PHY425
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Lab-2

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Aim: Solve the following problems using Numerical Methods

1) Compare plots for different RG (2,3,4,5) methods

$$\frac{dy}{dx} = -\frac{1+y^2}{1+x^2} \text{ with IC } y(0) = 1 \qquad \text{Solution : } \; y = \frac{1-x}{1+x}$$

2) Adaptive RKF45 method. Make such a way that h changes automatically

$$\frac{dy}{dx} = \frac{x}{3y^2}(1-y^3)$$ With IC, x=0, y=2 $$\text{Solution } y = \sqrt[3]{1+7\exp^{-0.5x^2}}$$

3) Find solution for Lotka-Volterra equations (known as predatory-prey equations)

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad \frac{dy}{dt} = \delta xy - \gamma y$$

x is number of deers, *y* is number of Lion.
α, β, γ, δ are positive real parameters describing the interactions of two species

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad \frac{dy}{dt} = \delta xy - \gamma y$$

$\alpha x :$ Deer reproduce exponential $\quad \beta xy :$ Deer and lion interaction leading to reduction of deer

$\delta xy :$ Lion reproduce based on food availability $\quad \gamma y :$ Natural death or outside movement

Simplified model: Many assumptions, like endless food availability, no natural calamity, and so on

Let's take these parameters as below and try to get solution

$$\alpha = 1.1, \ \beta = 0.4, \ \delta = 0.1, \text{ and } \gamma = 0.4$$

IC *x(0)*=100 and *y(0)*=5

Use 5th order RK to get *x* and *y* for *t* = 0, 200

Free to choose *h* (but should lead to proper solution)

Plot *x, y* w.r.t time scale on same plot.
Further plot *x vs y*.

Inputs for your code, Initial population can be changed for both deer and Lion.
a) Change *x(0)* from 100-1000 in steps of 200 and plot *x vs y* on same plot. Keep y(0) as 10
b) Change *y(0)* from 5-100 in steps of 20 and plot *x vs y* on same plot. Keep x(0) as 200

4) Radioactive material consist of material X which has decay constant k_1. X decays to another radioactive material Y, which further decay with decay constant k_2.

$$\frac{dX}{dt} = -k_1 X \text{ and } \frac{dY}{dt} = -\frac{dX}{dt} - k_2 Y$$

$$\text{I.C. } X(0) = 10^6 \text{ and } Y(0) = 0$$

Assume $k_1 = 1$

Solve using any method and get X, Y. Plot X and Y for

a) $k_2 = 0.25\, k_1$

b) $k_2 = k_1$

c) $k_2 = 2\, k_1$