

Aim: Solve the following problems using Numerical Methods

1) Find solution for Lotka-Volterra equations (known as predatory-prey equations)

$$\frac{dx}{dt} = \alpha x - \beta xy \quad \frac{dy}{dt} = \delta xy - \gamma y$$

x is number of deers, y is number of Lion.

$\alpha, \beta, \gamma, \delta$ are positive real parameters describing the interactions of two species

Use $\alpha = 1.2, \beta = 0.6, \delta = 0.3, \text{ and } \gamma = 0.8$

$x(0)=2, y(0)=1$, Plot x, y vs t from $t=0$ till 50 and plot x vs y

Play with α, β, γ and δ

This set should be able to provide proper distribution.

2) Lorenz equations for simple model for atmospheric fluid dynamics

$$\frac{dx}{dt} = -\sigma x + \sigma y, \quad \frac{dy}{dt} = rx - y - xz, \quad \text{and} \quad \frac{dz}{dt} = -bz + xy$$

I.C. $x(0) = y(0) = z(0) = 5$

Assume $\sigma = 10, b = 2.66667, \text{ and } r = 28$

t= 0 to 50

Use 4th order RK

Lorenz developed these equations to relate intensity of atmospheric fluid motion x, to temperature variations, y and z in horizontal and vertical directions.

Plot x vs t
x vs y
x vs z

3) Try to solve the pendulum with

$$\frac{d^2\theta}{dt^2} + \frac{g}{l} \sin \theta = 0 \qquad \theta(0) = 0.1, \frac{d\theta}{dt} = 0$$

Use RK5 method to solve

Length =1m. Also try with different length(5,10)
Change initial theta and see the result.
Also check how result is affected by *h* choice

4) Solve the following IVP from $t=1.0$ to $t=2.5$

$$\frac{dy}{dt} = \frac{-2y}{1+t}$$

Use fourth-order Adams method. Employ a step size of 0.25 and fourth-order RK method to predict the start up values if $y(0)=2$.

Use both explicit and implicit method.

5) Zombie population modeling

<http://mysite.science.uottawa.ca/rsmith43/Zombies.pdf>

$$\frac{dS}{dt} = \pi - \beta SZ - \delta S, \quad \frac{dZ}{dt} = \beta SZ + \zeta R - \alpha SZ \quad \text{and} \quad \frac{dR}{dt} = \delta S + \alpha SZ - \zeta R$$

Susceptible (S), Zombie (Z) and Removed (R)

Non – zombie – related death : δ

π birth rate is a constant,

β : encounter with a zombie

α : destroying the brain of zombie

ζ : humans in removed class resurrect and become zombie

$$\pi = 0, \quad \alpha = 0.005, \quad \beta = 0.0095, \quad \zeta = 0.0001 \quad \text{and} \quad \delta = 0.0001$$

$$S(0)=500, \quad Z(0)=5, \quad R(0)=0$$