Optimisation Assignment

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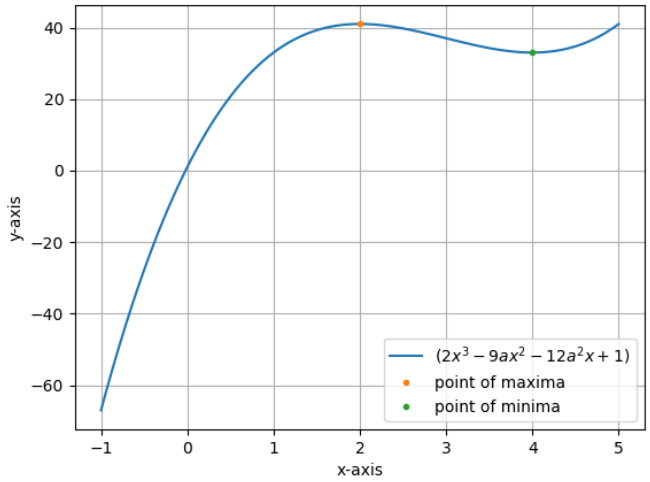
Contents

I. Problem

If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively. Such that $p^2 = q$,then a equals?
A) $\frac{1}{2}$ B) 3 C) 1 D) 2

II. Figure



III. Solution

Assume a=2 ,
Given p is maxima point and q is minima point

So., we can write the given equation as:

$$f(x) = 2x^3 - 18x^2 + 48x + 1 \tag{1}$$

we can find the maxima of eq(1) by using gradient ascent method

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{2}$$

$$x_{n+1} = x_n + \alpha(6x^2 - 36x + 48) \tag{3}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Maxima} = 40.99999999999584 \approx 41} \tag{4}$$

$$\boxed{\text{Maxima Point} = 1.9999991677483622 \approx 2} \tag{5}$$

Now,
we can find the minima of eq(1) by using gradient descent method

$$\implies x_{n+1} = x_n - \alpha \nabla f(x_n)$$

$$x_{n+1} = x_n - \alpha(6x^2 - 36x + 48) \tag{6}$$

Taking $x_0 = 3.5, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Minima} = 33.00000000000418 \approx 33} \tag{7}$$

$$\boxed{\text{Minima Point} = 3.9999991682037273 \approx 4} \tag{8}$$

so here we have calculated ,
Maxima point i.e $p = 2$
And , Minima $q = 4$
which satisfies the condition $p^2 = q$
Hence $a = 2$, option D is the correct answer.

https://github.com/aadrshptel/Fwc_module1/tree/main/Assignments/Matrix%20assignments/Optimisation/codes

Execute the code by using the command
python3 opt.py