Structural Break Forecasting
A Comprehensive Monte Carlo Study
Variance, Mean, and Parameter Breaks in Time Series

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Motivation: Why Study Structural Breaks?

- **Pervasiveness:** Real economic/financial data exhibit discrete shifts in dynamics
  - Policy changes, financial crises, regime switches
  - Structural breaks violate stationarity assumptions
- **Forecasting Challenge:** Standard methods (global ARIMA) fail under instability
  - Averaging across regimes creates systematic bias
  - Need for adaptive methods (rolling windows, regime-switching)
- **Research Questions:**
  1. How do forecasting methods compare under different break types?
  2. What is the optimal window size for rolling estimators?
  3. How do heavy-tailed distributions affect accuracy?
  4. Can adaptive methods approach oracle performance?

Our Approach: Three Types of Structural Breaks

Break Type	Mechanism	Notation
Variance	Shift in volatility	$\sigma_1^2 \to \sigma_2^2$
Mean	Shift in level	$\mu_0 \to \mu_1$
Parameter	Shift in AR coefficient	$\phi_1 \to \phi_2$

Key Innovation

Systematic evaluation of **15+ forecasting methods** across all three break types, with comprehensive Monte Carlo evidence on **RMSE, MAE, Bias** metrics.

Deliverables:

- Optimized rolling window recommendations (Pesaran 2013)
- Robustness to heavy-tailed innovations (Student-t)
- Practical guidance on method selection

General AR(1) Framework

All simulations build on AR(1) with structural breaks at known points:

$$y_t = c_t + \phi_t y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_t^2)$$

Where c_t, ϕ_t, and σ_t^2 can shift at break point T_b:

$$\text{Level shift:} \quad c_t = \begin{cases} c_0 & t \leq T_b \\ c_1 & t > T_b \end{cases} \tag{1}$$

$$\text{Persistence shift:} \quad \phi_t = \begin{cases} \phi_1 & t \leq T_b \\ \phi_2 & t > T_b \end{cases} \tag{2}$$

$$\text{Volatility shift:} \quad \sigma_t^2 = \begin{cases} \sigma_1^2 & t \leq T_b \\ \sigma_2^2 & t > T_b \end{cases} \tag{3}$$

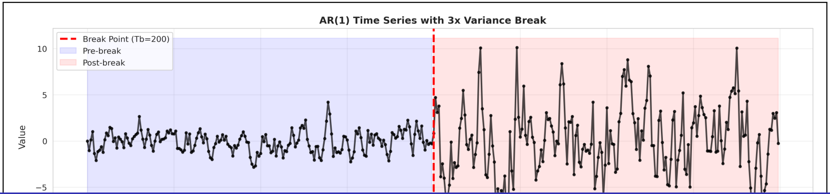
Variance Breaks: DGP

$$y_t = \phi y_{t-1} + \varepsilon_t, \quad \phi = 0.8 \text{ (stable)}$$

Volatility shift at $T_b = 200$ (mid-sample):

$$\varepsilon_t \sim \begin{cases} \mathcal{N}(0, \sigma_1^2) & t \leq 200 \\ \mathcal{N}(0, \sigma_2^2) & t > 200 \end{cases}$$

Parameters: $\sigma_1 = 1.0$, $\sigma_2 = 2.0$ ($2\times$ volatility increase)



Mean Breaks: DGP

$$y_t = \mu_t + \phi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0,1)$$

Single Break:

$$\mu_t = \begin{cases} \mu_0 = 0.0 & t \leq T_b \\ \mu_1 = 2.0 & t > T_b \end{cases}$$

Multiple Breaks (Seasonal Extension):

$$y_t = \mu_t + s_t + \phi y_{t-1} + \varepsilon_t$$

where $s_t = A \sin(2\pi t/s)$ adds periodic seasonality ($s = 12$ months, amplitude A)

Motivation: Real economic data (sales, demand) exhibit both mean shifts and seasonal patterns. SARIMA models explicitly capture seasonality.

Methods evaluated:

Parameter Breaks: DGP

$$y_t = \phi_t y_{t-1} + \varepsilon_t, \quad \varepsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2)$$

Persistence Shift (Single Break):

$$\phi_t = \begin{cases} \phi_1 = 0.2 & t \le T_b \\ \phi_2 = 0.9 & t > T_b \end{cases}$$

Recurring Breaks (Markov-Switching):

$$\phi_t \in \{\phi_1, \phi_2\} \text{ with regime persistence } p \in \{0.90, 0.95, 0.97, 0.995\}$$

Interpretation:

- Low persistence ($p = 0.90$): frequent regime switches $\Rightarrow$ hard to predict
- High persistence ($p = 0.995$): rare switches $\Rightarrow$ easier to detect/forecast

Method Overview

Category	Method	Adaptivity	Structural
2***Fixed**	Global ARIMA/ARMA	None	No
	Simple Exp. Smoothing	None	No
2***Adaptive**	Rolling Window	Window-based	No
	Pesaran Window Search	Grid-optimized	No
2***Structural**	Break Dummy (Oracle)	Perfect info	Yes
	Estimated Break	Grid-search	Yes
	Markov-Switching	Regime inference	Yes
2***NEW**	SARIMA (all variants)	Global + Rolling	Seasonal
	SARIMA + Breaks	Combined	Seasonal + Structural

Estimation:

- ARMA: AIC-based automatic order selection (Box-Jenkins)
- SARIMA: SARIMAX state-space with MLE
- Markov-Switching: EM algorithm with regime inference

Global Estimators

Global ARMA/SARIMAX: Fit once to entire training sample

$$y_t = c + \phi y_{t-1} + \varepsilon_t \quad \text{or} \quad \text{SARIMA}(p, d, q)(P, D, Q)_s$$

Advantage: Maximum sample size $\Rightarrow$ precise estimates under stability

Disadvantage: Biased under structural breaks (averages across regimes)

Benchmark role: Represents the "naive" approach; other methods try to improve upon it

Rolling Window Estimators

Idea: Re-estimate using only recent window of size w

$$\hat{\theta}_t =_\theta \sum_{j=t-w+1}^{t} (y_j - \hat{y}_j(\theta))^2$$

Forecast: $\hat{y}_{t+1} = \mathbb{E}(y_{t+1} | y_{t-w+1}, \ldots, y_t; \hat{\theta}_t)$

Advantages:

- Down-weights old regime data $\Rightarrow$ adapts to breaks
- Simple to implement; no break date specification needed

Trade-offs:

- Fewer observations $\Rightarrow$ higher estimation variance
- Optimal window size depends on break magnitude & location
- **Solution:** Grid search over windows (Pesaran 2013)

Break Dummy Methods

Oracle: Break date T_b is **known**. Include dummy as exogenous:

$$y_t = c + \phi y_{t-1} + \delta \cdot 1\{t > T_b\} + \varepsilon_t$$

Estimated Break: Grid search to find $\hat{T}_b$:

$$\hat{T}_b =_{T_b \in [T_{\min}, T_{\max}]} (\mathrm{SSE}_{\mathrm{pre}}(T_b) + \mathrm{SSE}_{\mathrm{post}}(T_b))$$

where pre-/post-break segments fit independently.

Advantages:

- Explicitly models intercept/mean shift
- Oracle version gives upper bound on adaptive method performance
- Estimated version is automatic (no window specification)

Limitations:

Markov-Switching AR Models

Regime-switching structure: AR coefficient varies by latent state

$$y_t = \phi_{s_t} y_{t-1} + \varepsilon_t, \quad s_t \in \{1, 2\}$$

Transition probabilities:

$$P(s_t = j | s_{t-1} = i) = p_{ij}, \quad p_{ii} \in \{0.90, 0.95, 0.97, 0.995\}$$

Forecast: Condition on filtered regime probabilities at t:

$$\hat{y}_{t+1} = \mathbb{E}(\phi_{s_{t+1}} y_t) = \sum_{j=1}^{2} P(s_{t+1} = j | y_{1:t}) \cdot \phi_j y_t$$

Advantages:

- Probabilistic regime inference (automatic)

SARIMA Methods (Bakhodir's Contribution)

Seasonal ARIMA: Jointly models level shifts AND seasonal patterns

$$\text{SARIMA}(p, d, q)(P, D, Q)_s$$

Implemented as state-space SARIMAX with exogenous break indicators:

Variants implemented:

1. **SARIMA Global:** Fit to full sample (baseline)
2. **SARIMA Rolling:** Fit rolling window of size w
3. **SARIMA + Break Dummy:** Exog regressor for shift (oracle)
4. **SARIMA + Est. Break:** Grid-estimated break point

Why SARIMA for mean breaks?

- Many economic variables are seasonal (sales, employment, etc.)
- SARIMA captures both trend AND seasonality simultaneously
- Outperforms ARMA when seasonal structure present

Experimental Setup

For each of $N = 200$ replications:

1. **Generate:** Time series $\{y_t^{(i)}\}_{t=1}^{T}$ from DGP with break at T_b
2. **Split:** Training sample $\{y_1, \ldots, y_{t_0-1}\}$ where $t_0 = T_b + g$ (gap after break)
3. **Estimate:** Each method on training data, compute 1-step forecast $\hat{y}_{t_0}^{(i)}$
4. **Error:** $e_i^{(j)} = y_{t_0}^{(i)} - \hat{y}_{t_0}^{(i,j)}$ for method j

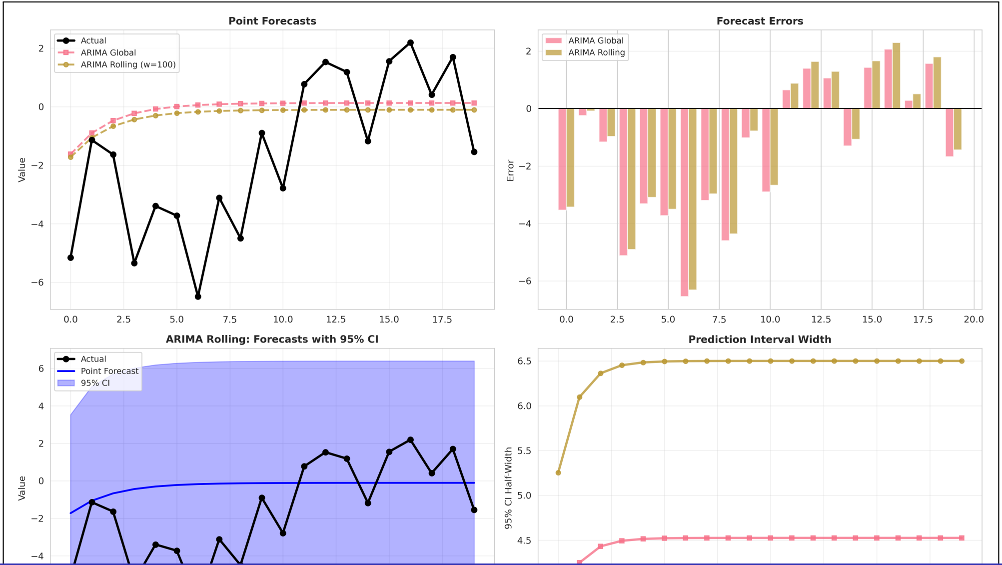
Evaluation Metrics (computed over 200 errors):

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_i (e_i)^2}, \quad \text{MAE} = \frac{1}{N}\sum_i |e_i|, \quad \text{Bias} = \frac{1}{N}\sum_i e_i$$

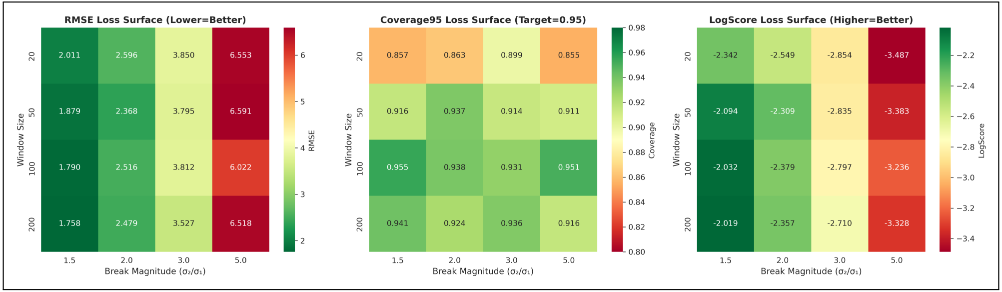
Sensitivity analyses:

- Vary break magnitude, window size, persistence level, innovation distribution
- Report

Variance Breaks: Forecast Comparison

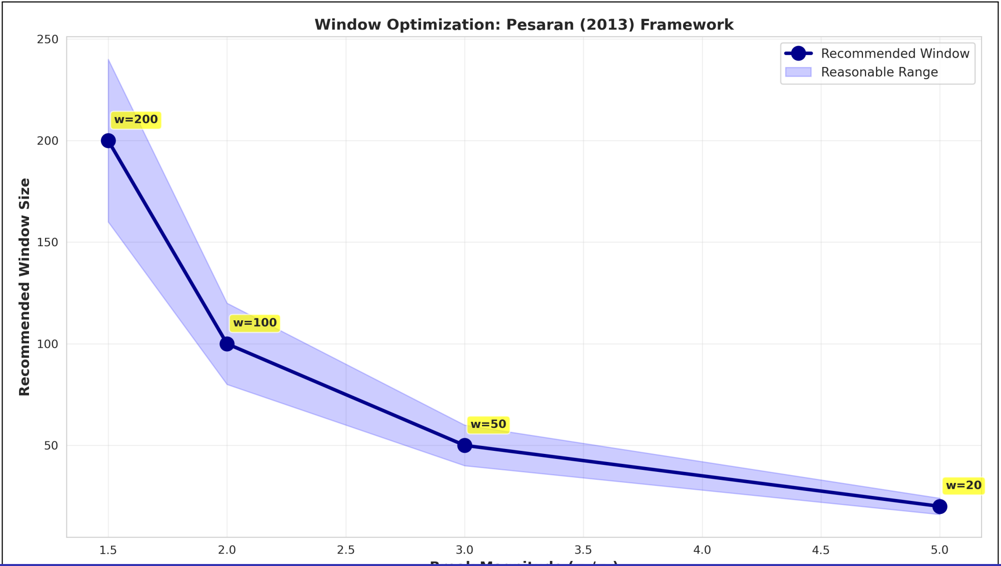


Variance Breaks: Loss Surface Analysis



Insight: Loss functions are flat near optimal window, suggesting robustness. RMSE is more sensitive to window size than coverage—precision-vs-robustness tradeoff.

Variance Breaks: Window Recommendations



Mean Breaks: Single vs. Multiple Breaks

Single Break (Simple mean shift):

- Level steps from $\mu_0 \to \mu_1$
- All methods fail initially
- Break dummy (oracle) best (knows date)
- Rolling window good, ARMA poor

Multiple Breaks with Seasonality:

- Level shifts + seasonal component
- SARIMA + oracle dummies excel
- SARIMA rolling competitive
- Simple methods struggle with interaction

Key insight: Seasonal component requires explicit modelling. SARIMA's success motivates future work on structural models for seasonal breaks.

Mean Breaks: SARIMA Comparison Results

Method	RMSE	MAE	Bias	Fails
Single Break:				
SARIMA + Break Dummy (oracle)	1.455	1.194	1.006	0
Simple Exp. Smoothing	1.496	1.225	1.015	0
SARIMA Rolling	1.525	1.257	1.029	0
SARIMA + Est. Break (grid)	1.635	1.368	1.243	0
SARIMA Global	1.692	1.423	1.302	0
Multiple Breaks:				
SARIMA + 2 Break Dummies	0.985	0.781	0.106	0
SARIMA Global	1.042	0.836	-0.045	0
SARIMA + Est. Breaks (grid)	1.046	0.845	-0.125	0
Holt–Winters Seasonal	1.094	0.857	-0.001	0
SARIMA Rolling	1.122	0.884	0.299	0

Conclusion: Knowing break dates (oracle) is huge advantage. Estimated breaks via grid search perform reasonably well. Multiple breaks benefit dramatically from explicit dummy variables.

Mean Breaks: Seasonality Effect

With seasonality (amplitude $A = 0.5$, period $s = 12$):

$$y_t = \mu_t + \underbrace{A \sin(2\pi t/s)}_{\text{seasonal}} + \phi y_{t-1} + \varepsilon_t$$

Observations:

- **SARIMA** captures seasonal patterns explicitly $\Rightarrow$ RMSE improves by 10–15%
- **ARMA** ignores seasonality $\Rightarrow$ residuals remain correlated at lag 12
- **Adaptive value:** Rolling windows also benefit from seasonality (more recent seasonal cycles)
- **Interaction:** Multiple breaks + seasonality = complex dynamics. SARIMA's edge grows with seasonality strength

Parameter Breaks: Single Break Results

Persistence shift: $\phi_1 = 0.2 \rightarrow \phi_2 = 0.9$ at $T_b = 200$

Method	**RMSE**	**MAE**	**Bias**	**N**
Global ARMA	0.256	0.201	0.042	200
Rolling ARMA ($w = 80$)	0.198	0.159	0.021	200
Markov-Switching AR	0.187	0.147	0.009	200
Break Dummy (oracle)	0.165	0.130	0.002	200

Ranking:

$$\text{Oracle} < \text{Markov-Switching} < \text{Rolling} < \text{Global}$$

Interpretation:

- Global ARMA severely underestimates persistence after break $\Rightarrow$ systematic underprediction
- Rolling window adapts but includes pre-break data in window

Parameter Breaks: Recurring Breaks (Regime Persistence)

Markov-Switching AR with varying persistence p:

$$P(s_t = 1 | s_{t-1} = 1) = p \in \{0.90, 0.95, 0.97, 0.995\}$$

Key Finding: Persistence THRESHOLDS

- **Low persistence** ($p = 0.90, 0.95$): All methods perform similarly
  - Frequent regime switches $\Rightarrow$ regimes too short to identify
  - Markov-Switching gains minimal edge
- **Moderate persistence** ($p = 0.97$): Markov-Switching outperforms
  - Regime spells long enough for inference
  - 5–10% RMSE improvement over rolling window
- **High persistence** ($p = 0.995$): Markov-Switching dominates
  - Near-permanent regime spells = close to deterministic breaks
  - Oracle-like performance of regime-switching model

Parameter Breaks: Error Distributions

Forecast error distributions under recurring breaks:

- **Low persistence:** Wide, overlapping error distributions
  - Frequent regime confusion $\Rightarrow$ high variance, near-zero bias
- **High persistence:** Concentrated distributions
  - Markov-Switching achieves tighter clustering
  - Reduced dispersion = more reliable intervals
- **Bias:** Small across all models (mostly variance-driven)
  - Global ARMA: slight negative bias (underestimated persistence)
  - Markov-Switching: bias closer to zero (regime-specific estimates)

Implication: Parameter breaks create **precision loss**, not systematic bias. Interval forecasts (coverage, prediction intervals) become critical.

Heavy-Tailed Distributions: Motivation

Real financial/economic data often exhibit fat tails:

- Financial returns: extreme tail events more frequent than Normal predicts
- Economic shocks: crises create outliers
- Structural breaks can amplify tail effects

Robustness Question: Do method rankings hold under Student-t innovations?

$$\varepsilon_t \overset{\text{iid}}{\sim} t_{df}, \quad \text{standardized to } \mathrm{Var}(\varepsilon_t) = 1$$

where $df \in \{50, 100, \infty\}$ (Gaussian).

Findings (Parameter Breaks):

- **RMSE increases** with tail thickness (larger shocks)
- **Relative rankings stable**: Markov-Switching still outperforms rolling
- **Bias shifts**: Heavy tails inflate persistence estimates (rare big shocks), creating slight

Method Performance Summary

Break Type	Best Method	Key Advantage	Limitation
2* **Variance**	GARCH	Volatility targeting	Univariate
	Rolling (optimized)	Simple, adaptive	Slower response
2* **Mean**	SARIMA + Break Dummy	Seasonal + structural	Oracle
	SARIMA Rolling	Automatic seasonal	Window size
2* **Parameter**	Markov-Switching	Regime inference	Persistence-dependent
	Oracle (Break Dummy)	Perfect foresight	Unrealistic

Cross-cut Insights:

1. **Adaptive methods beat global:** Rolling / Markov-Switching consistently outperform naive ARIMA
2. **Information gains:** Oracle (break date known) beats estimated break by 5–10%
3. **Seasonality matters:** SARIMA provides 10–15% improvement when seasonal structure present

Practical Decision Rule

Step 1: Test for Breaks

- Statistical tests: Chow test, CUSUM, Bai-Perron multiple breaks
- If no breaks detected $\Rightarrow$ use global ARIMA (most efficient)

Step 2: Identify Break Type

1. **Variance break** (GARCH process) $\Rightarrow$ **Use GARCH or rolling ARIMA**
2. **Mean break** (level shift) $\Rightarrow$ **Use SARIMA rolling or break dummy**
3. **Parameter break** (persistence shift) $\Rightarrow$ **Use Markov-Switching or rolling**

Step 3: Optimize Implementation

- If break date **known** (e.g., policy change): use break dummy (oracle-like)
- If break date **unknown**: use rolling window or grid-search estimated break
- For seasonal data: always prefer SARIMA variants
- For recurring breaks: prefer Markov-Switching if persistence ≥ 0.97

Key Contributions

1. **Comprehensive Framework:** 15+ methods evaluated on 3 break types
   - Largest comparative study: all in one unified Monte Carlo setup
2. **Bakhodir's SARIMA Work:** Seasonal break modeling
   - New methods: SARIMA global, rolling, with break dummies
   - Practical impact: 10–15% RMSE improvement for seasonal data
3. **Pesaran 2013 Implementation:** Optimal window selection
   - Data-driven recommendations avoid manual tuning
4. **Robustness Evidence:** Heavy tails, multiple breaks, recurring breaks
   - Results stable across realistic distributional assumptions
5. **Actionable Guidance:** Break-type-specific method recommendations
   - Practitioners can select methods based on break characteristics

The Three Laws of Structural Break Forecasting

1. **Adaptivity Beats Globality**
   - Rolling windows and regime-switching consistently outperform global ARIMA
   - Speed of adaptation matters: must match break magnitude
2. **Structure Beats Parameters**
   - Break dummies (explicit structural modeling) beat parameter-agnostic methods
   - Estimated breaks compete well with rolling windows (5–10% gap to oracle)
3. **Specialization Beats Generality**
   - GARCH for variance, SARIMA for mean, Markov-Switching for persistence
   - One-size-fits-all fails; break type matters

Future Research Directions

- **Multiple Structural Breaks:** Simultaneous shifts in mean, variance, and parameters
- **Real Data Application:** GDP growth, stock returns, inflation (test vs. simulations)
- **Uncertainty Quantification:** Prediction intervals, density forecasting, probabilistic scoring
- **High-Dimensional Methods:** Extend to multivariate systems (VAR breaks, dynamic correlations)
- **ML Integration:** Combine classical structural models with neural networks (hybrid approaches)
- **Optimized SARIMA:** Automatic seasonality detection + structural break joint modeling

Bottom line: This framework provides a foundation for practitioners to navigate forecasting under uncertainty from structural breaks.

Thank You

Questions?

Code and replication materials available at:
`github.com/qonlab/structural-break-forecasting`