28^{th} TJIMO

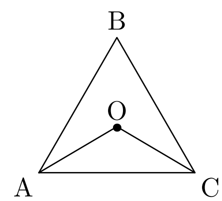
ALEXANDRIA, VIRGINIA

Round: **Team**

Problem 1. If the prime factorization of 60^6 is $2^a \cdot 3^b \cdot 5^c$, where a, b, c are positive integers, find $a+b+c$.

Problem 2. A bag contains marbles of three colors: 10 red marbles, p green marbles, and 55 blue marbles. The probability of randomly selecting a green marble from the bag is $\frac{p}{90}$. Find the probability of selecting a blue marble.

Problem 3. The area of equilateral triangle ABC (AB = BC = CA) is 300. O is the center of the triangle, as shown below.

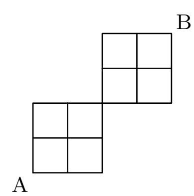


Find the area of triangle AOC.

Problem 4. Given that x, y are positive real numbers satisfying $x + \frac{1}{y} = 5$ and $y + \frac{1}{x} = 7$, find $\frac{x}{y}$.

Problem 5. Let right triangle ABC have $AB = 6$, $BC = 8$, and $CA = 10$. Let X be the midpoint of AB, let Y be the midpoint of BC, and let Z be the midpoint of CA. Find the area of XYZ.

Problem 6. Alex the ant is at point A on the below grid.



If he can only move up or to the right on the grid, find the number of distinct paths Alex can take to go from point A to B.

Problem 7. If x, y are distinct positive integers such that

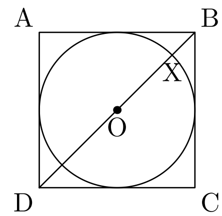
$$x^2 + 2xy = 40$$

$$y^2 + 2xy = 33$$

find $x + y$.

Time limit: 30 minutes

Problem 8. Suppose square $ABCD$ has side length of 10 and circle O is inscribed in the square, as shown below.



Segment BO intersects the circle again at X. Find the length of BX.

Problem 9. Joe lists all the three digit positive integers on a sheet of paper. However, he skips all the integers that are divisible by three and all the integers that are divisible by five. Determine how many integers Joe counts.

Problem 10. Compute the number of positive integers $1 \leq x \leq 143$ such that $x^2 + x^3$ is a perfect square.

Time limit: 30 minutes