Set 1

Problem 1. How many distinct arrangements of the word "MATH" are there?

Problem 2. Find the area of a square with perimeter 16.,

Problem 3. Compute the smallest positive integer that leaves a remainder of 3 when divided by 4 and a remainder of 4 when divided by 5.

Problem 4. If the sum of five consecutive integers is 25, what is the largest of the five integers?

Set 2

Problem 5. Compute $3 + 5 + 7 + ... + 25$.

Problem 6. If Kao has 3 fair, standard dice, what is the probability that the 3 numbers she rolls sum to 18?

Problem 7. Compute $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{9 \cdot 10}$.

Problem 8. Costco only sells pencils in packs of 3 and 5. What is the largest integer number of pencils that Akshaj cannot purchase?

Set 3

Problem 9. Jeffery has a very big lawn. If Akshaj can mow Jeffery's lawn in 8 hours and Michael can mow Jeffery's lawn in 12 hours, how many hours would it take Akshaj and Michael to mow Jeffery's lawn (working together)?

Problem 10. The Cookie Monster enjoys eating cookies every day. He starts out with 80 cookies and never adds any cookies to the jar. On the 1st day he eats 1 cookie, on the 2nd day he eats 2 cookies, and continues doing so such that on the nth day, he eats n cookies, unless there are less than n cookies remaining, in which case he simply eats all the remaining cookies. After how many days will the Cookie Monster's jar be empty?

Problem 11. What is the least possible integer for which 35% of that number is greater than 3?

Problem 12. Harry has a standard deck of cards (52 cards, 13 in each suit). He deals out a hand with 13 cards in it. He then separates his hand into piles based on suit. The pile with the most cards in it has Q cards. What is the smallest value Q can assume?

Set 4

Problem 13. How many digits are in the numerical value of the product $4^4 \cdot 5^5$?

Problem 14. There are 100 light bulbs lined up in a row in a long room. Each bulb has its own switch and is currently switched off. The room has an entry door and an exit door. There are 100 people lined up outside the entry door. Each bulb is numbered consecutively from 1 to 100 and so is each person. Person 1 enters the room, switches on every bulb, and exits. Person 2 enters and flips the switch on every second bulb (turning off bulbs 2, 4, 6...). Person 3 enters and flips the switch on every third bulb (changing the state on bulbs 3, 6, 9...). This continues until all 100 people have passed through the room. How many light bulbs are on at the very end?

Problem 15. The TJ Varsity Math team went out and ordered 9 eight-slice pizzas. Frankie came and ate $\frac{1}{6}$ of all the pizza slices. Oxahaj was next and he ate $\frac{2}{5}$ of all the pizza slices left. Samuel then ate $\frac{1}{2}$ of all the slices remaining. How many slices of pizza are left?

Problem 16. Sam buys some tacos from Taco Bell. Six of the tacos have beef, eight have chicken, eleven have pork, three have beef and pork, two have beef and chicken, five have chicken and pork, and two have beef, chicken, and pork. How many tacos did Sam buy?

Set 5

Problem 17. Harry lives on a coordinate plane. Starting from point A, he walks 4 units north, 3 units east, 8 units north, and then 6 more units east, in this order, ending at point B. What is the straight-line distance from A to B?

Problem 18. What is the probability that three standard six-sided dice show exactly two distinct face values when rolled?

Problem 19. If $\sqrt{a} + \sqrt{b} = \sqrt{13 + 2\sqrt{42}}$ for some positive integers a and b, what is $a^2 + b^2$?

Problem 20. If the greatest common divisor of two positive integers is 5 and their least common multiple is 30, what is their product?

Set 6

Problem 21. Points A, B, C, and D lie on a circle. Chords AC and BD intersect at a right angle at point E inside the circle. If $AE = 6$, $BE = 8$, and $CE = 4$, compute the length of CD.

Problem 22. What is the area enclosed by the x-axis, $y = 3x + 6$, and $y = -3x + 12$?

Problem 23. The Thorne Miniature room in the Chicago Art Institute consists of 64 tiny replicas of real-world rooms. If a 5 feet long table is represented by a 6 inch replica, and the scales are constant within the exhibition, what is the real life size area, in square inches, of a 25 square inch carpet replica?

Problem 24. How many positive integers less than or equal to 50 are relatively prime to 50?

Set 7

Problem 25. Suppose that the following are true:

- Hao always laughs in math class.
- Math class is on Wednesday and Friday.
- If Hao laughs in a class, he will laugh for the rest of the day.

How many of the following are always true?

- Hao only laughs two days a week.
- Hao will laugh on Friday.
- Hao will not laugh on Monday.
- Hao will laugh in Physics, which he has after math.

Problem 26. In how many ways can 1128 be written as a product of two positive integers if the order doesn't matter?

Problem 27. A circle is inscribed inside a square with perimeter 40, and a square is inscribed inside the circle. Compute the area of the smaller square.

Problem 28. Ray started with a positive integer x, divided it by 4, discarded any remainder, and obtained an integer y. If $y = 12$, what is the sum of all possible values of x?

Set 8

Problem 29. Big Ben is a huge philanthropist. He loves to give back to the community and sometimes even gives people things they didn't ask for. If he donated 100 dollars to charity on his first year of donation and increased the amount by 100% every year, in which year will the sum of his donations surpass \$10,000?

Problem 30. Determine the smallest positive integer that ends in 7 and is divisible by exactly 3 primes, all of which are distinct.

Problem 31. Harry decides to paint the entire outside of a $10 \times 12 \times 8$ rectangular prism. He then decides to cut the rectangular prism into $1 \times 1 \times 1$ cubes and picks one of them. What is the probability that the cube picked has at least 2 sides painted?

Problem 32. Estimate the total number of times the fifth letter of the alphabet (uppercase or lowercase) appears in the solutions to this Guts Round. Your answer will be considered correct if it is within 100 of the correct answer.