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Assignment 2

2. Show that colums of R_0^1 are orthogonal

proof:

$$R_0^1 = \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

If the colums are orthogonal then their dot product should be 0 and $|\text{Column}| = 1$

$\therefore C_1 \cdot C_2 = -C\theta S\theta + S\theta C\theta + 0 = 0$

& $|C_1| = C^2\theta + S^2\theta + 0^2 = 1$

$C_2 \cdot C_3 = -S\theta \times 0 + C\theta \cdot 0 + 0 \cdot 1 = 0$

& $|C_2| = S^2\theta + C^2\theta + 0^2 = 1$

$C_1 \cdot C_3 = C\theta \cdot 0 + S\theta \cdot 0 + 0 \cdot 1 = 0$

& $|C_3| = 0^2 + 0^2 + 1^2 = 1$

$\therefore$ all conditions are satisfied

$\therefore$ they are all orthogonal

3. Show that $|R_0^1| = 1$

proof:

$$|R_0^1| = \begin{vmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= C\theta(C\theta - 0) + S\theta(S\theta + 0) + 0$$

$$= C^2\theta + S^2\theta$$

$$\therefore \boxed{|R_0^1| = 1}$$

6. Show that $RS(a)R^T = S(Ra)$

<u>proof</u> :

let s be skew symmetric such that

$$S = \begin{bmatrix} 0 & -s_1 & s_2 \\ s_1 & 0 & -s_3 \\ -s_2 & s_3 & 0 \end{bmatrix}$$

a be a vector such that

$a = [a_x \ a_y \ a_z]^T$

$$\therefore \quad S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

w.k.t for any vector $p = [P_x \ P_y \ P_z]^T$

$S(a)p = a \times p$

& $\quad R(a \times b) = Ra \times Rb \quad \forall \ a\ b \in \mathbb{R}^3$
$\qquad\qquad\qquad\qquad\qquad\qquad R \in SO(3)$

$\therefore \ S(a)b = a \times b$

$\therefore \ RS(a)R^T b = R(a \times R^T b)$

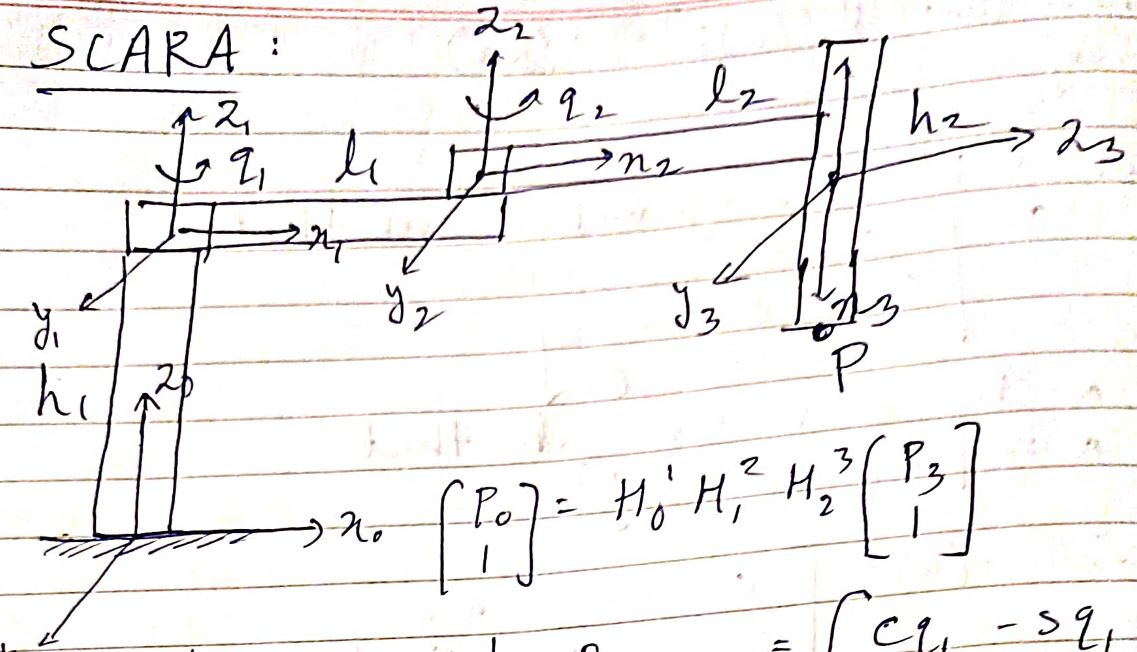
$\therefore \qquad\qquad = (Ra) \times (R R^T b)$

$\therefore \qquad\qquad = (Ra) \times b \qquad\qquad (RR^T = I)$

$\therefore \ RS(a)R^T b = S(Ra) b$

$\therefore \ \boxed{RS(a)R^T = S(Ra)}$

7. <u>SCARA</u> :



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_0^1 \longrightarrow R_0^1 = R_{z,q_1} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ h_1 \end{bmatrix}$$

$$\therefore H_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 \longrightarrow R_1^2 = R_{z,q_2} = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore H_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 & l_1 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

SCARA

$$H_2^3 \longrightarrow R_2^3 = R_{y,\pi/2} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} h_2 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore H_2^3 = \begin{bmatrix} 0 & 0 & 1 & l_2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_{ox} \\ P_{oy} \\ P_{oz} \\ 1 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & l_1 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & l_2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & l_1 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ -h_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 cq_2 + l_1 \\ l_2 sq_2 \\ -h_2 \\ 1 \end{bmatrix}$$

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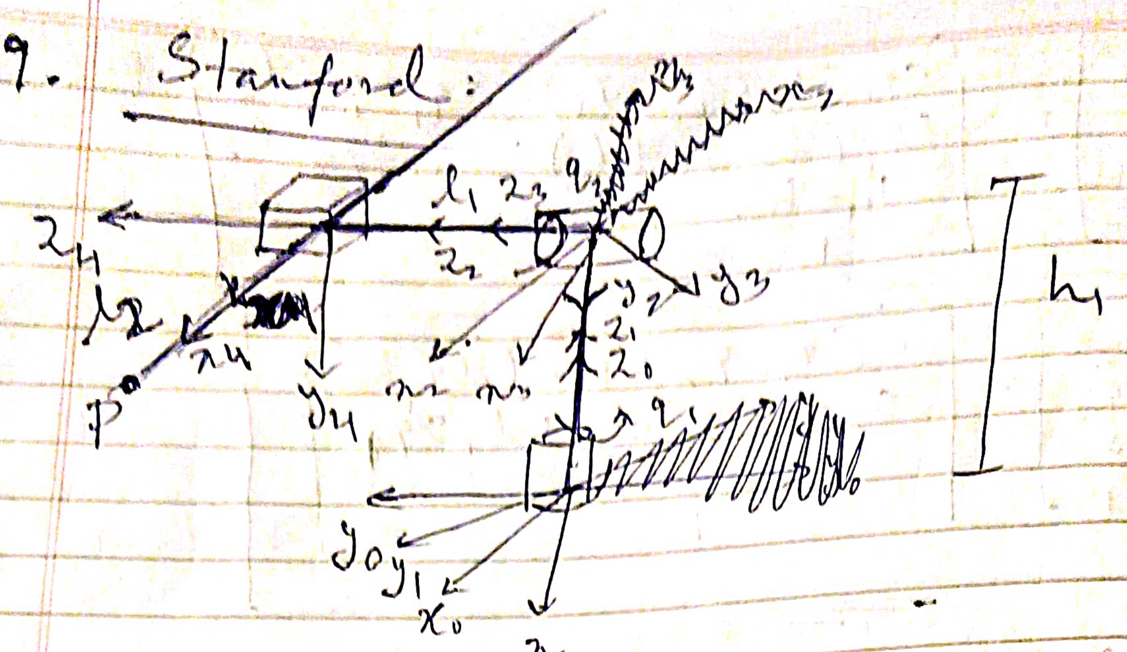
$$\therefore \begin{bmatrix} P_{ox} \\ P_{oy} \\ P_{oz} \\ 1 \end{bmatrix} = \begin{bmatrix} cq_1(l_2 cq_2 + l_1) - sq_1 sq_2 l_2 \\ sq_1(l_2 cq_2 + l_1) + cq_1 sq_2 l_2 \\ h_1 - h_2 \\ 1 \end{bmatrix}$$

$$\therefore \boxed{\begin{bmatrix} P_{ox} \\ P_{oy} \\ P_{oz} \\ 1 \end{bmatrix} = \begin{bmatrix} c(q_1 + q_2) l_2 + l_1 cq_1 \\ s(q_1 + q_2) l_2 + l_1 sq_1 \\ h_1 - h_2 \\ 1 \end{bmatrix}}$$

9. Stanford:



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 H_3^4 \begin{bmatrix} P_4 \\ 1 \end{bmatrix} \qquad P_4 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 \to d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 \to R_0^1 = R_{z,q_1} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 \to d_1^2 = \begin{bmatrix} 0 \\ 0 \\ h_1 \end{bmatrix}$$

$$H_1^2 \to R_1^2 = R_{x,\pi/2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\cancel{= \begin{bmatrix} cq_1(l_2cq_2 - l_2sq_2) - sq_1(l_1+l_2) \\ sq_1(l_2cq_2 - l_2sq_2) + cq_1(l_1+l_2) \\ -l_2sq_2 - l_2cq_2 \\ 1 \end{bmatrix}}$$

$$H_2^3 \begin{cases} \longrightarrow d_2^3 = [0 \ 0 \ 0]^T \\ \longrightarrow R_2^3 = R_{z,q_2} = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{cases}$$

$$H_3^4 \begin{cases} \longrightarrow d_3^4 = [0 \ 0 \ l_2]^T \\ \longrightarrow R_3^4 = I \end{cases}$$

$$\therefore \begin{bmatrix} P_{0x} \\ P_{0y} \\ P_{0z} \\ 1 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

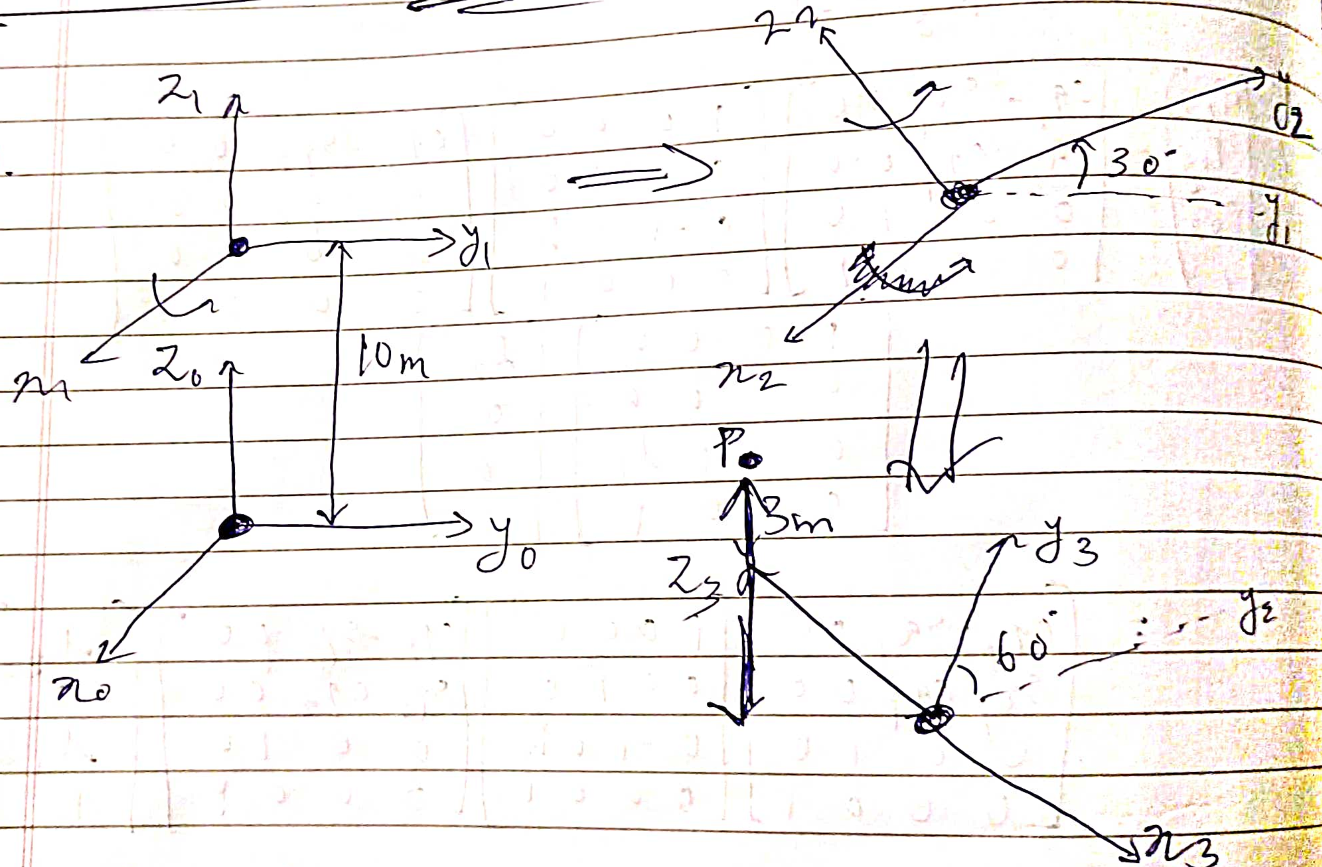
$$= \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ l_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2cq_2 \\ l_2sq_2 \\ l_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 cq_2 \\ l_1 \\ -l_2 cq_2 + h \\ 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} P_{0x} \\ P_{0y} \\ P_{0z} \\ 1 \end{bmatrix} = \begin{bmatrix} l_2 cq_1 cq_2 - l_1 sq_1 \\ l_2 sq_1 cq_2 + l_1 cq_1 \\ h - l_2 cq_2 \\ 1 \end{bmatrix}}$$

10.



$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 0 \end{bmatrix}$$

$$H_0^1 \rightarrow d_0^1 = \begin{bmatrix} 0 & 0 & 10 \end{bmatrix}^T$$

$$H_0^1 \rightarrow R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 \rightarrow d_1^2 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$H_1^2 \rightarrow R_1^2 = R_{x,\pi/6} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$H_2^3 \rightarrow d_2^3 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$H_2^3 \rightarrow R_2^3 = R_{z,\pi/3} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & 0 & 3 \end{bmatrix}^T$$

$$\therefore \begin{bmatrix} P_{0x} \\ P_{0y} \\ P_{0z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \\ 1 \end{bmatrix}$$