ROBOT

- Moving mechanical parts
- Electrical actuation
- Some autonomy (Usually sensing & some control actions based on the sensing implemented through codes)

Types of Robots:

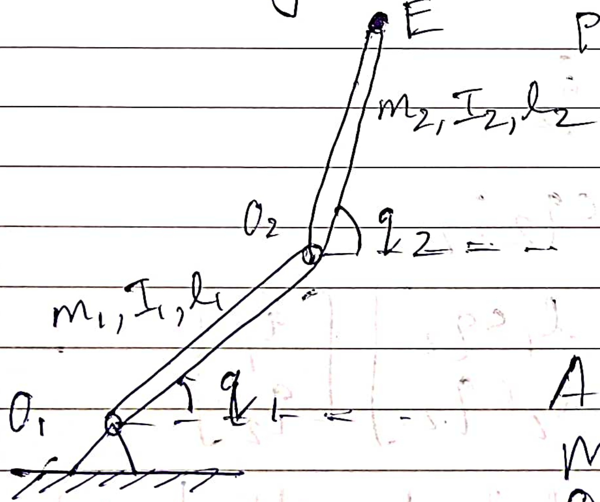
Manipulators, Mobile, Aerial, Higher level robots

Manipulators: Serial, Parallel

☆ 2R manipulator:

2 revolute joints

joints: prismatic, revolute



E → end effector (x, y)

$q_1 q_2$ → joint angles

Assume origin at O_1

Motors are connected at both O_1 & O_2 and we can control both torque & angles consider 3 tasks –

- Task 1 (T1) : Given arbitrary trajectory of end effectors (given x, y, fuction of time) make the robot follow the trajectory

- Task 2 (T2) : Given a location of a wall, make the robot touch the wall and apply a constant predefined force on it.

- Task 3 (T3) : Make a robot behave like a virtual spring that has stiffness 'k' and connects E to a given point $x_0\, y_0$

Now,

$$x = l_1 \cos q_1 + l_2 \cos q_2 = l_1 c q_1 + l_2 c q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2 = l_1 s q_1 + l_2 s q_2 \qquad — ①$$

differentiating ① –

$$\dot{x} = -l_1 s q_1 \dot{q}_1 - l_2 s q_2 \dot{q}_2$$

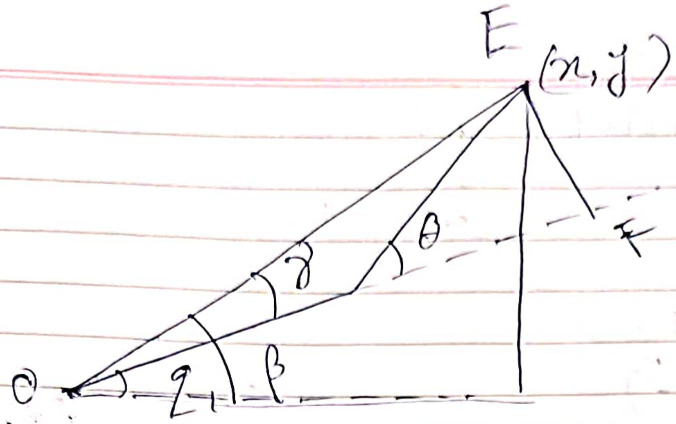
$$\dot{y} = l_1 c q_1 \cdot \dot{q}_1 + l_2 c q_2 \dot{q}_2$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_1 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \qquad — ②$$

Given x & y we need to be able to solve for q_1 & q_2

option 1 $\longrightarrow$ Solve numerically

option 2 $\longrightarrow$ Derive closed form expression



$$\boxed{\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)}$$

$(\theta = q_2 - q_1)$

$$q_1 = \beta - \gamma$$

$$= \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2\cos\theta}\right)$$

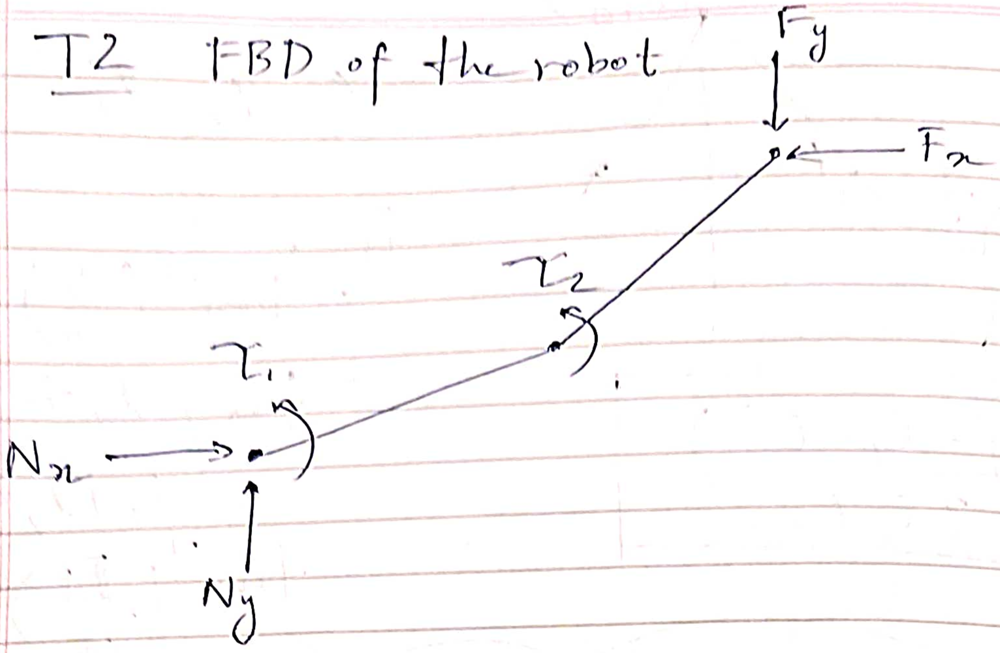
$$\tan^{-1}\left(\frac{EF}{OF}\right)$$

(3)

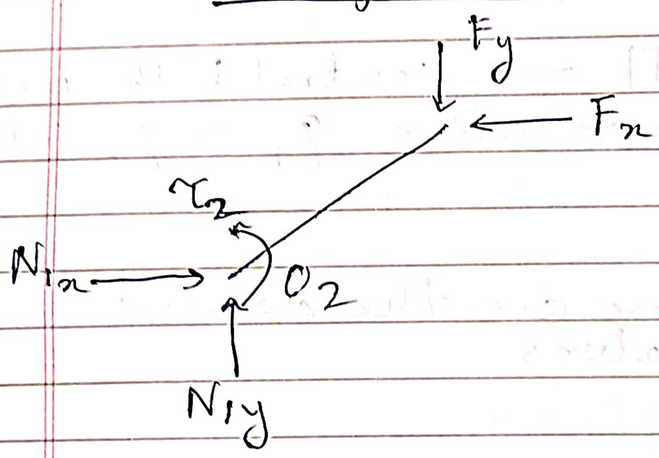
This is the answer to T1 $\longrightarrow$ control both motors in position control mode to achive q_1 & q_2 at each time step

☆ <u>Note</u> : $\left.\begin{matrix} x_d & y_d \\ q_{1d} & q_{2d} \end{matrix}\right\}$ These are the desired values

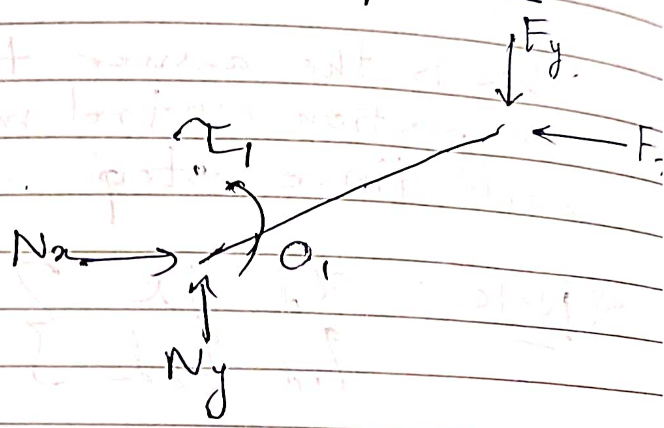
T2 FBD of the robot



FBD of link 2



FBD of link 1



Static equilim $\Longrightarrow \sum M_{O_2} = 0 \,,\ \sum M_{O_1} = 0$

$$\therefore \boxed{\begin{aligned} F_y l_2 \cdot c q_2 - F_x l_2 s q_2 &= \tau_2 \\ F_y l\, c q_1 - F_x l_1 s q_1 &= \tau_1 \end{aligned}} \quad \text{——— (4)}$$

(3) & (4) is the solⁿ for T2

$$\begin{bmatrix} -l_2 s q_2 & l_2 c q_2 \\ -l_1 s q_1 & l_1 c q_1 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} \tau_2 \\ \tau_1 \end{bmatrix}$$

Lagrangian

$$\mathcal{L} = K - V$$

$K \rightarrow$ Kinetic energy

$V \Rightarrow$ potential energy

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \left(\frac{\partial \mathcal{L}}{\partial q_i}\right) = Q_i'}$$

$q_i \rightarrow$ independent degrees of freedom generalized coordinates

$Q_i' \rightarrow$ generalized forces derived using virtual work principle

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation of } L_1} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2}_{\text{Link } L_2 \text{ rotation}} + \underbrace{\frac{1}{2} m_2 V_{c_2}^2}_{\text{translation}}$$

$$V_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_1 \, c(q_1 - q_2)$$

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

$$\mathcal{L} = K - V$$

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 c(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) s(q_2 - q_1) + m_1 g \frac{l_1}{2} c q_1 + m_2 g l_1 c q_1 = \tilde{\tau}_1$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 c(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) s(q_2 - q_1) + m_2 g \frac{l_2}{2} s q_2 = \tilde{\tau}_2$$

(6)

captures the dynamic effects

Task 3 T3

$F_x = kx$
$F_y = ky$

(more generally
$F_x = k(x - x_0)$
$F_y = k(y - y_0)$)

using (1) & (4)

$$k(l_1 s q_1 + l_2 s q_2) l_2 c q_2 - k(l_1 c q_1 + l_2 c q_2) l_2 s q_2 = \tau_2$$
$$k(l_1 s q_1 + l_2 s q_2) l_1 c q_1 - k(l_1 c q_1 + l_2 c q_2) l_1 s q_1 = \tau_1$$

(7)

answer to T3

This is spring effect torque which doesn't consider dynamics