1) Solve : $(x+2y+2)dx + (2x-y-3)dy = 0$

We have,

$$M\,dx + N\,dy = 0$$

We get,

$$M = (x+2y+2) \quad \&$$
$$N = (2x-y-3)$$

$$\frac{dM}{dy} = 2 \quad \& \quad \frac{dN}{dx} = 2$$

$$\therefore \frac{dM}{dy} = \frac{dN}{dx}$$

then, the solution is exact D.E.,

$$\cancel{\int M\,dx + \int N\,dy = 0}$$

$$\cancel{\int (x+2y+2)dx + \int (2x-y-3)dy = 0}$$

$$\cancel{\frac{x^2}{2} + 2yx + 2x + 2xy - \frac{y^2}{2} - 3y = 0}$$

$$\int M\,dx = \int (x+2y+2)\,dx = \frac{x^2}{2} + 2xy + 2x$$

$$\int (\text{terms in } N \text{ free from } x)\,dy = (-y-3)\,dy = \frac{-y^2}{2} - 3y$$

$$\therefore \int M\,dx + \int N\,dy = c.$$

$$\therefore \frac{x^2}{2} + 2xy + 2x - \frac{y^2}{2} - 3y = c.$$

$$\therefore \frac{x^2}{2} + (2y+2)x - \left(\frac{y^2}{2} + 3y\right) = c$$

is the soln of given D.E.

2) Solve : $(3x^2y + \frac{y}{x})dx + (x^3 + \log x)dy = 0$.

We have,

$$M\,dx + N\,dy = 0.$$

We get,

$$M = (3x^2y + \frac{y}{x}) \quad \&$$

$$N = (x^3 + \log x)$$

$$\therefore \frac{dM}{dy} = 3x^2 + \frac{1}{x}.$$

$$\therefore \frac{dN}{dy} = 3x^2 + \frac{1}{x}.$$

$$\therefore \frac{dM}{dy} = \frac{dN}{dx}.$$

then, the sol^n is exact D.E

$$\int M\,dx = \int (3x^2y + \frac{y}{x})dx$$

$$= 3y\frac{x^3}{3} + y(\log x) \quad \text{——— (1)}$$

$$\int (\text{terms is } N \text{ free from } x)\,dy = \int (\text{No term})\,dy = 0 \quad \text{——— (2)}$$

adding eq^n (1) & (2).

$$3y\frac{x^3}{3} + y(\log x) = c$$

is the sol^n of given D.E

3) Solve : $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0.$

$(y\cos x + \sin y + y)\,dx + (\sin x + x\cos y + x)\,dy = 0.$

We have,

$$M\,dx + N\,dy = 0.$$

We get,

$$M = y\cos x + \sin y + y.$$
$$N = \sin x + x\cos y + x$$

$\frac{dM}{dy} = \cos x + \cos y + 1$

$\frac{dN}{dx} = \cos x + \cos y + 1.$

$\therefore \frac{dM}{dy} = \frac{dN}{dx}$

$\therefore$ Eq^n is exact D.E.

$\int M\,dx = \int (y\cos x + \sin y + y)\,dx$

$= y\sin x + \sin y\, x + yx$ ——— ①

$\int$(terms in N free from x) dy $= \int$ (No term) dy

$= 0.$ ——— ②

eq^n ① add in eq^n ②.

$y\sin x + \sin y\, x + yx = c.$

$y\sin x + (\sin y + y)\,x = c.$

is the sol^n of given D.E.

4) Solve : $(1+e^{x/y})dx + e^{x/y}(1-\frac{x}{y})dy = 0$

We have,

$$M dx + N dy = 0$$

We get,

$$M = (1+e^{x/y})$$
$$N = e^{x/y}(1-\frac{x}{y})$$

$$\frac{dM}{dy} = e^{x/y}\left(\frac{-x}{y^2}\right)$$

$$\frac{dN}{dx} = e^{x/y}\left(\frac{-1}{y}\right) + \left(1-\frac{x}{y}\right)e^{x/y}\cdot\left(\frac{1}{y}\right)$$

$$= \frac{-1}{y}e^{x/y} + \frac{1}{y}e^{x/y} - \frac{x}{y^2}e^{x/y}$$

$$= \frac{-x}{y^2}e^{x/y}$$

$$\therefore \frac{\partial M}{dy} = \frac{\partial N}{\partial x}$$

$\therefore$ Eq is exact D.E.

$$\int M dx = \int (1+e^{x/y})dx = x + \frac{e^{x/y}}{1/y} \quad \text{——} ①$$

$$\int (\text{term in } N \text{ free from } x)\, dy = \int (\text{No term})dy = 0 \quad \text{——} ②$$

adding ① & ②

$$x + e^{x/y} y = c.$$

is the soln of given D.E.

$\frac{\partial M}{\partial N}$

5) Solve. $(y^2 e^{xy^2} + 4x^3)dx + (2xy e^{xy^2} - 3y^2)dy = 0$.

Given

$(y^2 e^{xy^2} + 4x^3)dx + (2xy e^{xy^2} - 3y^2)dy = 0$

$M\,dx + N\,dy = 0$

$M = y^2 e^{xy^2} + 4x^3$, $N = 2xy e^{xy^2} - 3y^2$

$\frac{dM}{dy} = y^2 e^{xy^2}(2xy) + e^{xy^2}(2y)$

$= 2y e^{xy^2}[1 + xy^2]$

$\frac{dN}{dx} = 2y[x e^{xy^2} \cdot y^2] + e^{xy^2}(1)$

$= 2y\, e^{xy^2}[1 + xy^2]$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ eq^n is Exact.

$\int M dx = \int (y^2 e^{xy^2} + 4x^3)dx = \frac{y^2 e^{xy^2}}{y^2} + \frac{4x^4}{4}$

$= e^{xy^2} + x^4$ —— ①

$\int \left(\begin{matrix}\text{terms in N} \\ \text{free from x}\end{matrix}\right) dy = \int -3y^2 dy = \frac{-3y^3}{3}$

$= -y^3$ —— ②

$e^{xy^2} + x^4 - y^3 = c$ is the req^n sol^n

6) Solve : $\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$

Given,

$\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$

$(\tan y - 2xy - y)dx = (x^2 - x\tan^2 y + \sec^2 y)dy$

$(\tan y - 2xy - y)dx - (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

$\therefore M\,dx + N\,dy = 0$

$\therefore M = \tan y - 2xy - y$

$N = -(x^2 - x\tan^2 y + \sec^2 y)$

$\frac{\partial M}{\partial y} = \sec^2 y - 2x - 1$ $\quad \frac{\partial N}{\partial x} = -2x + \tan^2 y$.

$1+\tan^2 y = \sec^2 y$

$$= -2x + \tan^2 y$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

eq^n is Exact

$$\int M\,dx = \int (\tan y - 2xy - y)\,dx$$
$$= (\tan y - y)\,x - 2y\frac{x^2}{2}$$
$$= (\tan y - y)\,x - x^2 y \quad \text{--- (1)}$$

$$\int \left(\begin{matrix}\text{terms in } N \\ \text{free from } x\end{matrix}\right) dy = -\int \sec^2 y\,dy$$
$$= -\tan y \quad \text{--- (2)}$$

$$(\tan y - y)\,x - x^2 y - \tan y = c$$

is the sol^n

1) Solve $\frac{dy}{dx} + 2y = x$.

$\frac{dy}{dx} = x - 2y$

$(x-2y)\,dx - dy = 0$

$M = x - 2y$

$N = -1$

$\frac{dM}{dy} = -2$

$\frac{dN}{dx} = 0$

$\frac{dM}{dy} \neq \frac{dN}{dx}$

$\therefore$ eq^n is not exact

Dep var y and its differential coefficient is of degree 1 and not multiply together

Given D.E is linear in y.

$\frac{dy}{dx} + Py = Q$

$\therefore \quad P = 2, \quad Q = x$

$IF = e^{\int P\,dx} = e^{\int 2\,dx} = e^{2x}$.

Sol^n is given by

(Dep var) (I.F) $= A + \int Q\,(IF)\,dx$

$y \cdot e^{2x} = A + \int x e^{2x}\,dx$

$\int u \cdot v\,dx = uv_1 - u'v_2 + u''v_2 - u'''v_2 + \ldots$

Sufix stands for integration

$'$ represends derivative.

LIATE

integration by generalized by parts.

$ye^{2x} = A + \left[\frac{xe^{2x}}{2} - \frac{(1)e^{2x}}{4} + 0 \right]$

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$y e^{2x} = A + e^{2x}\left(\frac{x}{2} - \frac{1}{4}\right)$ soln

2) Solve : $y \frac{dx}{dy} - x = 2y^3$.

$y \frac{dx}{dy} = 2y^3 + x$

$y\,dx - (2y^3 + x)\,dy = 0$

$M = y$ $\quad N = -(2y^3 + x)$

$\frac{dM}{dy} = 1$ $\quad \frac{dN}{dx} = -1$

$\frac{dM}{dy} \neq \frac{dN}{dx}$

eqn is not exact

$\frac{dx}{dy} - \frac{1}{y} x = \frac{2y^3}{y}$

$\frac{dx}{dy} - \frac{1}{y} x = 2y^2$ ——— ①

eqn ① is LDE in x

$\frac{dx}{dy} + Px = Q$

$P = \frac{-1}{y}$ $\quad Q = 2y^2$

$IF = e^{\int P dy} = e^{-\int \frac{1}{y} dy} = e^{-\log y}$

$= e^{\log y^{-1}} = y^{-1} = \frac{1}{y}$

Soln is given by

(dep. vari) (IF) = $A + \int Q\,(IF)\,dy$

$x\left(\frac{1}{y}\right) = A + \int 2y^2 \left(\frac{1}{y}\right) dy$

$= A + 2\int y\,dy$

$$= A + \frac{2y^2}{2}$$

$$x/y = A + y^2$$

is the solⁿ.

3) Solve $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$.

Given, $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$

is linear D.E in by

$$\frac{dy}{dx} = 3 - \frac{2}{x}y + \frac{1}{x^2} \quad \text{——} (1)$$

$$\frac{dy}{dx} + Py = Q.$$

$$\frac{dy}{dx} + \frac{2}{x}y = 3 + \frac{1}{x^2} \quad \text{——} (1)$$

$$P = 2/x \qquad Q = 3 + \frac{1}{x^2}$$

$$IF = e^{\int P dx} = e^{\int 2/x\, dx} = e^{2\log x}$$

$$= e^{\log x^2} = x^2$$

Solⁿ is

$$(\text{dep var})(IF) = A + \int Q\,(IF)\, dx$$

$$y\, x^2 = A + \int (3x^2 + 1)\, dx$$

$$= A + \frac{3x^3}{3} + x.$$

$$yx^2 = A + x^3 + x$$

4) Solve $y e^y dx = (y^3 + 2xe^y) dy$

Given,

$$ye^y dx - (y^3 + 2xe^y) dy = 0$$

$$M = ye^y, \quad N = -(y^3 + 2xe^y)$$

$$\frac{dM}{dy} = ye^y + e^y$$

$$\frac{dN}{dx} = -2e^y$$

$$\frac{dM}{dy} \neq \frac{dN}{dx} \quad \text{eq}^n \text{ is not exact}$$

$$\frac{dx}{dy} = \frac{y^3 + 2xe^y}{ye^y}$$

$$\frac{dx}{dy} = \frac{y^2}{e^y} + \frac{2}{y}x$$

$$\frac{dx}{dy} - \frac{2}{y}x = \frac{y^2}{e^y} \quad \text{——— ①}$$

It is LDE in x

$$\frac{dx}{dy} + Px = Q$$

$$\therefore P = \frac{-2}{y}, \quad Q = \frac{y^2}{e^y}$$

$$IF = e^{\int p\,dy}$$
$$= e^{\int -\frac{2}{y}dy}$$
$$= e^{-2\int 1/y\,dy}$$
$$= e^{-2\log y} = e^{\log y^{-2}} = y^{-2} = \frac{1}{y^2}$$

$$(\text{Dep.var})\,IF = A + \int Q\,(IF)\,dy$$

$$x\left(\frac{1}{y^2}\right) = A + \int \frac{y^2}{e^y} \cdot \frac{1}{y^2}\,dy$$

$$= A + \int e^{-y}\,dy$$

$$= A + \frac{e^{-y}}{-1} = A - e^{-y}$$

is the soln

5) Solve $(1+x^2)\frac{d^2y}{dx} + y = e^{\tan^{-1}x}$

$$\frac{dy}{dx} + \left(\frac{1}{1+x^2}\right)y = \frac{e^{\tan^{-1}x}}{(1+x^2)} \quad \text{—— ①}$$

It is LDE in y.

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{1}{1+x^2}, \quad Q = \frac{e^{\tan^{-1}x}}{(1+x^2)}$$

$$IF = e^{\int P dx} = e^{\int \frac{1}{1+x^2} dx}$$
$$= e^{\tan^{-1}x}$$

$$(\text{Dep var})(IF) = A + \int Q\,(IF)\,dx$$
$$y.\,e^{\tan^{-1}x} = A + \int \frac{e^{\tan^{-1}x}}{1+x^2} \cdot e^{\tan^{-1}x}\,dx$$
$$= A + \int \frac{e^{2\tan^{-1}x}}{1+x^2}\,dx \quad \text{—— ②}$$

Put $\tan^{-1}x = t$

$$\frac{1}{1+x^2}dx = dt.$$

$\therefore$ eqn ① becomes

$$y.\,e^{\tan^{-1}x} = A + \int e^{2t}\,dt = A + \frac{e^{2t}}{2}$$

$$y\,e^{\tan^{-1}x} = A + \frac{e^{2\tan^{-1}x}}{2}$$

is the required soln

6) Solve $\cos^2x \frac{dy}{dx} - \tan x = -y$.

$$\frac{dy}{dx} + \frac{y}{\cos^2x} = \frac{\tan x}{\cos^2x}$$

$$\frac{dy}{dx} + (\sec^2 x)y = \tan x \cdot \sec^2 x \quad \text{--- (1)}$$

eq^n (1) is L.D.E is y.

$P = \sec^2 x \qquad Q = \tan x \cdot \sec^2 x$

$IF = e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}$

sol^n is,

$(\text{Dep var})(IF) = A + \int Q(IF)\, dx$

$y\, e^{\tan x} = A + \int \tan x \sec^2 x \cdot e^{\tan x} dx \quad \text{--- (2)}$

put $\tan x = t$

$\sec^2 x\, dx = dt$

$\therefore \quad y e^{\tan x} = A + \int t e^t \, dt$

$= A + [t \cdot e^t - (1) e^t + 0]$

$= A + e^t [t - 1]$

$y e^{\tan x} = A + e^{\tan x} [\tan x - 1]$

$\therefore$ is the required sol^n

3] Equation Reducible to linear Eqⁿ.
(Bernolli's eqⁿ)

General form is,

$$\frac{dy}{dx} + Py = Qy^n \quad \text{or} \quad \frac{dx}{dy} + Px = Qx^n.$$

where P, Q are funⁿ of x or constant — where P, Q are funⁿ of y or constant.

1) Solve $\frac{dy}{dx} + 2y = xy^3$.

Given, $\frac{dy}{dx} + 2y = xy^3$ ——— ①

It is in Bernolli's form.
divide by y^3

$$\frac{1}{y^3}\frac{dy}{dx} + \frac{2y}{y^3} = x$$

$$y^{-3}\frac{dy}{dx} + 2y^{-2} = x \quad \text{——— ②}$$

put $y^{-2} = z$

$$-2y^{-3}\frac{dy}{dx} = \frac{dz}{dx}$$

$$y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dz}{dx}$$

$$-\frac{1}{2}\frac{dz}{dx} + 2z = x$$

$$\frac{dz}{dx} - 4z = -2x \quad \text{——— ④}$$

It is LDE in z

$P = -4 \qquad Q = -2x$

$IF = e^{\int P dx} = e^{\int -4 dx} = e^{-4x}$

Soln is,

$(\text{Dep}^t \text{ var})(IF) = A + \int Q(IF)\, dx$

$z \; e^{-4x} = A + \int -2x\, e^{-4x}\, dx$

$y^{-2} e^{-4x} = A - 2\left[\dfrac{x e^{-4x}}{-4} - \dfrac{(1) e^{-4x}}{16}\right]$

$y^2 = A e^{4x} - 2\left[\dfrac{-x}{4} - \dfrac{1}{16}\right]$

is the required soln

2) Solve $\dfrac{dx}{dy} - xy = y^3 x^2$

Given,

$\dfrac{dy}{dy} - xy = y^3 x^2$ —— ①

It is Bernoulli form

$\dfrac{1}{x^2}\dfrac{dx}{dy} - \dfrac{xy}{x^2} = y^3$

is, $x^{-2}\dfrac{dx}{dy} - x^{-1} y = y^3$ —— ②

put

$x^{-1} = z$

$-x^{-2}\dfrac{dx}{dy} = \dfrac{dz}{dy}$

$x^{-2}\dfrac{dx}{dy} = \dfrac{-dz}{dy}$

eqn (ii) becomes,

$-\dfrac{dz}{dy} - zy = y^3$

$\dfrac{dz}{dy} + yz = -y^3$ —— ③

eqn ③ in LDE in z

$P = y \qquad Q = -y^3$

$IF = e^{\int P dx} = e^{\int y dy} = e^{\frac{y^2}{2}}$

$(\text{Dep. var})(IF) = A + \int Q(IF)\, dy$

$z \; e^{y^2/2} = A + \int -y^3 e^{y^2/2} dy$

$= A - \int y^3 e^{y^2/2} dy$ —— ④

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put $\frac{y^2}{2} = t$

$\frac{1}{2}(2y\,dy) = dt \Rightarrow y\,dy = dt$

$$x.e^{-y^2/2} = A - \int y^2 . y e^{y^2/2} dy$$
$$= A - \int 2t\, e^t\, dt$$
$$= A - 2e^{y^2/2}\left[\frac{y^2}{2} - 1\right]$$ is the soln

Solve $\frac{dy}{dx} + y\tan x = y^3 \sec x$

Given,

$$\frac{dy}{dx} + y\tan x = y^3 \sec x \quad \text{——} (1)$$

Given, DE is reducible to LDE

$$\frac{dy}{dx} + Py = Qy^n$$

$$\frac{1}{y^3}\frac{dy}{dx} + \frac{y}{y^3}\tan x = \sec x$$

$$y^{-3}\frac{dy}{dx} + y^{-2}\tan x = \sec x \quad \text{——} (2)$$

put $y^{-2} = z$

diff. wrt x

$$-2y^{-3}\frac{dy}{dx} = \frac{dz}{dx}$$

$$y^{-3}\frac{dy}{dx} = \frac{-1}{2}\frac{dz}{dx}$$

$$\therefore \frac{-1}{2}\frac{dz}{dx} + z\tan x = \sec x$$

$$\frac{dz}{dx} - 2(\tan x)z = -2\sec x \quad \text{——} (3)$$

I f is LDE is z.

$$\frac{dz}{dx} - 2(\tan x) z = -2\sec x \quad \text{--- ③}$$

It is LDE in z

$\therefore P = -2\tan x$, $Q = -2\sec x$

$$IF = e^{\int P dx}$$
$$= e^{\int -2\tan x\, dx} = e^{-2\log \sec(x)}$$
$$= e^{\log(\sec x)^{-2}} = \frac{1}{\sec^2 x}$$

(Dep var) (IF) $= A + \int Q\,(IF)\, dx$

$$z \frac{1}{\sec^2 x} = A + \int -2\sec x \frac{1}{\sec^2 x} dx$$
$$= A - 2\int \cos x\, dx$$
$$= A - 2[\sin x] \text{ as reqn soln}$$

Solve $\cos\theta \frac{dr}{d\theta} - r\sin\theta = -r^2$

Given DE is,

$$\cos\theta \frac{dr}{d\theta} - r\sin\theta = -r^2$$

$$\frac{dr}{d\theta} - (\tan\theta) r = -r^2 \sec\theta \quad \text{--- ①}$$

It is in Bernoulli's form

$$\frac{1}{r^2}\frac{dr}{d\theta} - \frac{r}{r^2}\tan\theta = -\sec\theta$$

i.e, $r^{-2}\frac{dr}{d\theta} - r^{-1}\tan\theta = -\sec\theta$ --- ②.

put $r^{-1} = z$

diff w.r.t θ

$$-r^{-2}\frac{dr}{d\theta} = \frac{dz}{d\theta} \qquad r^{-2}\frac{dr}{d\theta} = -\frac{dz}{d\theta}$$

$$-\frac{dz}{d\theta} - z\tan\theta = -\sec\theta$$

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$$\frac{dz}{d\theta} + (\tan\theta)z = \sec\theta \quad \text{—— (3)}$$

It is LDE in z

$$\frac{dz}{d\theta} + (\tan\theta)z = \sec\theta \quad \text{—— (3)}$$

$P = \tan\theta$, $Q = \sec\theta$

$\therefore$ IF $= e^{\int P d\theta} = e^{\int \tan\theta d\theta}$

$= e^{\log(\sec\theta)}$

$= \sec\theta$

Sol^n is,

(Dep. var) . (IF) $= A + \int Q.(IF)\, d\theta$

$z . \sec\theta = A + \int \sec\theta . \sec\theta \, d\theta$

$= A + \int \sec^2\theta \, d\theta$

$r^{-1} . \sec\theta = A + \tan\theta$

$\frac{\sec\theta}{2} = A + \tan\theta$

is the req sol^n.

Type II - $f'(y)\frac{dy}{dx} + Pf(y) = Q$.

1 Solve $e^x\left(\frac{dx}{dy} + 1\right) = e^y$.

$$\frac{dz}{dy} + z = e^y$$

It is LDE is z

$P = 1$ $Q = e^y$

I.F $= e^{\int P dy} = e^{\int 1 dy} = e^y$

Sol^n is,

$z . e^y = A + \int e^y e^y dy$

$e^x . e^y = A + \int e^{2y} dy$

$$= \frac{A + e^{2y}}{2}$$

$\therefore e^x = Ae^{-y} + \frac{e^y}{2}$ is the soln of given eqn

Solve $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

Given, $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

$$= e^{2x-y} ~~(e^x - e^y)~~$$

$$= e^{2x}.e^{-y} - e^x$$

$$\frac{dy}{dx} + e^x = e^{2x}.e^{-y}$$

dividing by e^{-y}

$$e^y\frac{dy}{dx} + e^y e^x = e^{2x} \quad \text{--- (1)}$$

$$e^y = z$$

diff wrt x

$$e^y\frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dz}{dx} + e^x z = e^{2x} \quad \text{--- (2)}$$

eqn (2) is LDE in z

$\therefore P = e^x$, $Q = e^{2x}$

$$\text{If} = e^{\int P dx} = e^{\int e^x dx} = e^{e^x}$$

$$(\text{Dep var})(\text{IF}) = A + \int Q.(\text{IF})\,dx$$

$$z\, e^{e^x} = A + \int e^{2x}.e^{e^x}\,dx$$

put,

$e^x = t$ $\qquad$ $z e^{e^x} = A + \int e^x . e^{e^x} . e^x\,dx$

$e^x dx = dt$ $\qquad$ $= A + \int t.e^t\,dt$

$z\, \cancel{e^{e^x}} = \cancel{A}$ $\qquad$ $= A + [te^t - 1.e^t]$

$e^y.e^x = A + e^{e^x}[e^x - 1]$ is the soln of eqn