Problem 3

a) Using Induction,

let $k=1$, i.e., there is only one class in dataset.

n = total no. of rows in the dataset

if $k=1$ => The probability of that class being chosen

$$\frac{\cancel{n}}{\cancel{n}} = 1$$

$k=2$, two classes in the dataset.

The total probability of any of the class,

let k be the number of observations from class 1

and $n-k$ be the number of observations from class 2.

So, the sum of posterior probabilities

$$= \frac{k}{n} + \frac{(n-k)}{n} = \frac{k+n-\cancel{k}}{n} = \frac{\cancel{n}}{\cancel{n}} = 1$$

let say $k = z-1$, $z-1$ classes.

The sum of posterior probabilities is given by

$$\frac{1}{n}\sum_{i=1}^{z-1} k_i + (n-k_i) = 1$$

So, the sum of posterior probabilities of classes is equal to 1 by induction.

b) Given

$$P(X) = \frac{e^{(\beta_0 + \beta_1 X)}}{1 + e^{(\beta_0 + \beta_1 X)}}$$

$$\Rightarrow \quad e^{(\beta_0 + \beta_1 X)} = P(X)\left[1 + e^{(\beta_0 + \beta_1 X)}\right]$$

$$e^{(\beta_0 + \beta_1 X)} = P(X) + P(X)\left[e^{(\beta_0 + \beta_1 X)}\right]$$

$$P(X) = e^{(\beta_0 + \beta_1 X)} - P(X)\, e^{(\beta_0 + \beta_1 X)}$$

$$P(X) = e^{(\beta_0 + \beta_1 X)}\left[1 - P(X)\right]$$

Dividing both sides by $1 - P(X)$

$$\frac{P(X)}{1 - P(X)} = e^{(\beta_0 + \beta_1 X)}$$