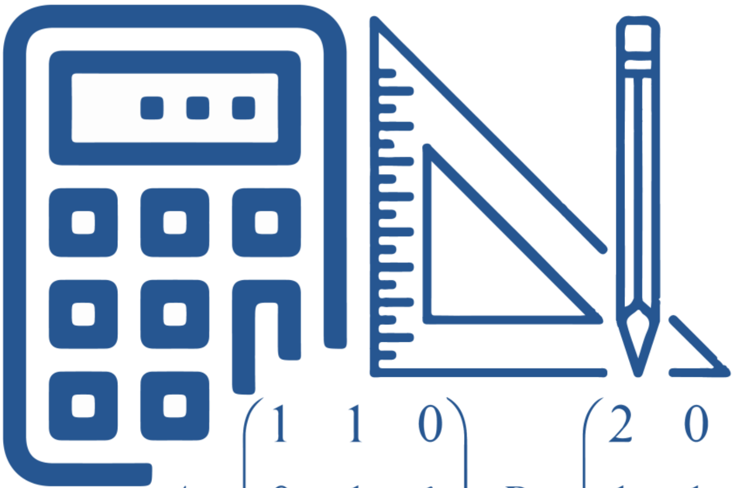
Engineering Mathematics – II



$$A=\begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix};B=\begin{pmatrix}2&0&0\\1&1&2\\2&0&1\end{pmatrix}$$

Unit 3- Linear differential equations with constant coefficients

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1 INTRODUCTION

Differential equations are widely used in fields of engineering and applied science. Mathematical formulations of most of the physical problems are in the forms of differential equations. Use of differential equations is most prominent in subjects like circuit analysis, theory of structures, vibrations, heat transfer, fluid mechanics etc. differential equations are of two types: ordinary and partial differentials equations.in ordinary equations, there is one dependent variable depending for its value on one independent variable. Partial differential equations will have more than one independent variables.

In that follows, we shall discuss ordinary and partial differential equations, which are of common occurrence in engineering fields, applications to some areas will be dealt.

2 PRELIMINARIES

2.1 SECOND DEGREE POLYNOMIALS AND THEIR FACTORIZATION:

a).

i). $D^2 - 2D - 3 = (D+1)(D-3)$

ii. $D^2 - 5D - 6 = (D+2)(D+3)$

iii. $D^2 - 2D + 1 = (D+1)^2$

iv. $D^2 - 2D + 1 = (D+1)^2$

v. $D^2 - 3D + 2 = (D+2)(D+1)$

vi. $D^2 - D - 2 = (D-2)(D+1)$

vii. $D^2 - 4D + 4 = (D-2)^2$

viii. $D^2 - a^2 = (D-a)(D+a)$

ix. $D^2 + a^2 = (D+ia)(D-ia)$

b)The root of $ax^2 + bx + c = 0$ are $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$,these roots are imaginary if $b^2 - 4ac < 0$.

i. $D^2 + 2D + 2 = 0 \Rightarrow D = \frac{-2\pm\sqrt{4-8}}{2} = -1 \pm i$

ii. $D^2 + D + 1 = 0 \Rightarrow D = \frac{-1\pm\sqrt{1-4}}{2} = \frac{-1}{2} \pm \frac{\sqrt{3}}{2} i$

If $D = \frac{-1}{2} \pm i\frac{\sqrt{3}}{2} = \alpha \pm \beta \; then \; \alpha = -\frac{1}{2}, \beta = \frac{\sqrt{3}}{2}$, β is always positive may be positive,negative or zero.

iii. $D^2 + 1 = 0 \Rightarrow D^2 = -1\ i.e D = \pm i \therefore \alpha = 0, \beta = 1$

iv. $D^2 + 4 = 0 \Rightarrow D^2 = -4\ i.e D = \pm 2i \therefore \alpha = 0, \beta = 2$

2.2 THIRD DEGREE POLYNOMIALS AND THEIR FACTORIZATION:

a).

i. $D^3 - a^3 = (D - a)(D^2 + aD + a^2)$

ii. $D^3 + 3D^2 + 3D + 1 = (D + 1)^3$

iii. $D^3 + a^3 = (D + a)(D^2 - aD + a^2)$

iv. $D^3 - 3D^2 + 3D - 1 = (D - 1)^3$

b).use of synthetic Division:

i). $\mathrm{f(D) = D^3 - 7D - 6 = 0, for D = -1, f(-1) = 0}$ $\therefore$ $\mathrm{(D + 1)}$isone if the factors.

-1 1 0 -7 -6

-1 1 6

1 -1 -6 0

$\therefore \mathrm{D^3 - 7D - 6 = 0 \Rightarrow (D + 1)(D^2 - D - 6) = 0}$

$\mathrm{(D + 1)(D - 3)(D + 2) = 0 \Rightarrow D = -1, -2,3}$

ii)for $\mathrm{D^3 - 2D + 4 = 0; D = -2}$ $\therefore \mathrm{f(-2) = 0}$ $\therefore \mathrm{(D + 2)}$

−2	1	0	−2	4
		−2	4	−4
	1	−2	2	0

$\therefore \mathrm{D^3 - 2D + 4 = 0 \Rightarrow (D + 2)(D^2 - 2D + 2) = 0}$

$\mathrm{D = -2 and D = 1 \pm i}, \alpha = 1, \beta = 1$

2.3 FOURTH DEGREE POLYNOMIALS AND THEIR FACTORIZATION:

a). $\mathrm{D^4 + a^4 = (D^2 - a^2)(D^2 + a^2) = (D - a)(D + a)(D + ia)(D - ia)}$

b) Making a perfect square by introducing a middle term:

i)for $D^4 + a^4 = 0$; consider$(D^2 - a^2)^2 = D^4 + 2a^2D^2 + a^4$

$D^4 + a^4 = (D^4 + 2a^2D^2 + a^4) - (2a^2D^2) = (D^2 + a^2)^2 - \left(\sqrt{2}aD\right)^2$

$D^4 + a^4 = (D^2 - \sqrt{2}aD + a^2)\,(D^2 + \sqrt{2}aD + a^2)$

ii) for $D^4 + 1 = D^4 + 2D^2 + 1 - 2D^2 = (D^2 + 1)^2 - (\sqrt{2}D)^2$

$D^4 + 1 = (D^2 - \sqrt{2}D + 1)(D^2 - \sqrt{2}D + 1)$

C) $D^4 + 8D^2 + 16 = (D^2 + 4)^2, 2D^2 + 1 = (D^2 + 1)^2 = (D + i)^2(D - i)^2$

$D^4 + 10D^2 + 9 = (D^2 + 9)(D^2 + 1) = (D + 3i)(D - 3i)(D + i)(D - i)$

d) i) $f(D) = D^4 - 2D^3 - 3D^2 + 4D + 4 = 0, For D = -1, f(-1) = 0$

∴ Factors are $(D + 1)^2\,(D - 2)^2 = 0.$

−1	1	−2	−3	4	4
		−1	3	0	−4
−1	1	−3	0	4	0
		−1	4	−4	
2	1	−4	4	0	
		2	−4		
	1	−2	0		

on a similar line,

ii) $D^4 - D^3 - 9D^2 - 11D - 4 = (D + 1)^3(D - 4)$

e) Perfect square of the type(a+b+c)^2

$i) D^4 + 2D^3 + 3D^2 + 2D + 1 = (D^2)^2 + 2.D^2.D + D^2 + 2D^2 + 2D + 1$

$=(D^2 + D)^2 + 2(D^2 + D) + 1$

$=[(D^2 + D) + 1]^2 = (D^2 + D + 1)^2$

ii) $D^4 - 4D^3 + 8D^2 - 8D + 4 = (D^2)^2 - 2.D^2.D + D^2 + 2D^2 - 2D + 4$

$=(D^2 - D)^2 + 4(D^2 - D) + 4$

$=[(D^2 - 2D) + 2]^2 = (D^2 - 2D + 2)^2$

2.4 FIFTH DEGREE POLYNOMIALS AND THEIR FACTORIZATION:

$D^5 - D^4 + 2D^3 - 2D^2 + D - 1 = D^4(D-1) + 2D^2(D-1) + 1(D-1)$

$= (D^4 + 2D^2 + 1)(D-1) = (D-1)(D^2+1)^2$

$= (D-1)(D+i)^2(D-i)^2$

3 THE NTH ORDER LINEAR DIFFERENTIAL EQUATION WITH CONSTANT COEFFICIENTS

A differential equation which contain the differential coefficient and the dependent variable in the first degree, does not involve the product of a derivative or with dependent variable and in which the coefficients are constant is called linear differential equation with constant coefficients.

The general form of such a different equation of order “n” is

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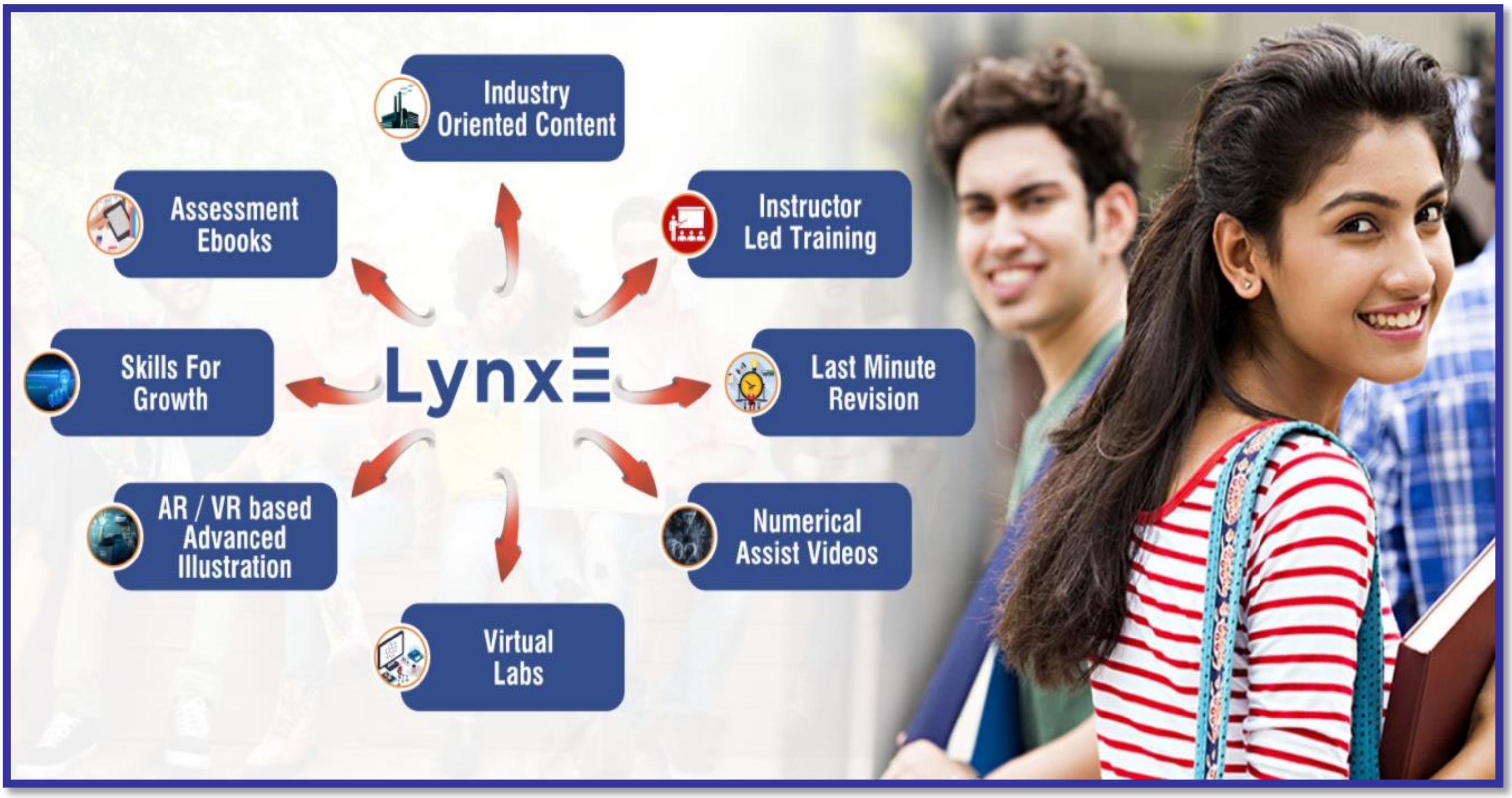

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