0. Triangle ABC has side lengths 3, 4, and 5. Triangle DEF has side lengths 6, 8, and 10. What is the ratio of the area of triangle DEF to triangle ABC?

   Answer. 4.

   Solution. Since triangle ABC is a right triangle with legs of lengths 3 and 4, its area is $\frac{3 \cdot 4}{2} = 6$. Similarly, triangle DEF is a right triangle with legs of lengths 6 and 8, so its area is $\frac{6 \cdot 8}{2} = 24$. The ratio of the areas is therefore $\frac{24}{6} = 4$.

1. At Michael's ice cream parlor, there are a total of 12 flavors of ice cream, and 6 different ice cream toppings. If an ice cream cone consists of 1 flavor of ice cream, and up to 2 toppings (meaning a customer can get 0, 1, or 2 toppings on their ice cream cone), assuming the order of toppings does not matter, how many different combinations of ice cream cones can a customer get?

   Answer. 264.

   Solution. If an ice cream cone has 0 toppings, since there are 12 flavors, there are 12 possible ice cream cones. If a customer gets 1 topping, there are $12 \cdot 6 = 72$ possible ice cream cones. If a customer gets 2 toppings, there are $12 \cdot 6 \cdot 5 = 360$ possible ice cream cones, however, since order does not matter, we must divide this number by 2, giving us 180 possible ice cream cones with 2 toppings. In total, this gives us $12 + 72 + 180 = 264$ possible ice cream cones.

2. In a certain fish tank, $\frac{1}{3}$ of the fish are clownfish, $\frac{1}{4}$ of the fish are basking sharks, $\frac{1}{5}$ of the fish are angelfish, and the rest are pufferfish. If there are 26 pufferfish in the tank, how many basking sharks are there in the tank?

   Answer. 30.

   Solution. The fraction of fish in the tank that are pufferfish is $1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} = \frac{60}{60} - \frac{20}{60} - \frac{15}{60} - \frac{12}{60} = \frac{13}{60}$. Since there are 26 pufferfish in the tank, which is $\frac{13}{60}$ of the total amount of fish in the tank, there are $\frac{26}{\frac{13}{60}} = 120$ total fish in the tank. Taking $\frac{1}{4}$ of this number gives us the number of basking sharks in the tank: $\frac{1}{4} \cdot 120 = 30$ basking sharks.

3. How many different rectangles are there with integer side lengths that have an area of 36? Note that a 1×2 rectangle is the same as a 2×1 rectangle.

   Answer. 5

   Solution. Factoring 36 gives us $2^2 \cdot 3^2$, giving us the following possible rectangles: 1×36, 2×18, 3×12, 4×9, 6×6, 9×4, 12×3, 18×2, and 36×1, giving us 9 rectangles. However, we must subtract 4, as 4 of the rectangles are simply repeats. Therefore, there are 5 different rectangles that have area 36.

4. Aaditya is downloading an app, and it is downloading at a constant rate. In 30 seconds, 20% of the app finished downloading. How much longer in seconds does Aaditya have to wait for the app to finish loading?

   Answer. 120.

Solution. 20% of the app downloads in 30 seconds, so 80% of the app will download in 120 seconds, since $\frac{80\%}{20\%} = \frac{120}{30}$. Since 20% of the app is already downloaded, we need 80% to finish downloading, which takes 120 seconds.

5. There are 30 people in Mrs. Coffey's geometry class. If 19 are taking Spanish, 14 are taking Latin, and everyone is taking at least one of the two languages, how many students are taking both languages?

   Answer. 3.

   Solution. By Principle of Inclusion Exclusion, there are $19 + 14 - 30 = 3$ people who are taking both languages.

6. Quintessa is drawing triangles on a piece of paper. The sum of all angles that she draws is 1080 degrees. How many triangles did she draw? Assume no triangles overlap.

   Answer. 6.

   Solution. Since the sum of the angles in one triangle is $180°$, the total number of triangles Quintessa drew is $\frac{1080}{180} = 6$ triangles.

7. Michael has a playlist with 10 songs on it. Three songs are by BTS, three songs are by Desiigner, and four songs are by Sia. He plays all 10 songs on shuffle, which plays the 10 songs in a random order. However, the function is designed so that no more than three songs by the same artist are played consecutively. How many possible orderings of the songs are there? Assume that all the songs are distinct.

   Answer. 3507840.

   Solution. The total number of orderings is 10! The number of orderings in which all all four Sia songs play consecutively is $7 \cdot 4! \cdot 6!$. Therefore, by complementary counting, the number of orderings that satisfy the conditions is $10! \cdot -7 \cdot 4! \cdot 6! = 7! \cdot (720 - 24) = 7! \cdot (696) = 3507840$.

8. How many integers less than 1000 are there such that the digits sum to 21?

   Answer. 28.

   Solution. We can list out the possible sets of digits that will give us a sum of 21: (9, 6, 6), (9, 7, 5), (9, 8, 4), (9, 9, 3), (8, 8, 5), (8, 7, 6), and (7, 7, 7). There are 6 possible ways to order (9, 7, 5), (9, 8, 4), and (8, 7, 6), 3 possible ways to order (9, 6, 6), (9, 9, 3), and (8, 8, 5), and 1 possible way to order (7, 7, 7), giving us a total of 28 numbers.