

Final Exam :

(Q1) Level 1 of Impurity :

$$I_p = P(E)^*(1 - P(E)) + P(M)^*(1 - P(M)) \\ + P(T)^*(1 - P(T)) + P(G)^*(1 - P(G))$$

$$\Rightarrow (2/7)(1 - 2/7) + (1/7)(1 - 1/7) + (2/7)(1 - 2/7) \\ + (2/7)(1 - 2/7)$$

$$\Rightarrow (0.28)(0.71) + (0.14)(0.85) + (0.28)(0.71) \\ + (0.28)(0.71)$$

$$I_p \Rightarrow 0.7154$$

$$\text{Average } I_p \Rightarrow (1/7) * I_p$$

$$\Rightarrow 1(0.7154)$$

Level 2 : Impurity

Left subtree :

$$I_p \Rightarrow P(T)^*(1 - P(T)) + P(G)^*(1 - P(G))$$

$$\Rightarrow (2/4)(1-2/4) + (2/4)(1-2/4)$$

$$I_{p1} \Rightarrow 1/2 \Rightarrow 0.5$$

Subtree 2:

$$I_p \Rightarrow P(E)(1-P(E)) + P(M)(1-P(M))$$

$$\Rightarrow (2/3)(1-2/3) + (1/3)(1-1/3)$$

$$I_{p2} \Rightarrow \approx 0.44$$

$$\text{Average } I_p \Rightarrow (4/7) I_{p1} + (3/7) I_{p2}$$

$$\text{Avg-} I_p \Rightarrow (4/7)(0.5) + (3/7)(0.44)$$

$$\Rightarrow 0.4742$$

Information gain in 2 levels:

$$\text{info-gain} \Rightarrow \text{Avg } I_p(\text{Level 1}) - \text{Avg } I_p(\text{Level 2})$$

$$\Rightarrow 0.7154 - 0.4743$$

$$\Rightarrow 0.2411$$