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DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination – Winter 2019

Course: B. Tech in **Sem: III**

Subject Name: Engineering Mathematics-III (BTBSC301) **Marks: 60**

Date: 10/12/2019 **Duration: 3 Hr.**

Instructions to the Students:

1. *Solve **ANY FIVE** questions out of the following.*
2. *The level question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) in front of the question.*
3. *Use of non-programmable scientific calculators is allowed.*
4. *Assume suitable data wherever necessary and mention it clearly.*

		(Level/CO)	Marks
Q. 1	**Attempt the following.**		12
A)	Find $L\left\{cosht \int_0^t e^u \, coshu \, du\right\}$.	**Analysis**	4
B)	If $f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$ is a periodic function with period 2π. Find $L\{f(t)\}$.	**Analysis**	4
C)	Using Laplace transform evaluate $\int_0^\infty e^{-at} \frac{sin^2 t}{t} \, dt$	**Evaluation**	4
Q. 2	**Attempt any three of the following.**		12
A)	Using convolution theorem find $L^{-1}\left\{\frac{1}{s(s+1)(s+2)}\right\}$	**Application**	4
B)	Find $L^{-1}\{\bar{f}(s)\}$, where $\bar{f}(s) = \log\left(\frac{s^2+1}{s(s+1)}\right)$	**Analysis**	4
C)	Using Laplace transform solve $y'' + 2y' + 5y = e^{-t}\sin t$; $y(0) = 0$, $y'(0) = 1$	**Application**	4
D)	Find $L^{-1}\left\{\frac{s^2+2s-4}{(s-5)(s^2+9)}\right\}$	**Analysis**	4
Q. 3	**Attempt any three of the following.**		12

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A)	Express the function $f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate that $\int_0^\infty \frac{\sin\lambda x \sin\lambda\pi}{1-\lambda^2} \, d\lambda$.	**Evaluation**	4
B)	Using Parseval's identity for cosine transform, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$.	**Application**	4
C)	Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2. \\ 0, & x > 2 \end{cases}$	**Analysis**	4
D)	If $F_s\{f(x)\} = \frac{e^{-as}}{s}$, then find $f(x)$. Hence obtain the inverse Fourier sine transform of $\frac{1}{s}$.	**Analysis**	4
Q. 4	**Attempt any three of the following.**		**12**
A)	Form the partial differential equation by eliminating arbitrary function f from $f(x^2+y^2+z^2, 3x+5y+7z) = 0$	**Synthesis**	4
B)	Solve $pz - qz = z^2 + (x+y)^2$	**Application**	4
C)	Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ where the boundary conditions are $u(0,t) = 0$, $u(l,t) = 0$ $(t > 0)$ and the initial condition $u(x,0) = x$; l being the length of the bar.	**Analysis**	4
D	Use the method of separation of variables to solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0) = 6e^{-3x}$	**Application**	4
Q. 5	**Attempt the following.**		**12**
A)	Determine the analytic function $f(z)$ in terms of z whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$	**Analysis**	4
B)	Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic.	**Analysis**	4
C)	Find the bilinear transformation which maps the points $z = 0, -1, -i$ onto the points $w = i, 0, \infty$. Also, find the image of the unit circle $\lvert z \rvert = 1$.	**Analysis**	4
Q. 6	**Attempt the following.**		**12**

A)	Use Cauchy's integral formula to evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $\lvert z \rvert = 3$.	**Evaluation**	4
B)	Find the poles of function $\frac{z^2-2z}{(z+1)^2(z^2+4)}$. Also find the residue at each pole.	**Analysis**	4
C)	Evaluate $\oint_C \frac{e^z}{\cos\pi z} dz$, where C is the unit circle $\lvert z \rvert = 1$.	**Evaluation**	4
	***** Paper End *****		