \ldots (1.39)$$

For differential equation of a wave, differentiate equation (1.38) w. r. t x

$$\frac{dy}{dx} = a\left(\frac{2\pi}{\lambda}\right) \cos \frac{2\pi}{\lambda} (vt - x)$$

$$\text{or,} \quad \frac{d^2y}{dx^2} = -a\left(\frac{2\pi}{\lambda}\right)^2 \sin \frac{2\pi}{\lambda} (vt - x) \quad \ldots (1.40)$$

Again differentiating equation (1.38) w.r.t t, we get,

$$\frac{dy}{dt} = a\left(\frac{2\pi}{\lambda}\right) v \cos \frac{2\pi}{\lambda} (vt - x)$$

$$\text{or} \quad \frac{d^2y}{dt^2} = -a\left(\frac{2\pi}{\lambda}\right)^2 v^2 \sin \frac{2\pi}{\lambda} (vt - x) \quad \ldots (1.41)$$

comparing equations (1.40) and (1.41)

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \quad \ldots (1.42)$$

$$\text{or} \quad \frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \quad \ldots (1.43)$$

This is the differential wave equations.

SOLVED PROBLEMS

***Problem 1.1 :** Equation of wave moving on a string is y = 8 sin π (0.02 x – 4.00 t). Here y and x are in cms and t in secs. Find amplitude, frequency and velocity of the wave. Two particles at any instant are situated at 200 cms apart. Calculate the phase difference between the two particles.*

Solution : The given equation is

$$y = 8 \sin \pi (0.02\,x - 4.00\,t)$$

This equation can be put in the following way :

$$y = 8 \sin 2\pi (0.01\,x - 2.00\,t) \quad \ldots (1)$$

comparing this equation with standard equation

$$y = -a \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right) \quad \ldots (2)$$

Amplitude a = 8 cm, periodic time

$$T = \frac{1}{2.00} = 0.50 \text{ sec.}$$

$$\text{Frequency } n = \frac{1}{T} = 2.0 \text{ sec}^{-1},$$

$$\text{wavelength } \lambda = \frac{1}{0.01} = 100 \text{ cm}$$

Now wave speed

$$v = n\lambda = 2.0 \times 100 = 200 \text{ cm/s.}$$

If the distance between two points be Δx, then phase difference between the two points is given by

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{100} \times 20.0 = \frac{2\pi}{5} \text{ radian}$$

$$= \frac{2}{5} \times 180 = \boxed{72^\circ} \quad [\because \pi \text{ radian} = 180^\circ]$$

***Problem 1.2 :** A simple progressive wave is represented by the equation y = 0.5 sin (314 t – 12.56 x) : where y and x are expressed in meters and t in secs. Find : (a) amplitude, (b) wavelength, (c) speed of the wave, (d) frequency, and (e) phase difference for the points 7.5 metres apart.*

Solution : Here y = 0.5 sin (314 t – 12.56 x)

The equation can be put in the following form

$$y = 0.5 \sin 12.56 \left(\frac{314}{12.56} t - x\right)$$

Comparing this equation with the standard equation

$$y = a \sin \frac{2\pi}{\lambda} (v t - x)$$

(a) amplitude a = 0.5 meters

(b) $\frac{2\pi}{\lambda} = 12.56$ i.e., $\lambda = \frac{2\pi}{12.56} = 0.5$ meters

(c) velocity $v = \frac{314}{12.56} = 25$ meters/sec.

(d) frequency $n = \frac{v}{\lambda} = \frac{25}{0.5} = \boxed{50 \text{ per sec}}$

Problem 1.3 : *A train of simple harmonic wave traveling in a gas along the positive direction of X axis with an amplitude 2 cm velocity 45 metres per sec. and frequency 75. Calculate the displacement, particle velocity and acceleration at a distance of 135 cm from the origin after an interval of 3 sec.*

Solution : The displacement of a particle y in a plane progressive wave is given by

$$y = a \sin \frac{2\pi}{\lambda} (v t - x)$$

Here a = 2 cm, v = 4500 cm/sec, n = 75,

$$x = 135 \text{ cm}, t = 3 \text{ sec}$$

and $$\lambda = \frac{v}{n} \times \frac{4500}{75} = 60 \text{ cm}$$

$$\therefore \quad y = 2 \sin \frac{2\pi}{60} (4500 \times 3 - 135)$$

$$= 2 \sin \frac{891\pi}{2} = 2 \sin \left(445\pi + \frac{\pi}{2}\right)$$

$$= -2 \sin \pi/2 = -2 \text{ cm}$$

The particle velocity

$$\frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda} (v t - x)$$

As above $\sin \frac{2\pi}{\lambda} (v t - x) = -1$

$$\therefore \cos \frac{2\pi}{\lambda} (v t - x) = 0$$

Hence, $$\frac{dy}{dt} = 0.0 \text{ cm/sec.}$$

Particle acceleration,

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} a \sin \frac{2\pi}{\lambda} (v t - x)$$

$$= -\frac{4\pi^2 v^2}{\lambda^2} y = -\frac{4\pi^2 (4500)^2}{(60)^2} \times (-2)$$

$$= \boxed{4.437 \times 10^5 \text{ cm/sec}^2}$$

1.6 ULTRASONIC WAVES [May 18]

- We all know that sound is due to the **Vibrations of Particles of the Medium.** Human ear can hear the sound waves of frequencies between 20 Hz to 20,000 Hz. This range of frequencies is known as the **Audible Range.**
- The sound waves whose frequencies are greater than 20,000 Hz are known as **Ultrasonic Waves.** The wavelength of ultrasonic waves is very small as compared to that of audible sound. The sound waves which have frequencies less than the audible range are called as **Infrasonic Waves.**
- The ultrasonic and infrasonic frequencies are inaudible to human beings but they are audible to some birds, dogs and insects. A dog can hear sound of frequencies above 20 kHz. Bats can hear sound waves of frequencies upto 100 kHz. This enables them to move freely even in the dark.
- As the ultrasonic waves have very high frequency they undergo very less diffraction and therefore less spreading. Because of this property, high energy can be concentrated in a very narrow beam and can cover very large distances with very less losses of energy. This special feature makes them useful in many applications.

1.7 PRODUCTION OF ULTRASONIC WAVES

- Ultrasonic waves cannot be produced by the ordinary method i.e. by using mechanical vibrations. This is because of the comparatively low natural frequencies of the moving parts. Hence other methods are used for the production of ultrasonic waves. The method chosen depends upon the output power required and the frequency range needed.
- A device which produces ultrasonic waves is called an **Ultrasonic Transducer.** To generate lower frequencies, a mechanical type device such as Galton's Whistle is used. Magnetostriction method is used when frequencies upto 300 kHz are needed, while piezo-electric generators are used mostly for frequencies above that.

1.7.1 Piezo-Electric Effect [Dec. 18, May 19]

- When opposite faces of a thin section of certain crystals like tourmaline, quartz etc. are subjected to distortion by applying **Pressure or Tension,** then **Equal and Opposite Charges** are developed on the faces perpendicular to the faces subjected to distortion. The magnitude of the potential difference developed is proportional to the amount of distortion produced.

The polarity of the charges gets reversed when the direction of the force of distortion is reversed. This phenomenon is known as **Piezo-Electric Effect**.

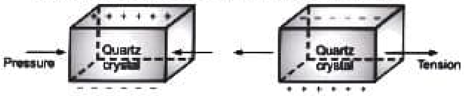


Fig. 1.5: Piezo-electric effect

- The converse of piezo-electric effect is also true i.e. if a potential difference is applied across the two faces of the crystal, it expands or contracts depending on the strength and direction of the applied field.
- Instead of steady voltage if an alternating voltage is applied across the faces of the crystal, then the crystal will expand and contract alternatively. This alternate expansion and contraction will make the crystal vibrate.
- If the frequency of the applied a.c. voltage happens to be equal to one of the modes of vibration of the crystal, resonance occurs and the crystal vibrates with maximum amplitude.

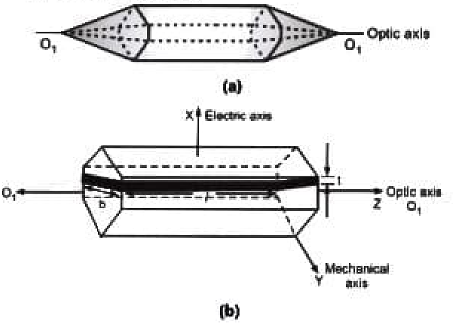


Fig. 1.6: (a) Natural quartz crystal and (b) Transverse section of quartz crystal cut along a plane perpendicular to the optic axis

Fig. 1.6 (a) shows a natural quartz crystal and Fig. 1.6 (b) shows a transverse section of the crystal.

- Consider a quartz crystal plate of thickness t and length *l* (along the optic axis). When an alternating voltage is applied across the faces of this crystal along the electrical axis, then alternating stress and strain is set up both in its thickness and length.
- If the frequency of the alternating voltage coincides with the natural frequency of vibration of the crystal, resonance occurs. The crystal vibrates with large amplitude. On maintaining suitable alternating potential, ultrasonic waves can be generated by this method.
- The frequency of the thickness vibrations,

$$f = \frac{P}{2t}\sqrt{\frac{E}{\rho}}$$

- The frequency of the length vibrations,

$$f = \frac{P}{2l}\sqrt{\frac{E}{\rho}}$$

where P = 1, 2, 3, ... etc. for **Fundamental**, **First Overtone** and **Second Overtone** respectively, E is the Young's modulus, ρ is the density of the crystal, t is the thickness and *l* is the length of the crystal.

Piezo-Electric Oscillator [Dec. 18]

- The experimental set up is as shown in Fig. 1.7.
- The high frequency alternating voltage applied to the crystal is obtained from an oscillatory circuit (inductance L_1 and a variable condenser C_1 in parallel).
- One end of the oscillatory circuit is connected to the plate of the valve and the other end to the grid. The quartz crystal is placed in between two metal plates A and B to form a parallel-plate capacitor with the crystal as a dielectric. This is connected in parallel to the variable condenser C_1.
- By adjusting the variable condenser, the frequency of the oscillatory circuit is tuned to the natural frequency of the crystal.
- At this stage, the crystal is set into mechanical vibrations and ultrasonic waves are generated. By this method, ultrasonic waves upto a frequency of 15×10^7 Hz can be obtained.

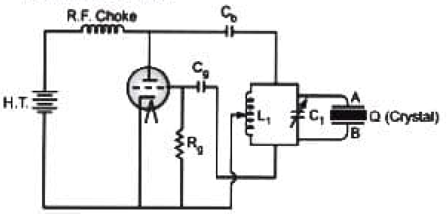


Fig. 1.7: Piezo-electric oscillator

Alternative Method for Piezo-electric Oscillator

Construction

- The piezo-electric oscillator uses basically a Hartley oscillator. The transistor is biased using the resistors R_1, R_2 and R_E. The combination of L_1, L_2 and C_1 works as tuning circuit which is couple with the transistor with a coupling capacitor C_2. The capacitor C_2 is used to provide positive feedback.

- The resonance frequency of the tank circuit is given by

$$f_r = \frac{1}{2\pi\sqrt{L_r C_1}}, \quad \text{where } L_r = L_1 + L_2$$

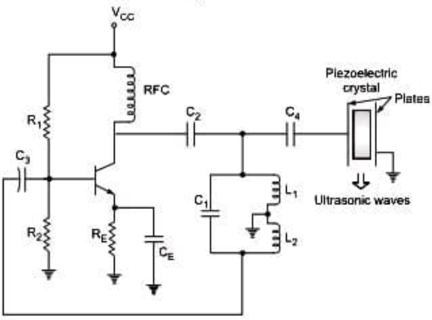


Fig. 1.8 : Piezo-electric oscillator

Working

- When the circuit is switched on, oscillating currents are produced in the tuning circuit.
- The oscillating currents generated by the tuning circuit are sustained and the electric signal obtained at the output is applied to the piezoelectric crystal through coupling capacitor C_4.
- When these high frequency electrical signals are applied to the crystal, because of reverse piezoelectric effect, the crystal undergoes alternate contraction and expansion. These vibrations produce ultrasonic waves.
- The frequency of ultrasonic waves can be changed by varying the values of components of the tuning circuit as per the relation $f = \frac{1}{2\pi\sqrt{L_1 C_1}}$. When frequency of oscillation of the tuning circuit becomes equal to the natural frequency of the crystal $f = \frac{P}{2l}\sqrt{\frac{E}{r}}$, resonance occurs and crystal oscillates with maximum amplitude and amplitude of ultrasonic waves will be maximum.
- Ultrasonic waves upto frequency of 1.5×10^8 Hz can be produced using piezoelectric oscillator.

1.7.2 Magnetostriction Effect [Dec. 17, May 18]

- According to this phenomenon, a rod of **Ferromagnetic Material** such as iron or nickel undergoes a **Change in its Length** when placed in a magnetic field parallel to its length.
- Instead of a steady field, if an alternating field is used, the rod expands and contracts in length alternately. This sets up a longitudinal vibration in the rod whose frequency is twice the frequency of the alternating magnetic field. If the natural frequency of the rod and the frequency of the alternating field is the same, resonance occurs and the amplitude of vibration of the rod is maximum.
- The range of frequency depends on the dimensions of the magneto strictive material. The longitudinal vibrations thus produced are exactly like those produced by a rod which is clamped at the mid point but has both ends free.
- The frequency of vibration of such a rod is

$$\boxed{f = \frac{1}{2l}\sqrt{\frac{E}{\rho}}}$$

where E is the Young's modulus,

l is the length of the rod, and

ρ is its density of rod.

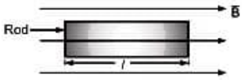


Fig. 1.9: Magnetostriction effect

Magnetostriction Oscillator

- This apparatus generates ultrasonic waves and is based on the principle of magnetostriction.
- Fig. 1.10 shows the experimental set up of magneto-striction oscillator. It consists of a permanently magnetised nickel or iron rod (magnetised initially by passing direct current in the coil which is wound around it). The rod is clamped at the centre. The two coils L_1 and L_2 are wound over the rod.
- The exciting coil L_1 is connected to the plate circuit of a valve while the coil L_2 is coupled to the plate via the grid circuit.
- By adjusting the variable condenser C, high frequency oscillation currents are set up in the plate circuit. This high frequency current flowing through the coil L_1 produces changes in the length of the rod. Due to this, the rod expands and contracts alternately and a vibration is set up in the rod.

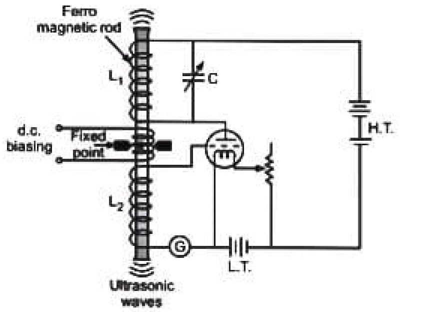


Fig. 1.10: Magnetostriction oscillator

- These vibrations in the length of the rod cause a variation in the magnetic flux through the coil L_2 and an e.m.f. is induced in it. This induced e.m.f. is fed to the grid which produces large variations in the plate current. Thus magneto strictive effect in the bar is increased.
- When the frequency of the circuit becomes equal to the natural frequency of the rod, resonance occurs and ultrasonic waves of maximum amplitude are produced. By adjusting the length of the rod and condenser capacity, high frequency oscillations of different frequencies can be obtained.

Alternate Method for Magnetostriction Oscillator

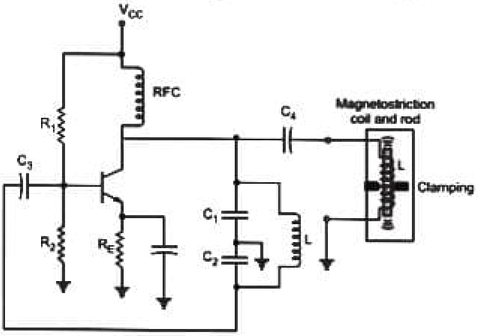


Fig. 1.11

Construction

- The magneto strictive oscillator uses basically a Colpitt's oscillator. The transistor is biased with the help of resistances R_1, R_2 and R_E. The inductance L and capacitors C_1 and C_2 form a tank circuit and C_3 is a feedback capacitor. The tank circuit is used for selecting resonance frequency.
- The appropriate frequencies at the end of the tank circuit are amplified and the oscillations corresponding to them are sustained. The resonance of tank circuit is given by $f_r = \frac{1}{2\pi\sqrt{LC_r}}$, where $C_r = \frac{C_1C_2}{C_1 + C_2}$. A ferromagnetic rod is kept surrounding to a coil in the shape of solenoid at the output of the oscillator.

Working

- When the circuit is switched on, oscillating currents are produced in the tuning circuit. The oscillations appearing at output terminal of oscillator circuit are fed to the magnetostriction coil through the coupling capacitor C_4. The magnetostriction coil is placed surrounding the ferromagnetic rod.
- The coil produces magnetic field which is alternately changing in opposite directions and is applied around the ferromagnetic rod. Due to magnetostriction effect, the changing magnetic field causes rod to contract and expand alternately. These vibrations of the rod travel in surrounding medium in the form of ultrasonic waves.
- The frequency of oscillating current in the tank circuit can be changed by varying the values of the components of the tank circuit.
- The frequency of vibrations of the rod is given by $f = \frac{p}{2l}\sqrt{\frac{E}{\rho}}$.
- where, p is integer, l is length of the rod, Y is Young's modulus and ρ is density of the rod.
- When frequency of the tuning circuit becomes equal to natural frequency of the rod, the rod vibrates with maximum amplitude and ultrasonic waves with maximum amplitude are obtained.

1.8 PROPERTIES OF ULTRASONIC WAVES

- As the wavelength of the waves is very small, ultrasonic waves suffer least diffraction. They can be **Transmitted Over Longer Distances** as a highly directional beam without appreciable loss of energy.
- Ultrasonic waves are **Highly Energetic** and may have intensities upto 10 kW/m^2.
- On passing through liquids, ultrasonic waves are propagated longitudinally forming nodes and antinodes. This produces **Cavitation Effect**.
- In solids, ultrasonic waves propagate both longitudinally and transversely. In transverse waves, there exist no nodes or antinodes.

- Velocity of ultrasonic waves depends on the **Temperature of the Medium** through which it is propagating.
- When ultrasonic waves are passed through a liquid kept in a rectangular vessel, they are reflected from the bottom of the vessel. The directed and reflected rays get superimposed resulting in a stationary wave. Due to the formation of the stationary wave the density of the node is greater than that at the antinode. Now, if a parallel beam of light is passed at right angle to the wave the liquid acts as a diffraction grating. This is called as **Acoustical Grating.**

1.9 APPLICATIONS OF ULTRASONIC WAVES

1.9.1 Scientific Applications

- **Echo Sounding:** Ultrasonic sound waves are used for sound signalling, depth sounding, determining the position of ice bergs, submarines etc. These applications make use of the **echo principle**. The high frequency sound waves can be readily formed into a narrow beam and can be focused in any desired direction. Because of this, these waves can travel many kilometres in water before being absorbed. Ultrasonic waves of 50 kHz frequency are generated by a crystal vibrator. The moment signal is sent from the transmitter, a deflection of the spot on the C.R.O. screen is observed. The beam travels to the receiver or obstacle and reflects back. When the reflected beam returns, it is indicated by a deflection of the spot on the CRO. The time interval between the two deflections can be measured. Knowing the velocity of the ultrasonic waves and the time interval, the position of the receiver or obstacle can be determined. This is the underlying principle in echo sounding.
- **Depth Sounding:** This application of ultrasonic waves makes use of the **Echo Principle**. The depth of sea or the depth of water below a ship can be calculated using ultrasonic waves. Because of their high frequency and short wavelengths, ultrasonic waves are not absorbed by water so strongly as lower frequency waves. Waves of frequency of about 40 kHz are used. They are produced by a crystal transducer and are directed towards the bottom of the sea, at regular intervals. The reflected waves from the bottom of the sea are received by the same crystal causing it to vibrate. The vibrations generate a small e.m.f. across its faces which is recorded on a sensitive CRO. The time interval between the emission of initial wave pulse and the e.m.f. generated due to the reflected waves is recorded. If v is the velocity of ultrasonic waves in sea water and t is the time interval between sending and receiving of the wave, then

$$\text{Depth of sea} = \frac{\text{Velocity of sound in sea} \times \text{time}}{2}$$

$$d = \frac{vt}{2}$$

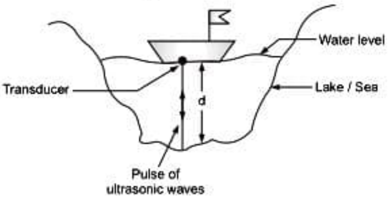


Fig. 1.12

- **Determining the Position of Icebergs, Submarines, a Shoal of Fish in Sea, etc.:** The principle employed here is the **echo principle**. Ultrasonic waves are reflected from objects even if they are very small. Hence the presence of submerged objects in the sea like icebergs, submarines etc. can be detected. Pulses of ultrasonic signals are sent out at short intervals. The reflected echo is received in the ultrasonic receiver and the time interval between the transmitted and received signals is noted. This is the two way travel time from the source to the target. Knowing the velocity of sound in water, the distance between the source and the target can be calculated.
- **Cleaning and Removing Dirt:** Clothes or utensils that have to be cleaned are subjected to ultrasonic waves. These waves will put the dirt particles or water particles into vibration. As a result, these particles loosen their attachment with the surface and fall-off. The same treatment is used for removing soot and dust from the chimney.

1.9.2 Engineering Applications

- **Non-Destructive Testing**
  - Non-destructive testing is characterized by low intensity of the sound wave used. Here sound wave is not expected to cause any change in the chemical or physical characteristics of the specimen material.
  - Such applications are found in testing, inspection and quality control.

- In this case, ultrasonic waves are propagated into the specimen under inspection. When the ultrasonic waves are incident on the defect, reflection of the wave from the interface (between material and defect) in the object takes place. Thus, the defects are located without any real damage to the specimen.
- Ultrasonic waves may be used for a large number of non-destructive testing on different materials. Some of these are
  - (i) Ultrasonic flaw detection
  - (ii) Ultrasonic study of structure of matter.

- **Flaw Detection**
  - The strength of components plays a significant role in most of the engineering applications. Any kind of defect greatly reduces the strength of materials. These defects can be as large as cracks or as tiny as cavities.
  - A high frequency pulse from pulse generator is impressed on a quartz crystal which is placed on the specimen under test. The crystal (transducer) first acts as a transmitter sending out high frequency waves into the specimen.
  - Then it acts as a receiver to receive the ultrasonic echo pulses reflected from the flaw and from the far end of the specimen. The received ultrasonic echo pulses are transformed by the transducer into corresponding electric echo pulses of the same frequency. These are then amplified and displayed on the C.R.O. screen as a series of pulses.
  - The first pulse corresponds to the transmitted wave, the next pulse corresponds to the reflected wave. i.e. first one from the flaw and the second one from the far end of the specimen. Each reflected pulse is indicated at a particular time after the initial transmitted pulse. The time interval between the transmitted and reflected pulse represents the distance travelled by the wave. From this the exact position of the flaw is located.

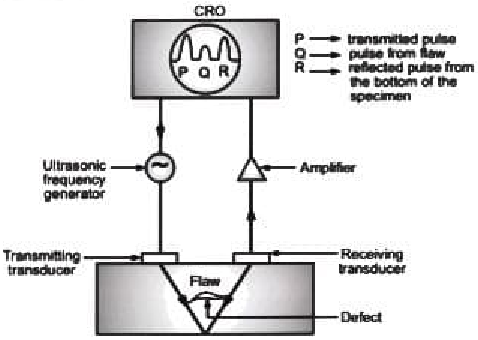


Fig. 1.13

- **Cavitation**
  - When an ultrasonic transducer is placed in a liquid, it produces ultrasonic vibrations in it. This results in the development and implosion of bubbles. These bubbles are known as the ***cavitation bubbles*** and they are formed as follows.
  - When a liquid is subjected to a powerful ultrasonic radiation, tension develops at some point in a liquid. The excess stress tears apart the liquid producing a hollow bubble that sucks in dissolved gases and vapours.
  - The life time of the bubble is very short, and it collapses very quickly. During the implosion of the bubble, the pressure of the shock wave that is formed near the bubble reaches several hundreds of atmospheres.
  - The formation and implosion of bubbles accounts for erosion and pitting of an ultrasonic transducer kept in the liquid. The bubbles have two effects : (i) they produce a dense cloud in front of the transducer and block the propagation of ultrasonic waves, (ii) frequent implosion of bubbles destroys the surface of the transducer causing pits.
  - Even though cavitation bubbles formed by ultrasonic vibrators in liquids block the wave propagation, it has some successful industrial applications like ultrasonic cleaning, ultrasonic emulsification etc.

- **Ultrasonic Cleaning**
  - Ultrasonic cleaning is achieved through a combined effect of cavitation and acceleration of

the cleaning liquid. Ultrasonic waves in liquid produce cavitation i.e. tiny space in the liquid. The vacuum created in these spaces exerts a strong pull on exposed solid surfaces. This detaches any particles of dust attached to them.

- The transducer is placed at the bottom of the tank in which the cleaning solution (either a water detergent solution or standard solvents) is taken.
- For the cleaning of metallic parts, low frequency waves are used while for cleaning fibres, high frequency waves are used. The specimen to be cleaned is kept immersed in the cleaning solution in the tank.

- **Ultrasonic Emulsification**
  - It has been observed that intense ultrasonic waves can thoroughly mix immiscible liquids like oil and water to form a stable emulsion. The emulsification results because of the cavitation bubbles imploding at the boundary surfaces between a liquid and vibrator, and also between liquid and walls of the container.
  - The two liquids which are to be emulsified are taken in a container. This container is placed in a liquid bath which is subjected to strong ultrasonic vibrations by a transducer. Then emulsification due to gas bubbles takes place at the surface of the container containing the two liquids.
- **Measurement Gauge**
  - Ultrasonic thickness measurement is based on the **Echo Principle**. A piezoelectric transducer attached to the test piece converts the electric pulse into ultrasonic waves. The transducer can be attached to the test piece directly or it can be coupled to the piece by an incompressible medium such as oil or water.
  - The ultrasonic waves propagate into the test piece. They travel through the sample and are reflected back from the opposite surface. The same transducer then receives the reflected echo and converts it to an electrical pulse. The time taken for the pulse to travel through the sample is related to the thickness and the velocity in the material. The thickness T can be expressed by the formula,

$$\text{Thickness} = \frac{\text{Velocity of sound in specimen} \times \text{time}}{2}$$

$$T = \frac{vt}{2}$$

where, v is the velocity of the ultrasound in the material and t is the time between the pulse being transmitted and the echo being received.

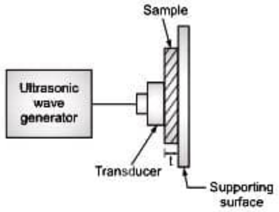


Fig. 1.14

- If the velocity of ultrasonic waves in the material is not known, then the value has to be determined experimentally by using the same material whose thickness is already known. From the known values of T and t, the velocity v of ultrasound in the material can be calculated.
- The advantage of using ultrasonic waves for thickness measurement is that the thickness can be measured from one side of the test piece. There is no need for drilling holes or otherwise inflicting damage to the piece.

1.9.3 Medical Applications

Certain tumors which cannot be detected by X-rays can be detected by ultrasonic waves. Joints affected by rheumatic pains get great relief when exposed to ultrasonic waves. Surgical use of ultrasonic waves include the selective cutting of the tissue during an operation. Ultrasonic waves are very useful for dental cutting because

- They make the cutting almost painless.
- They cut the hard material very easily, and
- They do not require any mechanical device for cutting purpose. Thus, ultrasonography now has become an important tool which help physicians in diagnosing and treating medical ailments.

***Problem 1.4** : Calculate the natural frequency of the thickness vibrations for quartz plate of thickness 5.5×10^{-3} m, given that Young's modulus along X-axis is 8×10^{10} N/m^2 and density of crystal is 2.65×10^3 kg/m^3.*

Data: $t = 5.5 \times 10^{-3}$ m, $E = 8 \times 10^{10}$ N/m^2, $\rho = 2.65 \times 10^3$ kg/m^3.

Formula: The fundamental frequency of thickness vibrations is given by

$$n = \frac{1}{2t}\sqrt{\frac{E}{\rho}}$$

Solution:

$$n = \frac{1}{2 \times 5.5 \times 10^{-3}}\sqrt{\frac{8 \times 10^{10}}{2.65 \times 10^{3}}}$$

$$n = \frac{1}{2 \times 5.5 \times 10^{-3}} \times 5.5 \times 10^{3}$$

$$= 500 \times 10^{3} \text{ Hz}$$

$$\boxed{n = 500 \text{ kHz}}$$

***Problem 1.5 :** Calculate the frequency of the fundamental note emitted by a piezoelectric crystal, using the following data. Vibrating length = 3 mm, Young's modulus = 8×10^{10} N/m² and density of crystal = 2.5 g/cm³.*

Data: l = 3 mm, E = 8×10^{10} N/m², ρ = 2.5 g/cm³.

Formula: The fundamental frequency of length vibration is given by

$$n = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

Solution:

$$n = \frac{1}{2 \times 3 \times 10^{-3}}\sqrt{\frac{8 \times 10^{10}}{2.5 \times 10^{3}}}$$

$$= 0.943 \times 10^{6} \text{ Hz}$$

$$= \boxed{943 \text{ kHz}}$$

***Problem 1.6 :** An ultrasonic source of 0.07 MHz sends down a pulse towards the seabed which returns after 0.65 sec. The velocity of sound in sea water is 1700 m/s. Calculate the depth of sea and the wavelength of the pulse.*

Data: f = 0.07 MHz, t = 0.65 sec, v = 1700 m/s

Formulae: (i) Depth of sea = $\frac{\text{Velocity of sound in sea} \times \text{Time}}{2}$

$$= \frac{vt}{2}, \text{ (ii) } \lambda = \frac{v}{g}$$

Solution: (i) $v = \frac{1700 \times 0.65}{2} = \boxed{552.5 \text{ m}}$

(ii) Wavelength of the pulse,

$$\lambda = \frac{v}{f} = \frac{1700}{0.07 \times 10^{6} \text{ Hz}} = \boxed{2.4 \text{ cm}}$$

***Problem 1.7 :** Calculate the natural frequency of 40 mm length of a pure iron rod; given that density of pure iron is 7.25×10^{3} kg/m³ and its Young's modulus is 115×10^{3} N/m².* **[Dec. 17]**

Data: l = 40 mm, $\rho = 7.25 \times 10^{3}$ kg/m³.

Formula:

$$f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

Solution:

$$f = \frac{1}{2 \times 40 \times 10^{-3}}\sqrt{\frac{115 \times 10^{3}}{7.25 \times 10^{3}}}$$

$$= \boxed{49.75 \text{ kHz}}$$

***Problem 1.8 :** An ultrasonic source of 70 kHz sends down a pulse towards the sea bed which returns after 0.5 sec. The velocity of sound in sea water is 1400 cm/sec.*

(a) What is the depth of the sea ?

(b) What is the wavelength of the pulse in water ?

Data: f = 70 kHz = 70×10^{3} Hz; t = 0.5 sec, v = 1400 m/sec

Formulae: (i) $\lambda = \frac{v}{g}$

(ii) Depth of sea $= \frac{\text{Velocity of water} \times \text{Time}}{2} = \frac{vt}{2}$

Solution: (i) $D = \frac{1400 \times 0.5}{2} = \boxed{350 \text{ m}}$

(ii) Wavelength, $\lambda = \frac{v}{f} = \frac{1400}{70 \times 10^{3}} = \boxed{20 \times 10^{-3} \text{ m}}$

***Problem 1.9 :** Calculate the thickness of a quartz plate required to produce ultrasonic waves of frequency 2 MHz.*

Given: Density of crystal = 2650 kg/m³

Young's modulus= 8×10 N/m²

Data:

$$F = 2 \text{ MHz}$$

$$\rho = 2650 \text{ kg/m}^3$$

$$E = 8 \times 10^{10} \text{ N/m}^2$$

***Formula:** The natural frequency,*

$$n = \frac{P}{2t}\sqrt{\frac{E}{\rho}}$$

Solution: Take P = 1

$$t = \frac{1}{2 \times 2 \times 10^{6}}\sqrt{\frac{8 \times 10^{10}}{2650}}$$

$$= 1.37 \times 10^{-3} \text{ m}$$

$$\boxed{t = 1.37 \times 10^{-3} \text{ m}}$$

***Problem 1.10 :** Calculate the natural frequency of a cast iron rod of 2.6 cm length.*

***Given:** Density of rod = 7.23×10^{3} kg/m³*

Young's modulus= 1.16×10^{11} N/m³

Data: l = 2.6 cm ρ = 7.23 × 10^3 kg/m^3 E = 1.16 × 10^{11} N/m^2

Formula: $$n = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

Solution: $$n = \frac{1}{2 \times 2.6 \times 10^{-2}}\sqrt{\frac{1.16 \times 10^{11}}{7.23 \times 10^3}}$$

$$n = 770.29 \times 10^2 \text{ Hz} = \boxed{77.03 \text{ kHz}}$$

Problem 1.11 : *A quartz crystal of thickness 0.001 metre is vibrating at resonance. Calculate the fundamental frequency given that Y for quartz is 7.9 × 10^{10} N/m^2 and ρ for quartz is 2650 kg/m^3.*

Data: t = 0.001 m, λ = 7.9 × 10^{10} N/m^2, ρ = 2650 kg/m^3

Solution: $$f = \frac{1}{2t}\sqrt{\frac{\lambda}{\rho}}$$

$$= \frac{1}{2 \times 0.001}\sqrt{\frac{7.9 \times 10^{10}}{2650}} = 2729.9 \text{ kHz}$$

$$= \boxed{2730 \text{ kHz}}$$

Problem 1.12 : *Calculate the thickness of a quartz plate required to produce ultrasonic waves of frequency 2 MHz.*

Given : Density of crystal = 2650 kg/m^3

Young's modulus = 8 × 10 N/m^2

Data :

$$F = 2 \text{ MHz}$$
$$\rho = 2650 \text{ kg/m}^3$$
$$E = 8 \times 10^{10} \text{ N/m}^2$$

Formula : The natural frequency,

$$n = \frac{P}{2t}\sqrt{\frac{E}{\rho}}$$

Solution : Take P = 1

$$t = \frac{1}{2 \times 2 \times 10^6}\sqrt{\frac{8 \times 10^{10}}{2650}}$$

$$= 1.37 \times 10^{-3} \text{ m}$$

$$t = \boxed{1.37 \times 10^{-3} \text{ m}}$$

Problem 1.13 : *Calculate the natural frequency of a cast iron rod of 2.6 cm length.*

Given :

Density of rod = 7.23 × 10^3 kg/m^3

Young's modulus = 1.16 × 10^{11} N/m^3

Data :

$$l = 2.6 \text{ cm}$$
$$\rho = 7.23 \times 10^3 \text{ kg/m}^3$$
$$E = 1.16 \times 10^{11} \text{ N/m}^2$$

Formula : $$n = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

Solution : $$n = \frac{1}{2 \times 2.6 \times 10^{-2}}\sqrt{\frac{1.16 \times 10^{11}}{7.23 \times 10^3}}$$

$$n = 770.29 \times 10^2 \text{ Hz} = \boxed{77.03 \text{ kHz}}$$

Problem 1.14 : *A quartz crystal of thickness 0.001 metre is vibrating at resonance. Calculate the fundamental frequency given that Y for quartz is 7.9 × 10^{10} N/m^2 and ρ for quartz is 2650 kg/m^3.*

Data : t = 0.001 m, λ = 7.9 × 10^{10} N/m^2, ρ = 2650 kg/m^3

Solution : $$f = \frac{1}{2t}\sqrt{\frac{\lambda}{\rho}}$$

$$= \frac{1}{2 \times 0.001}\sqrt{\frac{7.9 \times 10^{10}}{2650}}$$

$$= 2729.9 \text{ kHz}$$

$$= \boxed{2730 \text{ kHz}}$$

1.10 INTRODUCTION TO DIELECTRICS

- A **Dielectric** is an insulating material in which all the electrons are tightly bound to the nucleus of an atom. There are no free electrons available for the conduction of electricity. Thus the electrical conductivity of dielectrics is very less, ideally it is zero.
- The distinction between a dielectric material and an insulator lies in the application to which it is employed. The best examples of dielectrics are glass, polymer, mica, oil and paper.
- The insulating materials are used to prevent the electrical flow of electric current to undesired locations, whereas the dielectrics are used to store electrical energy.

1.11 DIELECTRIC PARAMETERS

1.11.1 Dielectric Constant [May 18]

- It is observed that the storing of capacity of a capacitor increases if the space between its plate is filled with dielectric material. If C_o is the capacitance in vacuum and C the capacitance when the space is filled with a dielectric material then the **Dielectric Constant** of the material is

$$K = \frac{C}{C_o} \quad \dots (1.44)$$

- Thus, the dielectric constant of a material is the ratio of the capacitance of a capacitor completely filled with that material to the capacitance of the same capacitor in vacuum.

In other words, the ratio of permittivity of medium to that of the vacuum is,

i.e. $$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \qquad \text{... (1.45)}$$

where,
K_r = dielectric constant
ε_r = relative permittivity.
ε = permittivity of material
ε_0 = permittivity of vacuum

1.11.2 Electric Displacement [May 18]

- The **Electric Displacement** field in a material is defined as,

$$D = \varepsilon_0 E + P \qquad \text{... (1.46)}$$

where, ε_0 is the permittivity of the free space or vacuum, E is the electric field and P is the polarization density of the electric material.

- The electric displacement field is a vector field having unit c/m^2.
- The electric displacement field is a vector field which describes the displacement effects of an electric field on the charges within a dielectric material, such as polarization charges or bound charges.
- In short it is the charge per unit area that would be displaced across a layer of conductor placed across an electric field.

1.11.3 Polarization

- When an electric field E is applied to a dielectric material consisting of positive and negative charges, the positive charges move opposite to the direction of the field while negative charges in the direction of the field.
- The displacement of the charges creates a local dipole in the dielectric. This process is known as **Dielectric Polarization**.

Fig. 1.15 shows the concept of polarization.

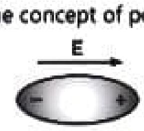


Fig. 1.15 : Polarization

- The polarization density is defined as induced dipole moment per unit volume.

1.11.4 Polarizability [May 18]

- When a dielectric material is placed in an electric field, the displacement of electric charges give rise to the creation of dipole in the material. The polarization P is directly proportional to the applied electric field E.

i.e. $$P \propto E$$

or $$P = \alpha E \qquad \text{... (1.47)}$$

where α is a proportionality constant known as **Polarizability** and the unit is fm^2.

- If the dielectric material contains N dipoles per unit volume, then,

$$P = N \alpha E \qquad \text{... (1.48)}$$

1.12 TYPES OF POLARIZATION

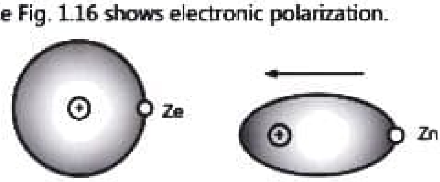
- When an electric field is applied to a dielectric material, it creates or realigns the dipoles resulting in polarization. The main types of polarization are categorised as below
  - Electronic polarization (P_e)
  - Ionic polarization (P_i)
  - Orientation polarization (P_o)

1.12.1 Electronic Polarization

- A dielectric material has large number of atoms having nuclei at the centre and electrons around it in different orbits. When an electric field is applied, the nucleus moves away from the field while the electrons towards the field. Therefore, there is a displacement which is less than the dimensions of the atom, the type of polarization is called **Electronic Polarization**.
- The electronic polarization is independent of temperature. The polarization is given by,

$$\bar{P}_e = N\alpha_e\bar{E} \qquad \text{... (1.49)}$$

- The Fig. 1.16 shows electronic polarization.



(a) E = 0 (b) E = E

Fig. 1.16 : Electronic polarization