28^{th} TJIMO

Alexandria, Virginia

Round: **Individual**

Problem 1. Compute $2^{0+1\cdot 6} - 2^0 \cdot 1 + 7$.

Problem 2. The sum of the digits of the year 2016 is $2 + 0 + 1 + 6 = 9$. Find the next year for which the sum of the digits is also 9.

Problem 3. Compute $1 + 2 + 3 + \cdots + 63$.

Problem 4. A certain circle has an area whose value is twice the value of its circumference. Compute the diameter of the circle.

Problem 5. Polya has 5 shirts, each of a different color: red, green, blue, yellow, and black. He also has 4 pairs of pants of different colors: red, orange, yellow, green. If Polya does not want to wear pants of the same color as his shirt, compute the number of ways he can choose a shirt and a pair of pants.

Problem 6. Cantor normally takes 30 minutes to walk to school at 3 miles per hour. One day, he left home 10 minutes later than usual. Compute the speed, in miles per hour, at which he must travel to still get to school on time. Express your answer as a decimal to the nearest tenth.

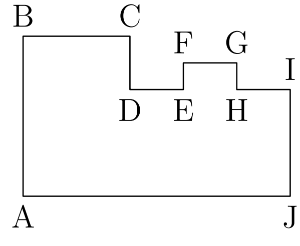
Problem 7. Compute the number of positive divisors of 2016.

Problem 8. Fermat is selling his very last and little mat. He decides to give a discount at 10% off of the original price. After 2 weeks, no one bought his mat. Furious, he decided to give an additional 30% off the discounted price. Finally, Andrew Wiles bought the mat for \$20. Find, to the nearest cent, the original price of the mat.

Problem 9. Al, Bob, Carl, David, and Edward are standing side by side for a picture. Carl is to the left of Al, Bob is to the right of Edward, David is to the left of Edward, Edward is next to Al, and David is not next to Al. Furthermore, David is complaining that the people on his sides are not giving him enough space. Determine, from left to right, the order in which they are standing.

Problem 10. Compute the units digit of 2017^{2016}.

Problem 11. Find the perimeter of the figure below, where $AB = 6$, $AJ = 10$, $GH = 1$, and all interior angles either measure $90°$ or $270°$.



Time limit: 60 minutes.

Problem 12. At a vending machine, 3 bags of chips and 2 bottles of water cost \$2.35, and 2 bags of chips and 3 bottles of water cost \$2.65. Compute the cost of 1 bag of chips and 1 bottle of water.

Problem 13. Lewis has 25 mL of a 20% acid solution and wants to make a 40% acid solution. However, he accidentally adds 10 mL of additional pure water before realizing he was supposed to add acid. Determine the amount of pure acid (in mL) he now needs to add in order to make his 40% acid solution.

Problem 14. A scalene triangle has all integer side lengths, with two sides having lengths 1105 and 2016. Compute the number of possible lengths for the third side.

Problem 15. Two standard six-sided dice are rolled. Compute the probability that the product of the numbers rolled is divisible by 2 or 3. Express your answer as a common fraction.

Problem 16. A regular n-sided regular polygon has 2015 diagonals. Compute the number of diagonals in a regular $n+1$-sided polygon.

Problem 17. Zermelo, Zorn, and Zsigmondy are in a race. If Zermelo always beats Zsigmondy, but otherwise ties are allowed, find the number of possible results. For example, Zorn and Zermelo at a tie for first and Zsigmondy second is one such result.

Problem 18. Dr. Os's class can normally be split evenly into equal groups of 7 when everyone is present. When 2 people are absent and they try to split into groups of 9, they are 1 person short of having equal groups. Find the minimum number of students in the entire class.

Problem 19. Two tanks are used to collect water to drain a full swimming pool. When only tank A is opened, it takes 2 hours for the pool to drain completely. When only tank B is opened, it takes 3 hours for the pool to drain completely. Compute how many **minutes** it will take to drain the pool if both tanks are opened from the start.

Problem 20. Rectangle $CDEF$ is inscribed in right triangle ABC with right angle C such that D is on AC, E is on AB, and F is on BC. If $AD = 3, CD = 4$, and $DE = 5$, find the area of $\triangle ABC$. Express your answer as a common fraction.

Problem 21. Alexander and Thomas are in the same history class. They are both on track to get either an A, A-, or B+. The probability that Alexander gets an A is 0.4, the probability that Thomas gets an A is 0.5, and the probability that neither gets an $A-$ but at least one gets an A is 0.6. Compute the probability that at least one of them gets an A but neither gets a B+. Express your answer as a decimal to the nearest tenth.

Problem 22. Compute the smallest positive integer value of n such that $n!$ (n factorial) ends in 16 consecutive zeros, where $n! = 1 \cdot 2 \cdot \ldots \cdot n$.

Time limit: 60 minutes.

Problem 23. Two cars are traveling towards each other, with car A traveling at 70 km/h and car B traveling at 65 km/hr. Initially, they are 15 km apart, and a bird begins to fly from car A to car B. When the bird reaches car B, it immediately starts flying back to car A. The bird continues to do this until the cars collide. Given that the bird flies at a speed of 20 km/h, compute the total distance the bird will fly before stopping. Express your answer as a common fraction.

Problem 24. Two circles with equal radii are placed inside a 16×18 rectangular box as shown in the diagram. Each circle touches two walls of the box. The two circles are also tangent to each other at one point inside the box. Compute the radius of the circles.

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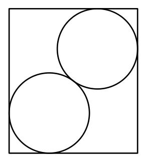
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Problem 25. Find the least positive integer whose sum of positive divisors is 42.

Problem 26. In the figure below (not to scale), BD bisects $\angle ABX$, CD bisects $\angle ACX$, and EG bisects $\angle AED$. Furthermore, DE is parallel to CX. If $AB = 27$, $AC = 36$, and $AE = 12$, compute the length of AG.

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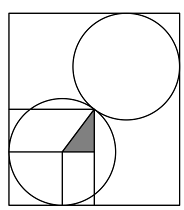
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Problem 27. If $\frac{1}{x} + \frac{1}{y} = 5$ and $x + y = 10$, find the value of $x^2 + y^2$.

Problem 28. Aaditya wants to find three positive numbers that sum to a number less than 1. He decides to use a random number generator three times to get three numbers between 0 and 1. If the random number generator has a continuous, uniform distribution, compute the probability that Aaditya finds three numbers that add up to a number less than 1. Express your answer as a common fraction.

Problem 29. Quadrilateral $ABCD$ has sides $AB = 1008$, $BC = 2016$, and $CD = 2016$. If $\angle BAD$ is a right angle and $m\angle ADC = \frac{1}{2} m\angle BCD$, compute $m\angle BCD$ in degrees.

Time limit: 60 minutes.

Problem 30. Let $P(n)$ be the probability that two randomly chosen positive integers less than n are relatively prime. As n increases, $P(n)$ approaches a fixed constant. Compute this constant. You may find the fact $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$ useful. Express your answer in simplest form.

Time limit: 60 minutes.