Lab-1

Aim: Implement algorithm of what we learn in the class

1) $$\frac{dy}{dx} = -xy^2,\ y(0) = 2$$

Calculate y(0.2) by Taylor's series method

$$y(x) = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + \frac{x^4}{4!}y_0''''$$

$$y' = -xy^2$$

$$y'' = -y^2 - 2xyy'$$

$$y''' = -4yy' - 2xy'^2 - 2xyy''$$

$$y'''' = -6y'^2 - 6y'y'' - 6xy'y'' - 2xyy'''$$

2) Use Euler's Method to evaluate 1) from 0 till 10 in different steps. Compare the result with its analytical solution.

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1}), \text{ where } x_n = x_{n-1} + h$$

3) Use Huen's Method to evaluate 1) from 0 till 10 in different steps. Compare the result with its analytical solution and Euler's Method.

$$y_n = y_{n-1} + h\frac{f(x_{n-1}, y_{n-1}) + f(x_n, y_n^0)}{2}, \text{ where } x_n = x_{n-1} + h$$

4) Use Mid-point method to evaluate 1) from 0 till 10. Compare result with analytical, Euler and Huen's Method

$$y_{n+1/2} = y_n + \frac{h}{2}f(x_n, y_n)$$

$$y'_{n+1/2} = f(x_{n+1/2}, y_{n+1/2})$$

$$y'_{n+1} = y_n + hf(x_{n+1/2}, y_{n+1/2})$$

5) Also find solution for

$$\frac{dy}{dx} = -\frac{1+y^2}{1+x^2} \text{ with IC } y(0) = 1 \qquad \text{Solution}: \; y = \frac{1-x}{1+x}$$

a) Simple Huen method without iteration
b) Huen with iteration
c) Mid-point method

Pseudo-code

Euler

```
float euler(double x, double y, double h){
  return y + h *func(x,y);
}
      for(int i=1; i<20; ++i){
         x[i]=x[0]+i*h;
         y[i]= euler(x[i-1], y[i-1],h);
       }
```

Heuns with iteration

```
float  ihuens(double x, double y, double h){

  float y0= y + h*func(x,y);
Loop Start (1-1000)
  float y1 =  y + h*(func(x,y) + func(x+h,y0))*0.5;
 If (fabs(y1  - y0) >0.01) {
    y0 = y1;}
   else break out of loop and return y1;
}
```

Huens

```
float huens(double x, double y, double h){

  float y0= y + h*func(x,y);
  return y + h*(func(x,y) + func(x+h,y0))*0.5;
}
```

Mid-point Method

```
float  y0= y + 0.5*h*func(x,y);


return y + h*func(x+0.5*h,y0);
```