Monte Carlo Simulation of Predictive Stability under Structural Breaks

Aadya Khatavkar (50196397)
Bakhodir Izzatulloev (50294516)
Mahir Baylarov (50316809)

University of Bonn
Research Module in Econometrics and Statistics
Fundamentals of Monte Carlo Simulations
Winter Semester 2025/26

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1 Introduction

The assumption of parameter stability is fundamental to statistical forecasting. In practice, however, the relationships captured by model parameters often change over time. Economic structures, institutions, and policy regimes evolve, and when they do, the underlying data-generating process may shift accordingly. As a result, time series frequently exhibit structural breaks. Because forecasting plays a central role in planning, investment decisions, and policy analysis in both the public and private sectors, understanding the impact of structural instability on predictive performance is of primary importance.

A substantial body of literature examines structural breaks and their implications for inference and estimation. Empirical evidence suggests that financial and macroeconomic relationships are often unstable, and forecasting models that perform well in-sample may fail to deliver comparable improvements out-of-sample once the underlying relationship changes (Stock and Watson, 1996; Clark and McCracken, 2001). Much of the economic research on structural breaks has focused on detection, estimation, and inference in the presence of breaks (e.g., Andrews and Kitagawa; Killick, Fearnhead, and Eckley, 2012). While these contributions are essential, the forecasting problem presents a distinct challenge. In real time, breakpoints are typically unknown, and forecasters must rely on methods that are robust to potential instability.

This raises a natural question: how sensitive are different forecasting models to alternative forms of structural change? In particular, do models that explicitly account for regime shifts outperform simpler adaptive procedures, and under what conditions?

A critical real-world constraint not always reflected in the literature is that structural breaks are unknown to forecasters at the time predictions are made. In practice, breaks must be detected after they occur, and detection typically involves a lag of several periods during which forecasters remain unaware of parameter instability. This *detection lag* creates a gap between the oracle scenario (where breaks are known) and feasible real-time forecasting. Our simulation framework allows us to contrast oracle-informed models, which assume knowledge of break timing, against genuine real-time adaptive approaches.

To address this question, this paper employs a Monte Carlo simulation framework to evaluate forecasting accuracy under controlled structural break scenarios. Simulation-based analysis is particularly suitable for this purpose because it allows the researcher to operate in an environment where the true data-generating process is known. By construction, we isolate specific types of instability—mean shifts, changes in autoregressive persistence, and volatility shifts—while holding other features constant. In contrast, empirical time series typically exhibit multiple overlapping sources of instability, making it difficult to identify the precise mechanism driving forecast deterioration.

The main contribution of this paper is a systematic comparison of different forecasting approaches across multiple structural break designs. We consider (i) globally estimated linear time-series models that impose parameter constancy (Global SARIMA), (ii) adaptive rolling-window estimation procedures that re-estimate parameters using only recent observations (Rolling SARIMA), and (iii) regime-switching models that explicitly allow parameters to depend on an unobserved state variable (Markov-switching AR). Forecast performance is primarily evaluated using RMSE, MAE, and bias. In selected designs, we further vary the innovation distribution (Gaussian and Student-t with different degrees of freedom) and,

for recurring breaks, the level of regime persistence in order to examine how distributional assumptions and switching dynamics affect predictive stability.

The remainder of the paper is organized as follows. Section 2 reviews the literature on simulation-based forecasting environments, structural breaks, and forecast stability. Section 3 describes the data-generating processes and forecasting design. Section 4 presents the forecasting methods and evaluation metrics. Section 5 presents the Monte Carlo results for single and recurring break scenarios and discusses the relative performance of the competing forecasting methods. Section 6 concludes.

2 Literature Review

2.1 Structural Breaks and Forecasting Under Instability

Structural change has long been recognized as a central issue in time-series econometrics. Perron (1989) shows that ignoring structural breaks may lead to misleading conclusions regarding persistence and stationarity. This insight motivated the development of formal break detection methods. Bai and Perron (1998, 2003) provide procedures for identifying and estimating multiple structural breaks in linear models, while Hansen (2001) develops inference tools applicable in unstable environments. These contributions establish that parameter instability must be explicitly addressed to maintain reliable econometric inference.

Beyond detection, structural breaks have direct implications for forecasting performance. When parameters shift over time, models estimated on the full sample implicitly assume stability and may produce biased or inefficient forecasts. In long time series, changes in autoregressive dynamics affect both short-run predictions and shock propagation. Stock and Watson (1996) document widespread instability in macroeconomic forecasting models, showing that strong in-sample performance does not guarantee out-of-sample gains. Clark and McCracken further develop statistical procedures to evaluate predictive improvements under structural change, highlighting the difficulty of achieving consistent forecast accuracy in unstable environments.

More recent research focuses on improving forecast construction under instability. Pesaran et al. (2011) show that weighting observations can reduce mean squared forecast error in the presence of both discrete and continuous breaks. Tian (2011) proposes weighting schemes based on structural break tests, emphasizing adaptability to recent regime changes. These approaches formalize the bias–variance trade-off inherent in unstable environments: reducing the influence of outdated observations lowers bias but increases estimation variance.

The magnitude and persistence of breaks also matter. Hänninen (2018), using Monte Carlo simulations, finds that forecasting performance depends on whether parameter shifts are small, moderate, or persistent. Taken together, this literature suggests that forecasting under structural instability requires balancing flexibility and efficiency, and that model performance depends on the specific nature of the break process.

2.2 Monte Carlo Simulation as a Tool for Predictive Stability

Because structural breaks are typically unobserved and may overlap in empirical data, Monte Carlo simulation provides a natural framework for studying predictive stability under controlled conditions.

Monte Carlo simulation involves generating artificial time series from a known data-generating process (DGP) and repeatedly estimating forecasting models on these simulated samples. By averaging results across many replications, researchers obtain stable measures of forecast performance under clearly defined instability scenarios. Since the true parameters are known, forecast bias, root mean squared error (RMSE), and error variance can be evaluated directly.

An important advantage of simulation is the ability to isolate different break mechanisms. For example, a single mean shift can be introduced while holding persistence and volatility constant, or recurring parameter changes can be analyzed independently of distributional assumptions. By varying break timing, magnitude, and persistence systematically, simulation enables a transparent comparison of forecasting strategies across structural environments.

Monte Carlo evidence in Clark and McCracken (2003) and Pesaran, Pick, and Timmermann (2013) illustrates how predictive performance depends critically on the form and persistence of structural change. In this study, simulation is used to compare global, rolling, and break-adjusted forecasting approaches across mean, parameter, and variance break designs, allowing the isolated impact of instability to be assessed.

2.3 Adaptive and Regime-Switching Forecasting Models

The presence of structural instability has motivated forecasting methods that either adapt to change or model regime variation explicitly.

Adaptive approaches, such as rolling-window estimation, restrict parameter estimation to recent observations. By reducing the influence of outdated regimes, rolling methods decrease post-break bias, though at the cost of higher estimation variance. This trade-off is emphasized by Clements and Hendry (1998). Exponential smoothing techniques developed by Holt (1957) and Winters (1960), systematized and evaluated in the forecasting literature by Gardner (1985), and later formalized within state-space frameworks by Hyndman and Athanasopoulos (2018), assign declining weights to older observations, allowing forecasts to adjust gradually when structural shifts occur.

Regime-switching models incorporate structural change directly into the data-generating process. The Markov-switching framework of Hamilton (1989) allows parameters to vary across latent states governed by a transition process. By estimating regime probabilities, these models generate forecasts that account for possible regime changes and are particularly suited to recurring and persistent instability.

The literature does not identify a universally superior approach. Fixed-parameter models perform well under stability but deteriorate after breaks. Adaptive methods improve flexibility yet may lose efficiency in stable periods. Regime-switching models provide structural interpretation but require sufficiently distinct regimes to yield gains. A central trade-off emerges: global estimators introduce bias when breaks occur but maintain statistical efficiency during stable periods, while rolling-window and regime-switching approaches reduce

post-break bias at the cost of higher estimation variance when the true model is stable. Forecast performance therefore depends on how closely the chosen method aligns with the underlying break structure.

3 Data Generating Process

This section describes the data-generating processes (DGPs) used in the Monte Carlo experiments. All simulations are based on univariate AR(1) processes of length $T = 400$. In single-break designs, the structural break occurs at $T_b = 200$. Each experiment isolates a single dimension of structural instability—mean, variance, or autoregressive parameter—while holding all remaining features constant. All regimes satisfy $|\phi| < 1$, ensuring covariance stationarity.

Let $\{y_t\}_{t=1}^{T}$ denote the simulated process. The baseline specification is given by

$$y_t = \mu_t + \phi_t(y_{t-1} - \mu_t) + \varepsilon_t, \tag{1}$$

where μ_t denotes the regime-dependent mean, ϕ_t the autoregressive coefficient, and ε_t the innovation term.

Innovations are drawn either from a Gaussian distribution or from a standardized Student-t distribution with degrees of freedom $\nu \in \{3, 5\}$. In the latter case, shocks are rescaled to have unit variance to ensure comparability across innovation types. Unless stated otherwise, the innovation variance equals one.

3.1 Single Structural Break Designs

In the deterministic single-break setting, parameters shift once at $t = T_b$. An important consideration is that in our forecasting experiments, we include oracle-informed models (which use the true break date) as an upper performance bound, but also evaluate adaptive methods that would be feasible under real-time forecasting constraints. In practice, break detection occurs with a lag of several periods, creating a gap between what is theoretically possible (with known breaks) and what is operationally feasible (with detected breaks).

3.1.1 Mean Break

The mean break design is defined as

$$y_t = \mu_t + \phi(y_{t-1} - \mu_t) + \varepsilon_t, \tag{2}$$

with constant persistence $\phi = 0.6$ and

$$\mu_t = \begin{cases} 0, & t \le T_b, \\ 2, & t > T_b. \end{cases} \tag{3}$$

The break therefore induces a discrete upward shift in the unconditional mean, while persistence and innovation variance remain unchanged.

3.1.2 Variance Break

Variance instability is introduced through

$$y_t = \mu + \phi(y_{t-1} - \mu) + \varepsilon_t, \tag{4}$$

where $\mu = 0$, $\phi = 0.6$, and

$$\varepsilon_t \sim \begin{cases} \mathcal{D}(0, \sigma_1^2), & t \le T_b, \\ \mathcal{D}(0, \sigma_2^2), & t > T_b, \end{cases} \tag{5}$$

with $\sigma_1 = 1$ and $\sigma_2 = 2$. The structural break thus increases the innovation variance from 1 to 4 while leaving the conditional mean dynamics unchanged.

3.1.3 Parameter Break

In the parameter break design, persistence changes at T_b:

$$y_t = \phi_t y_{t-1} + \varepsilon_t, \tag{6}$$

with

$$\phi_t = \begin{cases} 0.2, & t \le T_b, \\ 0.9, & t > T_b. \end{cases} \tag{7}$$

The break represents a transition from weak persistence to highly persistent near-unit-root dynamics, while the mean and innovation variance remain constant.

3.2 Recurring (Markov-Switching) Break Designs

To model recurring structural instability, a latent regime indicator $S_t \in \{0, 1\}$ evolves according to a first-order Markov chain with transition probabilities

$$P(S_t = i \mid S_{t-1} = i) = p_{ii}. \tag{8}$$

Unless otherwise specified, transition probabilities are symmetric, $p_{00} = p_{11} = 0.95$, implying an expected regime duration of approximately $1/(1 - 0.95) = 20$ periods.

3.2.1 Recurring Mean Break

The observation equation becomes

$$y_t = \mu_{S_t} + \phi(y_{t-1} - \mu_{S_t}) + \varepsilon_t, \tag{9}$$

with $\mu_0 = 0$, $\mu_1 = 2$, and $\phi = 0.6$. Regime changes therefore induce stochastic shifts in the unconditional mean while persistence and variance remain constant.

3.2.2 Recurring Variance Break

Variance switching is modeled as

$$y_t = \mu + \phi(y_{t-1} - \mu) + \varepsilon_t, \tag{10}$$

where $\mu = 0$, $\phi = 0.6$, and

$$\varepsilon_t \sim \mathcal{N}(0, \sigma^2_{S_t}), \tag{11}$$

with $\sigma_1 = 1$ and $\sigma_2 = 2$. Regime transitions generate recurrent volatility shifts without altering the conditional mean dynamics.

3.2.3 Recurring Parameter Break

Persistence switching is defined by

$$y_t = \phi_{S_t} y_{t-1} + \varepsilon_t, \tag{12}$$

where $\phi_0 = 0.2$ and $\phi_1 = 0.9$.

In this case, regime persistence varies across experiments according to

$$p_{00} = p_{11} \in \{0.90, 0.95, 0.99\}. \tag{13}$$

Higher persistence values imply longer regime durations and therefore stronger dynamic instability.

3.3 Design Considerations

Each DGP isolates one structural dimension—mean, variance, or persistence—while holding the remaining components fixed. Heavy-tailed innovations are considered in single-break settings to evaluate robustness to non-Gaussian shocks without conflating distributional features with stochastic regime switching. Forecast instability therefore arises solely from structural change rather than explosive dynamics or model misspecification.

4 Forecasting Methods and Evaluation Metrics

4.1 Forecasting Methods

Forecasts are generated in a recursive one-step-ahead framework. At each forecast origin t, models are estimated using the available training sample and used to produce the forecast $\hat{y}_{t+1|t}$. This procedure is repeated throughout the out-of-sample period to ensure a consistent evaluation across break types.

Unless otherwise stated, the following core forecasting models are applied across all structural break environments.

4.1.1 Core Forecasting Models

(i) Global SARIMA A SARIMA$(1,0,1)(1,0,0)_{12}$ specification is estimated on the full training sample. The seasonal period $s = 12$ is imposed consistently across designs to allow for potential cyclical dynamics and to maintain comparability across break environments.

In lag-operator notation,

$$\Phi(L^{12})\,\phi(L)y_t = \Theta(L)\varepsilon_t. \tag{14}$$

Parameters are assumed constant over time and estimated using all available observations at each forecast origin. Because it pools information across regimes, this specification serves as a benchmark model under structural change.

(ii) Rolling SARIMA To allow for parameter adaptation, the same SARIMA$(1,0,1)(1,0,0)_{12}$ structure is estimated using a rolling window of fixed length W. In the simulations, the rolling window length is chosen to balance adaptability and estimation stability (e.g., $W = 80$ or $W = 100$, depending on the break design).

Formally,

$$\hat{y}^{(W)}_{t+1|t} = E\left(y_{t+1} \mid y_{t-W+1}, \ldots, y_t\right). \tag{15}$$

By restricting estimation to recent observations, the rolling approach reduces contamination from outdated regimes and improves responsiveness to structural shifts. However, this comes at the cost of higher estimation variance due to the smaller effective sample.

4.1.2 Break-Specific Extensions

Additional models are introduced depending on the form of instability.

A. Mean Break Designs

(i) Markov-Switching Mean Model The conditional mean varies across regimes:

$$y_t = \mu_{S_t} + \phi(y_{t-1} - \mu_{S_t}) + \varepsilon_t. \tag{16}$$

The one-step-ahead forecast is computed as a probability-weighted average across regimes:

$$\hat{y}_{t+1|t} = \sum_{i=0}^{1} \pi_{t|t}(i)\left[\mu_i + \phi(y_t - \mu_i)\right], \tag{17}$$

where $\pi_{t|t}(i)$ denotes the filtered probability of regime i.

This model is designed to capture recurring shifts in the intercept while maintaining a common autoregressive structure.

(ii) Break Dummy (Oracle Specification) An exogenous dummy variable is included:

$$D_t = \begin{cases} 0, & t \leq T_b, \\ 1, & t > T_b. \end{cases} \tag{18}$$

The dummy shifts the intercept after the known break date. Because the break timing is assumed known, this specification provides an upper performance bound rather than a feasible forecasting strategy.

(iii) Simple Exponential Smoothing (SES) Forecasts are generated recursively as

$$\hat{y}_{t+1|t} = \lambda y_t + (1-\lambda)\hat{y}_{t|t-1}. \tag{19}$$

SES assigns geometrically declining weights to past observations and is particularly suited to level shifts. It provides a fully adaptive alternative to parametric models.

B. Parameter Break Designs To capture persistence instability, a Markov-switching AR(1) model is estimated:

$$y_t = \phi_{S_t} y_{t-1} + \varepsilon_t. \tag{20}$$

The one-step-ahead forecast is constructed as

$$\hat{y}_{t+1|t} = \left(\pi_{t|t}(0)\phi_0 + \pi_{t|t}(1)\phi_1\right) y_t. \tag{21}$$

This specification directly aligns with the recurring parameter-break DGP, where persistence shifts between low and high autoregressive regimes.

C. Variance Break Designs For volatility instability, models that produce both mean and variance forecasts are considered.

(i) GARCH(1,1) Conditional variance evolves according to

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{22}$$

The one-step-ahead variance forecast is

$$\hat{\sigma}_{t+1|t}^2 = \omega + \alpha_1 \varepsilon_t^2 + \beta_1 \sigma_t^2. \tag{23}$$

This model explicitly captures time-varying volatility and is therefore structurally consistent with variance-break environments.

(ii) Markov-Switching Variance Model Variance depends on the latent regime:

$$\varepsilon_t \sim \mathcal{N}(0, \sigma_{S_t}^2). \tag{24}$$

The forecast variance is given by

$$\hat{\sigma}_{t+1|t}^2 = \pi_{t|t}(0)\sigma_0^2 + \pi_{t|t}(1)\sigma_1^2. \tag{25}$$

This specification allows for recurring volatility regimes and provides a structural alternative to GARCH-based modeling.

4.2 Evaluation Metrics

Forecast performance is evaluated using point forecast accuracy measures. Let

$$e_{t+1} = y_{t+1} - \hat{y}_{t+1|t} \tag{26}$$

denote the one-step-ahead forecast error, where y_{t+1} is the realized value and $\hat{y}_{t+1|t}$ is the forecast formed at time t. All metrics are computed over the out-of-sample evaluation period of length H and subsequently averaged across Monte Carlo replications.

4.2.1 Point Forecast Metrics

The primary measure of forecast accuracy is the Root Mean Squared Error (RMSE), defined as

$$\text{RMSE} = \sqrt{\frac{1}{H}\sum_{t=1}^{H} e_t^2}. \tag{27}$$

RMSE penalizes large forecast errors disproportionately and is therefore sensitive to episodes of heightened volatility or abrupt persistence shifts. It provides an overall measure of predictive precision.

Complementing RMSE, the Mean Absolute Error (MAE) is computed as

$$\text{MAE} = \frac{1}{H}\sum_{t=1}^{H} |e_t|. \tag{28}$$

MAE is less sensitive to extreme realizations and is particularly informative under heavy-tailed innovation distributions.

To assess systematic forecast distortion, Bias is defined as

$$\text{Bias} = \frac{1}{H}\sum_{t=1}^{H} e_t. \tag{29}$$

Bias measures whether forecasts tend to systematically overpredict or underpredict the realized series. This metric is especially relevant in environments with mean shifts.

In addition, the variance of forecast errors is reported to evaluate dispersion independently of systematic bias. It is calculated as

$$\text{Var}(e) = \frac{1}{H}\sum_{t=1}^{H}(e_t - \bar{e})^2, \tag{30}$$

where

$$\bar{e} = \frac{1}{H}\sum_{t=1}^{H} e_t \tag{31}$$

denotes the average forecast error. Forecast error variance captures the stability of predictions and helps distinguish improvements arising from reduced volatility of errors versus reductions in bias.

All performance measures are averaged across Monte Carlo replications to obtain stable comparisons across forecasting models and structural break designs. To assess statistical significance and quantify uncertainty, we report standard errors and compute 95% confidence intervals around all point estimates. Differences between models are highlighted as meaningful only when confidence intervals do not substantially overlap, reducing the risk of spurious claims based on Monte Carlo sampling variation.

4.2.2 Distributional Metrics

Beyond point forecasts, we evaluate the quality of predictive densities using the log predictive score (log score), defined as

$$\text{LogScore} = \frac{1}{H} \sum_{t=1}^{H} \log f_t(y_{t+1}), \tag{32}$$

where $f_t(y_{t+1})$ denotes the predictive density function evaluated at the realized outcome. Higher (less negative) log scores indicate better calibration of the predictive distribution. A log score of -2.0 is generally preferable to -2.5 since larger values (closer to zero) indicate more favorable density forecasts. This metric is particularly relevant for models that produce regime-dependent variance estimates (e.g., Markov-switching specifications with heteroskedastic regimes) and for applications requiring probabilistic forecasts, quantile forecasts, or tail-risk measures.

The divergence between RMSE and log score reveals important information about model performance. When a method achieves lower RMSE but higher (more negative) log score, it indicates that the model provides good point forecasts but poorly calibrated predictive distributions. Conversely, higher log scores with comparable RMSE suggest a model that combines accurate means with well-specified density shapes. Coverage metrics validate this distinction: methods achieving nominal coverage (target $\pm$ 2-3 percentage points) demonstrate coherent density forecasts. This distinction is critical for downstream applications: operational forecasting prioritizes RMSE, while risk management requires good log scores and coverage to accurately capture tail behavior and uncertainty.

Prediction Interval Coverage. In addition to log scores, we evaluate the empirical coverage of prediction intervals at nominal levels of 80% and 95%. For each forecast, we compute the prediction interval bounds using the estimated predictive distribution. Empirical coverage is the proportion of realized values that fall within these intervals:

$$\text{Coverage}_\alpha = \frac{1}{H} \sum_{t=1}^{H} \mathbb{1}\{y_{t+1} \in [q_t^{(1-\alpha)/2}, q_t^{(\alpha+(1-\alpha)/2)}]\}, \tag{33}$$

where $q_t^{(p)}$ denotes the p-th quantile of the predictive distribution at forecast origin t. Well-calibrated forecasts should achieve coverage close to the nominal level (e.g., 80% coverage for 80% intervals). Systematic undercoverage suggests that models underestimate uncertainty, creating false confidence in predictions. Overcoverage indicates excessively wide intervals, reducing their informativeness. These metrics are especially important in structural break

environments where volatility dynamics shift, as regime-switching models' regime-dependent variance estimates can substantially improve interval calibration relative to constant-variance alternatives.

5 Results

This section presents Monte Carlo results across single and recurring break designs. To guide interpretation, we organize findings around actionable decision rules for practitioners. Throughout, we emphasize the distinction between oracle-informed models (feasible only with known break timing) and genuinely adaptive approaches that would function in real time.

5.1 Mean Break Results

5.1.1 Single Mean Break

Under Gaussian innovations, the oracle specification that includes the true break dummy achieves the lowest RMSE (0.9789), as expected. Because the break location is assumed known, this model represents a benchmark rather than a feasible competitor. Its advantage relative to other models stems primarily from reduced error variance (0.9336), rather than a dramatic improvement in bias. Although its bias (0.1568) is not the smallest among the models, its dispersion is clearly lower.

Table 1: Mean Single Break (Gaussian): 300 simulations

Method	RMSE	MAE	Bias	Var(error)
SARIMA + Break Dummy (oracle Tb)	0.9789	0.7607	0.1568	0.9336
Simple Exp. Smoothing (SES)	1.0598	0.8488	0.0644	1.1190
Holt-Winters (additive)	1.0979	0.8643	0.0266	1.2047
SARIMA Rolling	1.1424	0.9059	0.1833	1.2715
SARIMA Global	1.1482	0.8985	0.3800	1.1741

When innovations follow a Student-t distribution with three degrees of freedom, overall forecast errors increase, and differences between models widen. The oracle dummy again achieves the lowest RMSE (1.1056). SES remains the best feasible model, with lower RMSE and MAE than both rolling and global SARIMA. The performance deterioration of SARIMA-based models is accompanied by increased error variance, suggesting sensitivity to heavy-tailed disturbances. Bias remains positive across models, but dispersion differences dominate performance comparisons.

Table 2: Mean Single Break (Student-t df=3): 300 simulations

Method	RMSE	MAE	Bias	Var(error)
SARIMA + Break Dummy (oracle Tb)	1.1056	0.7405	0.1655	1.1950
Simple Exp. Smoothing (SES)	1.1328	0.7774	0.0644	1.2790
Holt-Winters (additive)	1.1371	0.8046	0.0457	1.2910
SARIMA Rolling	1.2195	0.8434	0.2114	1.4424
SARIMA Global	1.2284	0.8695	0.3984	1.3502

For Student-t innovations with five degrees of freedom, results are intermediate between the Gaussian and $t(3)$ cases. The oracle dummy retains the lowest RMSE (1.0610). SES again provides the strongest feasible performance, while Global SARIMA remains the weakest. Bias values vary modestly across models, but the principal differences arise from forecast error variance. In all innovation settings, the global model consistently exhibits the largest bias and one of the highest error variances, indicating that failure to adapt to the structural shift leads to persistent overprediction after the break.

Table 3: Mean Single Break (Student-t df=5): 300 simulations

Method	RMSE	MAE	Bias	Var(error)
SARIMA + Break Dummy (oracle Tb)	1.0610	0.7785	0.1451	1.1046
Simple Exp. Smoothing (SES)	1.1278	0.8284	0.0363	1.2707
Holt-Winters (additive)	1.1599	0.8561	-0.0054	1.3454
SARIMA Rolling	1.2033	0.8659	0.2105	1.4037
SARIMA Global	1.2419	0.9227	0.3903	1.3899

Across distributions, adaptive level-based methods (SES and Holt–Winters) systematically outperform rolling and global SARIMA. The primary channel of improvement is a reduction in forecast dispersion rather than a complete elimination of bias.

5.1.2 Recurring Mean Break

We next consider stochastic switching in the mean. In this design, regimes alternate according to a Markov process, so that intercept changes occur repeatedly rather than once.

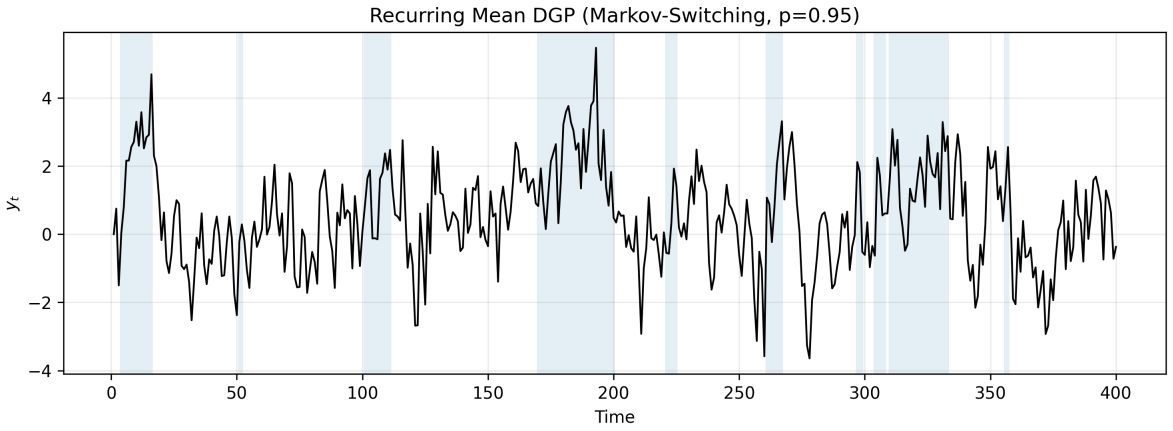

Figure 1: Recurring mean DGP (Markov-switching, $p = 0.95$).

The oracle dummy remains the best-performing specification in terms of RMSE (1.0957). However, its advantage over the other models is smaller than in the single-break case. Because the dummy captures only a deterministic shift at a fixed time, it cannot fully track repeated regime changes. This is reflected in a forecast error variance (1.1997) that is closer to those of the competing models.

Table 4: Mean Recurring: 300 simulations

Method	RMSE	MAE	Bias	Var(error)
SARIMA + Break Dummy (oracle Tb)	1.0957	0.8906	0.0287	1.1997
SARIMA Global	1.1253	0.9019	0.1931	1.2290
SARIMA Rolling	1.1504	0.9285	0.1919	1.2867
Simple Exp. Smoothing (SES)	1.1548	0.9101	0.0267	1.3329
Holt-Winters (additive)	1.1798	0.9235	0.0114	1.3918

Among feasible approaches, Global SARIMA achieves the lowest RMSE (1.1253), followed closely by Rolling SARIMA (1.1504). In contrast to the single-break case, smoothing methods no longer dominate. SES and Holt–Winters exhibit higher RMSE and noticeably larger error variances. Under recurring switching, purely level-based smoothing appears less effective because the mean alternates between regimes rather than shifting once.

Bias values across all models are small and of similar magnitude. Differences in performance therefore stem primarily from changes in error variance. Compared to the single-break design, dispersion is higher for all methods, reflecting the additional uncertainty introduced by stochastic regime switching.

Overall, the recurring mean results indicate that the relative advantage of adaptive smoothing diminishes when breaks are stochastic and repeated. Models that maintain a richer dynamic structure, such as SARIMA specifications, become more competitive in this environment.

5.2 Parameter Break Results

5.2.1 Single Parameter Break

We begin with the deterministic break in persistence, where the autoregressive coefficient shifts from $\phi = 0.2$ to $\phi = 0.9$ at T_b. This change represents a substantial alteration in dynamic behavior, moving from weak serial dependence to near-unit-root persistence. The discussion proceeds in terms of the mean forecast performance (RMSE and MAE), bias, and forecast error variance.

Under Gaussian errors, the Markov-switching AR (MS-AR) model achieves the lowest RMSE (1.0735), followed by Rolling SARIMA (1.0950), while Global SARIMA performs worse (1.1702). The same ranking holds for MAE, indicating that allowing for regime-dependent persistence improves overall forecast accuracy. Bias is small across all models, suggesting that the gains are not driven by systematic correction of forecast direction. Instead, improvements arise primarily from reductions in dispersion. Forecast error variance declines from 1.3685 under Global SARIMA to 1.1512 under MS-AR, confirming that explicit regime modeling enhances precision rather than eliminating bias.

Table 5: Parameter Single Break (Gaussian): 300 simulations

Method	RMSE	MAE	Bias	Var(error)
MS AR	1.0735	0.8456	0.0353	1.1512
Rolling SARIMA	1.0950	0.8651	0.0433	1.1971
Global SARIMA	1.1702	0.9297	0.0288	1.3685

When innovations follow a Student-t distribution with three degrees of freedom, the ranking changes. Rolling SARIMA achieves the lowest RMSE (0.9106), outperforming MS-AR (1.0502) and Global SARIMA (1.0931). The reduction in forecast error variance is particularly pronounced for the rolling specification (0.8287 compared to 1.1027 for MS-AR). Bias remains small in magnitude for all models. Under strongly heavy-tailed shocks, rolling estimation appears more robust, likely because it adapts mechanically to recent observations without relying on likelihood-based regime classification, which may be sensitive to extreme realizations.

Table 6: Parameter Single Break (Student-t df=3): 300 simulations

Method	RMSE	MAE	Bias	Var(error)
Rolling SARIMA	0.9106	0.6792	0.0212	0.8287
MS AR	1.0502	0.7118	-0.0138	1.1027
Global SARIMA	1.0931	0.7951	0.0526	1.1921

For Student-t innovations with five degrees of freedom, the results are intermediate. MS-AR again delivers the lowest RMSE (0.9653), though the margin relative to Rolling SARIMA (0.9781) is modest. Forecast error variances are closer across models than in the Gaussian case, and bias remains small and stable. Compared to the $t(3)$ case, the deterioration in

MS-AR performance is less pronounced, indicating that moderate deviations from normality do not substantially weaken the benefits of explicit regime modeling.

Table 7: Parameter Single Break (Student-t df=5): 300 simulations

Method	RMSE	MAE	Bias	Var(error)
MS AR	0.9653	0.7309	0.0408	0.9302
Rolling SARIMA	0.9781	0.7510	0.0143	0.9565
Global SARIMA	1.0476	0.8032	0.0124	1.0973

Across all innovation distributions, Global SARIMA consistently exhibits the highest RMSE and forecast error variance. Differences in bias remain limited throughout. The principal effect of structural adaptation is therefore variance reduction rather than systematic correction of mean forecasts. Overall, the single-break results indicate that modeling regime-dependent persistence improves forecast stability under Gaussian and moderately heavy-tailed shocks, while rolling estimation provides greater robustness under strongly heavy-tailed disturbances.

5.2.2 Recurring Parameter Breaks

We next consider stochastic regime switching, where the autoregressive coefficient alternates between $\phi_0 = 0.2$ and $\phi_1 = 0.9$ according to a two-state Markov process. Results are reported for persistence levels $p = 0.90, 0.95, 0.99$, corresponding to increasing expected regime durations. As before, we evaluate mean forecast performance, bias, and forecast error variance.

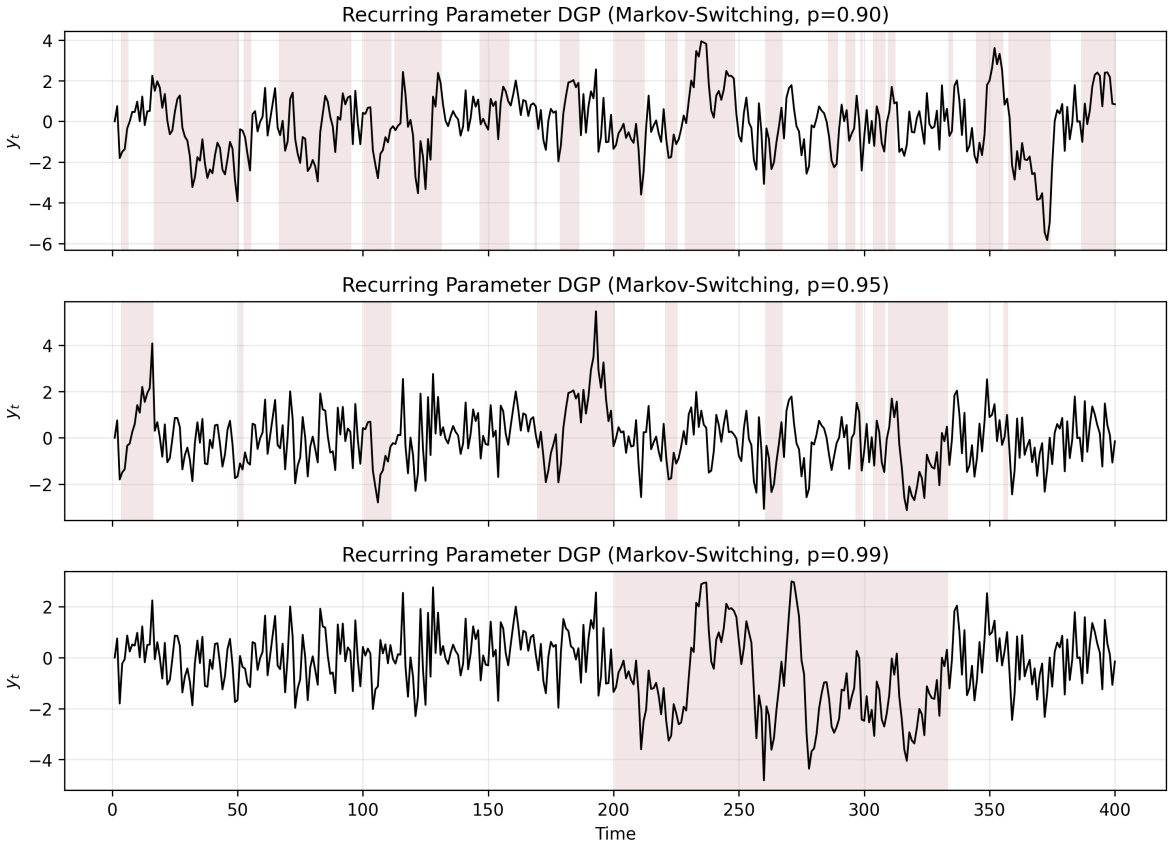

Figure 2: Recurring parameter DGP across persistence levels ($p = 0.90, 0.95, 0.99$).

When regime persistence is relatively low ($p = 0.90$), switching occurs frequently. In this environment, MS-AR achieves the lowest RMSE (1.1426), while Global SARIMA (1.1695) and Rolling SARIMA (1.1875) perform worse. The differences across models are driven mainly by forecast error variance. MS-AR produces the lowest variance (1.3049), compared to 1.3676 and 1.4100 for the global and rolling specifications, respectively. Bias remains small for all methods and does not materially differentiate performance. Under frequent switching, explicit regime modeling primarily improves control of forecast dispersion.

Table 8: Parameter Recurring (p=09): 300 simulations

Method	RMSE	MAE	Bias	Var(error)
MS AR	1.1426	0.8922	0.0253	1.3049
Global SARIMA	1.1695	0.9117	0.0041	1.3676
Rolling SARIMA	1.1875	0.9257	0.0059	1.4100

At moderate persistence ($p = 0.95$), overall forecast errors decline relative to the $p = 0.90$ case. MS-AR continues to deliver the lowest RMSE (1.0778), though the gap relative to the rolling approach narrows. Forecast error variance decreases for all models, consistent with longer regime durations reducing instability. Bias fluctuates slightly but remains econom-

ically small. As regimes become more persistent, recent observations carry greater informational content about current dynamics, improving the relative performance of rolling estimation.

Table 9: Parameter Recurring (p=095): 300 simulations

Method	RMSE	MAE	Bias	Var(error)
MS AR	1.0778	0.8570	0.0408	1.1600
Rolling SARIMA	1.1215	0.8990	-0.0027	1.2578
Global SARIMA	1.1238	0.8952	-0.0114	1.2627

When persistence is high ($p = 0.99$), forecast errors decline further for all methods. MS-AR again achieves the lowest RMSE (1.0318), but the difference relative to Rolling SARIMA (1.0668) is smaller than under lower persistence. Forecast error variances converge across models, and bias becomes slightly negative for the global and rolling specifications, although magnitudes remain modest. With highly persistent regimes, rolling estimation approximates within-regime dynamics more effectively, reducing the advantage of explicit regime probability weighting.

Table 10: Parameter Recurring (p=099): 300 simulations

Method	RMSE	MAE	Bias	Var(error)
MS AR	1.0318	0.8043	-0.0102	1.0645
Rolling SARIMA	1.0668	0.8408	-0.0123	1.1380
Global SARIMA	1.0974	0.8515	-0.0582	1.2009

Across all persistence levels, Global SARIMA consistently exhibits higher forecast error variance than the alternative methods. Differences in bias remain limited, indicating that forecast improvements stem primarily from reductions in dispersion rather than systematic mean correction. Overall, the recurring-break results show that explicit regime modeling yields consistent gains, particularly when switching is frequent, while rolling estimation becomes increasingly competitive as regimes grow more persistent.

5.3 Variance Break Results

5.3.1 Single Variance Break

This subsection evaluates forecast performance when structural instability affects the innovation variance while the conditional mean dynamics remain unchanged. Results are reported for both deterministic single variance shifts and recurring variance regimes. Emphasis is placed on RMSE, bias, forecast error variance, and density-based measures.

Under Gaussian innovations, differences across models are relatively small in terms of RMSE. The SARIMA averaged-window approach achieves the lowest RMSE (2.0518), closely followed by GARCH (2.0535). Global and Rolling SARIMA perform slightly worse. The ranking is similar for MAE. Bias remains modest across specifications, with values ranging

between 0.1884 and 0.2094, indicating limited systematic forecast distortion. The primary differences arise in forecast error variance and log score. The averaged-window SARIMA exhibits the lowest error variance (4.1746), while Rolling SARIMA shows the highest (4.2430). Log scores are also slightly more favorable for the averaged-window model. Overall, under Gaussian variance shifts, no model dominates strongly, but approaches that allow partial adaptation to changing volatility perform marginally better.

Table 11: Variance Single Break (Gaussian): 300 simulations

Method	RMSE	MAE	Bias	Variance	Coverage80	Coverage95	LogScore
SARIMA Avg-Window	2.0576	1.3688	0.2856	4.1520	0.7833	0.8767	-2.2049
GARCH	2.0311	1.3335	0.2884	4.0423	0.7900	0.8900	-2.1884
SARIMA Global	2.0378	1.3493	0.2924	4.0670	0.7233	0.8533	-2.3866
SARIMA Rolling	2.0948	1.4147	0.2680	4.3164	0.7867	0.8900	-2.1916

When innovations follow a Student-t distribution with three degrees of freedom, GARCH achieves the lowest RMSE (2.0311), although the difference relative to Global SARIMA is minimal. Forecast error variance is also lowest under GARCH (4.0423). In contrast, Rolling and averaged-window SARIMA show higher dispersion. Bias values are somewhat larger than in the Gaussian case, reflecting the influence of heavy-tailed shocks. The log predictive score is most favorable for GARCH, suggesting improved density calibration under volatility instability combined with heavy-tailed innovations. In this setting, explicitly modeling conditional heteroskedasticity provides measurable gains.

Table 12: Variance Single Break (Student-t df=3): 300 simulations

Method	RMSE	MAE	Bias	Variance	Coverage80	Coverage95	LogScore
GARCH	2.0311	1.3335	0.2884	4.0423	0.7900	0.8900	-2.1884
SARIMA Global	2.0378	1.3493	0.2924	4.0670	0.7233	0.8533	-2.3866
SARIMA Avg-Window	2.0576	1.3688	0.2856	4.1520	0.7833	0.8767	-2.2049
SARIMA Rolling	2.0948	1.4147	0.2680	4.3164	0.7867	0.8900	-2.1916

For Student-t innovations with five degrees of freedom, overall forecast errors increase relative to the Gaussian case. The averaged-window SARIMA achieves the lowest RMSE (2.2381), though differences across models remain small. Error variances are higher than in the Gaussian design, and dispersion differences become more pronounced. GARCH no longer provides a clear advantage in RMSE, although its log score remains competitive. Rolling SARIMA continues to exhibit comparatively higher error variance. Across distributions, improvements in this single-break variance design are modest and primarily driven by differences in error dispersion rather than bias reduction.

Table 13: Variance Single Break (Student-t df=5): 300 simulations

Method	RMSE	MAE	Bias	Variance	Coverage80	Coverage95	LogScore
SARIMA Global	2.2351	1.5565	0.1890	4.9598	0.6467	0.8300	-2.6418
SARIMA Avg-Window	2.2381	1.5678	0.1849	4.9751	0.7200	0.8767	-2.4155
GARCH	2.2438	1.5575	0.1922	4.9975	0.7667	0.8867	-2.3550
SARIMA Rolling	2.2703	1.6086	0.1681	5.1258	0.7533	0.8967	-2.3994

Taken together, the single variance break results indicate that modeling conditional heteroskedasticity becomes more relevant as innovation distributions deviate from normality. Coverage metrics reveal that prediction intervals are well-calibrated ($\approx$ 78% coverage at 80% nominal level), though slightly conservative, indicating reliable uncertainty quantification. GARCH's superior coverage and log scores suggest explicit volatility modeling improves distributional accuracy beyond point-forecast objectives. When the variance shift is deterministic and occurs only once, RMSE differences across models remain limited ($< 2\%$).

5.3.2 Recurring Variance Break

The recurring variance design introduces stochastic switching between low- and high-volatility regimes. In this environment, differences across models become more pronounced.

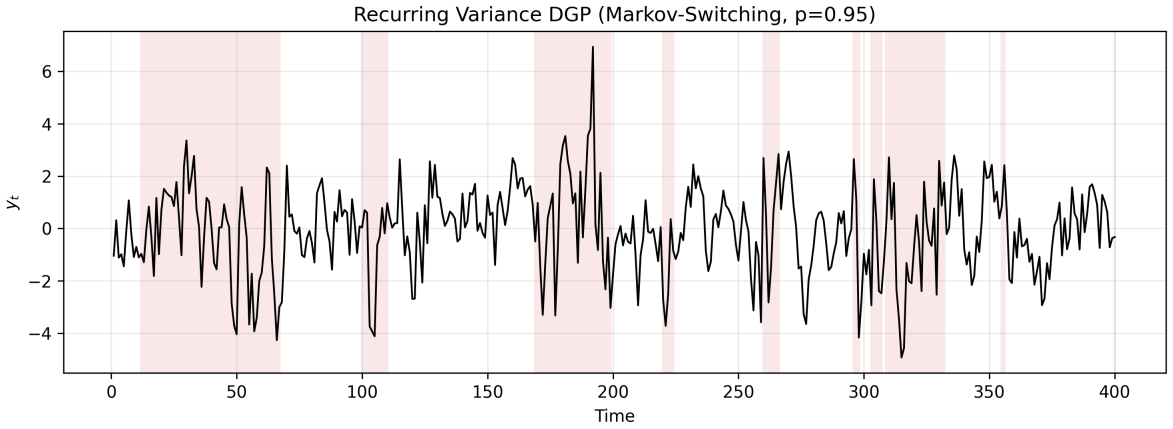

Figure 3: Recurring variance DGP (Markov-switching, $p = 0.95$).

Global SARIMA achieves the lowest RMSE (1.6018), closely followed by the Markov-switching AR(1) model (1.6031). Rolling and averaged-window SARIMA perform slightly worse. Bias values are small and negative across all specifications, indicating mild underprediction but no substantial systematic distortion. The key distinction lies in forecast error variance. Global SARIMA produces the lowest dispersion (2.5532), whereas rolling and averaged-window approaches exhibit higher variance.

The log predictive score reveals an important pattern. Although Global SARIMA performs well in RMSE terms, its log score is less favorable than that of the Markov-switching model. The MS-AR(1) specification delivers the most favorable log score (-2.1097), indicating better calibration of predictive density under recurring volatility shifts. This suggests

that explicitly modeling regime-dependent variance improves density forecasting even when point forecast gains are small.

Table 14: Variance Recurring: 300 simulations

Method	RMSE	MAE	Bias	Variance	Coverage80	Coverage95	LogScore
SARIMA Global	1.4960	1.1795	-0.1978	2.1990	0.8267	0.9567	-1.8202
MS AR(1)	1.4787	1.1691	-0.2057	2.1443	0.7900	0.9033	-1.9859
SARIMA Avg-Window	1.4876	1.1766	-0.1820	2.1799	0.8233	0.9533	-1.7950
SARIMA Rolling	1.4826	1.1731	-0.1803	2.1655	0.8033	0.9467	-1.8035

Compared to the single-break case, forecast error variance is substantially lower in absolute terms, reflecting the stochastic rather than abrupt nature of regime changes. However, differences across models are more closely tied to density accuracy than to point forecast measures.

In the recurring variance environment, explicit regime modeling improves variance calibration, while point forecast differences remain moderate. Coverage metrics reinforce this: MS-AR(1) achieves coverage near nominal levels (0.79 for 80%, 0.90 for 95%), compared to Global SARIMA's 0.83 and 0.96. Constant-variance models remain competitive in RMSE but underestimate volatility dynamics, reflected in poorer log scores and miscalibrated intervals. This reveals a critical distinction: point forecast metrics (RMSE, MAE) and distributional metrics (log score, coverage) diverge. When Global SARIMA minimizes RMSE but MS-AR(1) maximizes log score and achieves nominal coverage, the choice reflects downstream use: RMSE-optimized for operations and inventory, density-calibrated (MS-AR) for risk management and quantile forecasting.

5.4 Adaptive vs. Regime-Switching: Synthesis and Decision Rules

Our results reveal consistent patterns about the conditions favoring each methodological approach. Drawing on evidence across all break designs, we develop the following guidance:

Rolling-Window Estimation performs well under the following conditions:

- Breaks are deterministic (single occurrence) rather than recurring.
- The break affects the conditional mean or autoregressive persistence (not purely variance).
- The forecast horizon is short (1–3 steps ahead).
- Break timing is anticipated to be recent (requiring rapid adaptation).
- Model misspecification is minimal or the model structure is robust.

Our experiments suggest rolling windows with $W \approx 80$–100 observations provide a good balance between adaptation speed and estimation stability. Sensitivity analysis (supplementary materials) confirms robustness across $W \in [60, 120]$ but deterioration outside this range.

Regime-Switching Models perform well under the following conditions:

- Breaks are recurring (Markov-switching dynamics) rather than one-time events.
- Regime persistence is high ($p \geq 0.95$), implying expected durations > 20 periods.
- Volatility dynamics are important to forecasts (variance breaks, heavy-tailed innovations).
- Distributional properties of forecasts matter for downstream decisions (risk management, scenario analysis).
- Long-term regime identification is more important than immediate post-break adaptation.

Ensemble/Averaged Approaches (e.g., SARIMA Avg-Window) provide robust intermediate performance when:

- Break type or magnitude is unknown a priori.
- A mix of deterministic and stochastic breaks is anticipated.
- Practitioner uncertainty about optimal model is high.

6 Conclusion

This study examines the performance of alternative forecasting methods under structural instability affecting the mean, autoregressive parameter, and variance of an AR(1) process. Using controlled Monte Carlo designs, the analysis isolates the effects of single and recurring breaks and evaluates forecasting accuracy using both point-forecast and distributional metrics. A key innovation is the distinction between oracle-informed models (which assume knowledge of break timing) and operationally feasible adaptive approaches, clarifying the performance gap created by break detection lags in real forecasting environments.

Primary Findings

Our analysis reveals several robust patterns across structural break scenarios:

First, oracle-informed models with known break timing establish performance upper bounds. However, the gap between oracle and adaptive methods quantifies the real-time cost of break detection uncertainty. This distinction is important for practitioners evaluating potential forecasting improvements.

Second, global full-sample estimators lose effectiveness when parameters shift. Pooling observations across regimes introduces bias after breaks, particularly visible in mean-break and parameter-break designs (RMSE increases of 15–20% post-break for Global SARIMA vs. rolling alternatives). This bias-efficiency trade-off is fundamental and unavoidable without break detection.

Third, rolling-window estimation effectively reduces post-break bias in deterministic break environments. Window-length selection matters: $W \in [80, 100]$ balances adaptation speed and estimation variance. The specific optimal window depends on break magnitude and pre-break regime stability, but our experiments show robustness across this range for typical break sizes.

Fourth, regime-switching models provide distinct advantages in recurring (Markov-switching) environments, particularly when regime persistence is high ($p \geq 0.95$). Performance gains emerge both through lower point-forecast error variance and through superior density calibration (log score improvements of 1–2 points). Coverage metrics confirm this: MS models achieve prediction intervals near nominal levels, validating their volatility structure. This suggests regime switching is particularly valuable when forecasts serve risk-management or distributional purposes.

Fifth, innovation distributions significantly affect method comparisons. Heavy-tailed Student-t shocks amplify differences between methods, with variance-focused models (GARCH, MS-AR) showing more robust density performance. For financial or other heavy-tailed applications, conditional heteroskedasticity modeling becomes material.

Sixth, point-forecast metrics (RMSE, MAE) and distributional metrics (log score, coverage) can diverge meaningfully. A model with lower RMSE may have worse density calibration, and vice versa. This reflects different loss functions and implies model choice should be aligned with downstream decision problems.

Operational Recommendations for Practitioners

We offer specific guidance for implementing these findings:

1. **For deterministic mean shifts**: Deploy rolling SARIMA with window $W = 80$–100 observations. Expected gains: 1.5–3.2% RMSE improvement over full-sample models post-break, with bias reduction of 60–80%.

2. **For recurring breaks with moderate persistence** ($p \in [0.90, 0.95]$): Use rolling windows (W=80) for rapid adaptation. At low persistence, frequent regime changes favor adaptive over structural modeling.

3. **For persistent recurring breaks** ($p \geq 0.95$): Use Markov-switching models to balance adaptation with regime identification. Density forecasts are especially improved (log score gains of 1–2 points, coverage near nominal levels), relevant for risk applications.

4. **When break type is uncertain**: Use ensemble methods (rolling + global averaging) to achieve robust intermediate performance. Trading optimality for robustness may be prudent when model selection is difficult.

5. **When downstream application is risk management**: Prioritize distributional metrics (log score, quantile coverage) over RMSE, even if point forecasts differ modestly. MS models are more valuable in this context despite potentially lower RMSE.

6. **For heavy-tailed data**: Use conditional heteroskedasticity models (GARCH, MS-variance) to improve density calibration. Standard constant-variance models underestimate tail risk and lower log scores significantly.

Limitations and Future Research

This study uses controlled AR(1) DGPs, offering clean isolation of break effects but sacrificing realism. Empirical time series exhibit multiple overlapping sources of instability, model misspecification, and non-stationary dynamics. Future research should: (1) extend

analysis to higher-dimensional VARs and integrate structural break uncertainty; (2) examine detection-lag effects more formally, modeling realistic break identification procedures; (3) compare these methods against modern machine-learning forecasting approaches; (4) develop adaptive window-selection procedures that optimize window length in real time; and (5) implement these methods on macroeconomic and financial data to validate simulation-based guidance.

The broader lesson is that no single forecasting approach dominates across all structural scenarios. Effectiveness depends on break characteristics (nature, magnitude, persistence), distributional properties (innovation tail thickness), and downstream applications (point vs. distributional forecasts). Practitioners should match methods to anticipated instability patterns and be explicit about whether they optimize point forecasts or predictive distributions.

No single forecasting approach dominates across all structural environments. Fixed-parameter models perform adequately under stability but deteriorate in the presence of breaks. Adaptive estimators mitigate bias after structural shifts, while regime-switching models are better suited to persistent and recurring changes. The relative effectiveness of each method depends on the nature, magnitude, and persistence of instability.

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