

# Optimisation Assignment

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### I. PROBLEM

If the function  $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ , where  $a > 0$ , attains its maximum and minimum at  $p$  and  $q$  respectively. Such that  $p^2 = q$ , then  $a$  equals?  
 A)  $\frac{1}{2}$    B) 3   C) 1   D) 2

### II. SOLUTION

Given that Maxima and Minima lie at  $P$  and  $Q$  respectively.

which means  $\frac{df(x)}{dx} = 0$  at  $P$  and  $Q$ .

which means  $P$  and  $Q$  will be the roots of  $f'(x)$ .

So Now ,

$$\frac{df(x)}{dx} = f'(x) = 6x^2 - 18ax + 12a^2 = 0 \quad (1)$$

$$f'(x) = x^2 - 3ax + 2a^2 = 0 \quad (2)$$

$$f'(x) = x^2 - 2ax - ax + 2a^2 = 0 \quad (3)$$

$$\Rightarrow x(x - 2a) - a(x - 2a) = 0 \quad (4)$$

$$\Rightarrow (x - a)(x - 2a) = 0 \quad (5)$$

$$\Rightarrow x = 2a, a \quad (6)$$

$$(7)$$

So, the roots are  $2a, a$ .

Now,

Case 1 : When  $P = 2a$  and  $Q = a$ . As  $p^2 = q$ ,

$$\Rightarrow 4a^2 = a$$

so,  $a = 1/4$

Now,

Case 2 : When  $P = a$  and  $Q = 2a$ .

As  $p^2 = q$ ,

$$\Rightarrow a^2 = 2a$$

so,  $a = 2$

Finally  $a = 1/4, 2$

Hence we can conclude from the given options that  $a = 2$ , i.e Option D is the correct option.