# $28^{\text{th}}$ TJIMO

ALEXANDRIA, VIRGINIA

Round: **Individual**

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**Problem 1.** Compute $2^{0+1\cdot 6} - 2^0 \cdot 1 + 7$.

**Problem 2.** The sum of the digits of the year 2016 is $2+0+1+6=9$. Find the next year for which the sum of the digits is also 9.

**Problem 3.** Compute $1+2+3+\cdots+63$.

**Problem 4.** A certain circle has an area whose value is twice the value of its circumference. Compute the diameter of the circle.

**Problem 5.** Polya has 5 shirts, each of a different color: red, green, blue, yellow, and black. He also has 4 hats of different colors: red, orange, yellow, green. Compute the number of ways he can choose a shirt and a hat.

**Problem 6.** Cantor normally takes 30 minutes to walk to school at 3 miles per hour. One day, he left home 10 minutes later than usual. Compute the speed, in miles per hour, at which he must travel to still get to school on time. Express your answer as a decimal to the nearest tenth.

**Problem 7.** Compute the number of positive divisors of 1024.

**Problem 8.** Fermat was selling his very last and little mat at an original price of \$20. He then decided to give a discount at 10% off of the original price. After 2 weeks, no one bought his mat. Furious, he decided to give an additional 30% off the discounted price. Finally, after both discounts, Andrew Wiles bought the mat. Find the price Andrew Wiles paid. Express your answer in dollars and cents.

**Problem 9.** Al, Bob, Carl, David, and Edward are standing side by side for a picture. Carl is to the left of Al, Bob is to the right of Edward, and David is to the left of Edward. Furthermore, David is complaining that the people on both of his sides are not giving him enough space. Find the person standing farthest to the left. (Note: "Carl is to the left of Al" does not necessarily mean Carl is adjacent to Al.)

**Problem 10.** Mario has a humongous jug with 640 ounces of apple juice. Every day, Mario drinks half of the contents in the jug. Thus, after the first day, there will be 320 ounces remaining. Compute the number of days after which there will be 5 ounces of apple juice left.

**Problem 11.** Define the operation $a \triangle b = a + 2b$ for any real $a, b$. If $a \triangle (b \triangle a) = (ka) \triangle b$ for all positive integers $a$ and $b$, compute $k$.

**Problem 12.** At a vending machine, 3 bags of chips and 2 bottles of water cost \$2.35, and 2 bags of chips and 3 bottles of water cost \$2.65. Compute the cost of 1 bag of chips and 1 bottle of water.

**Problem 13.** Compute the number of unordered sets of three prime numbers sum to 38. (The set $\{1, 2, 3\}$ is considered the same as the set $\{3, 2, 1\}$.)

*Time limit: 60 minutes.*

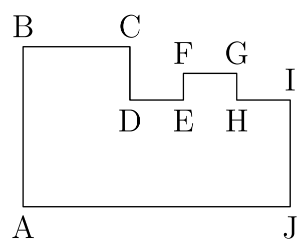
**Problem 14.** A palindrome is a positive integer that reads the same forwards and backwards. For example, 2002 and 1111 are palindromes, while 2017 is not. Compute the number of 5-digit palindromes.

**Problem 15.** Determine the greatest integer from the following set: $\{7^{2000},\ 2^{6000},\ 7^{1000} \times 3^{2000},\ 3^{1000} \times 5^{2000},\ 3^{4000}\}$

**Problem 16.** Find $\alpha$ such that the following system of equations have no solutions: $\begin{cases} 2x + 3y = 7 \\ 5x + \alpha y = 10 \end{cases}$

**Problem 17.** Lewis has 25 mL of a 20% acid solution and wants to make a 40% acid solution. However, he accidentally adds 10 mL of additional pure water before realizing he was supposed to add acid. Determine the amount of pure acid (in mL) he now needs to add in order to make his 40% acid solution.

**Problem 18.** Find the perimeter of the figure below, where $AB = 6$, $AJ = 10$, $GH = 1$, and all interior angles either measure $90°$ or $270°$.




**Problem 19.** Compute the units digit of $2017^{2016}$.

**Problem 20.** Two standard six-sided dice are rolled. Compute the probability that the product of the numbers rolled is divisible by 2 or 3. Express your answer as a common fraction.

**Problem 21.** Zermelo, Zorn, and Zsigmondy are in a race. If Zermelo always beats Zsigmondy, but otherwise ties are allowed, find the number of possible results. For example, Zorn and Zermelo at a tie for first and Zsigmondy second is one such result.

**Problem 22.** Dr. Os's class can normally be split evenly into equal groups of 7 when everyone is present. When 2 people are absent and they try to split into groups of 9, they are 1 person short of having equal groups. Find the minimum number of students in the entire class.

**Problem 23.** Rectangle $CDEF$ is inscribed in right triangle $ABC$ with right angle $C$ such that $D$ is on $AC$, $E$ is on $AB$, and $F$ is on $BC$. If $AD = 3, CD = 4$, and $DE = 5$, find the area of $\triangle ABC$. Express your answer as a common fraction.

*Time limit: 60 minutes.*

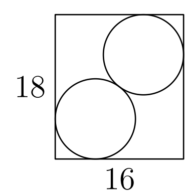
**Problem 24.** If $\frac{1}{x} + \frac{1}{y} = 5$ and $x + y = 10$, find the value of $x^2 + y^2$.

**Problem 25.** Two tanks are used to collect water to drain a full swimming pool. When only tank A is opened, it takes 2 hours for the pool to drain completely. When only tank B is opened, it takes 3 hours for the pool to drain completely. Compute how many **minutes** it will take to drain the pool if both tanks are opened from the start.

**Problem 26.** Compute the smallest positive integer value of $n$ such that $n!$ ($n$ factorial) ends in 16 consecutive zeros, where $n! = 1 \cdot 2 \cdot \ldots \cdot n$.

**Problem 27.** Two cars are traveling towards each other, with car A traveling at 70 km/h and car B traveling at 65 km/hr. Initially, they are 15 km apart, and a bird begins to fly from car A to car B. When the bird reaches car B, it immediately starts flying back to car A. The bird continues to do this until the cars collide. Given that the bird flies at a speed of 20 km/h, compute the total distance the bird will fly before stopping. Express your answer as a common fraction.

**Problem 28.** Two circles with equal radii are placed inside a $16 \times 18$ rectangular box as shown in the diagram. Each circle touches two walls of the box. The two circles are also tangent to each other at one point inside the box. Compute the radius of the circles.




**Problem 29.** Find the least positive integer whose sum of positive divisors is 42.

**Problem 30.** Aaditya has a random number generator that randomly selects a real number $x$ uniformly at random between 0 and 1. He uses his random number generator twice to obtain two real numbers $a$ and $b$. Compute the probability that $\dfrac{1}{6} < \dfrac{a}{a+b} < \dfrac{5}{6}$.

*Time limit: 60 minutes.*