BIO-3513 COMPUTATIONAL BIOLOGY

Modelling the Effect of Sleep Timing on Alertness Using a Simple ODE-Based Approach

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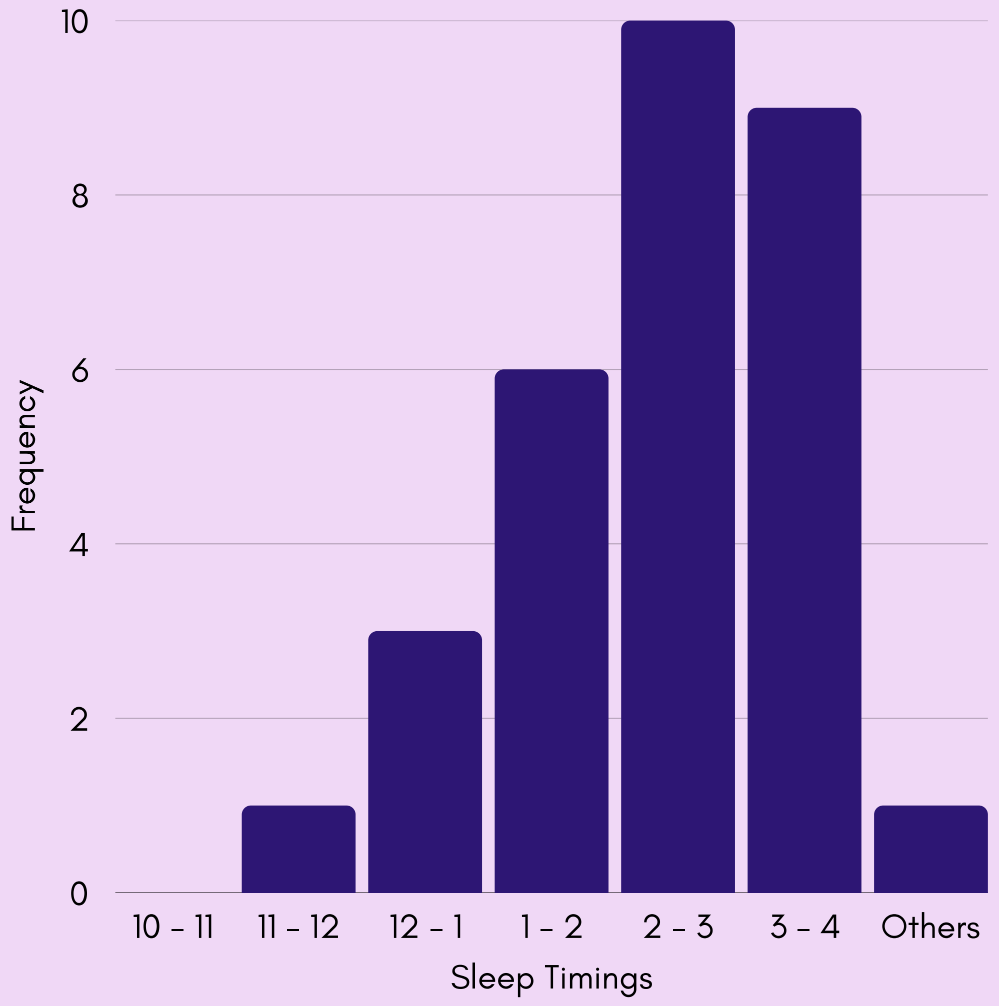
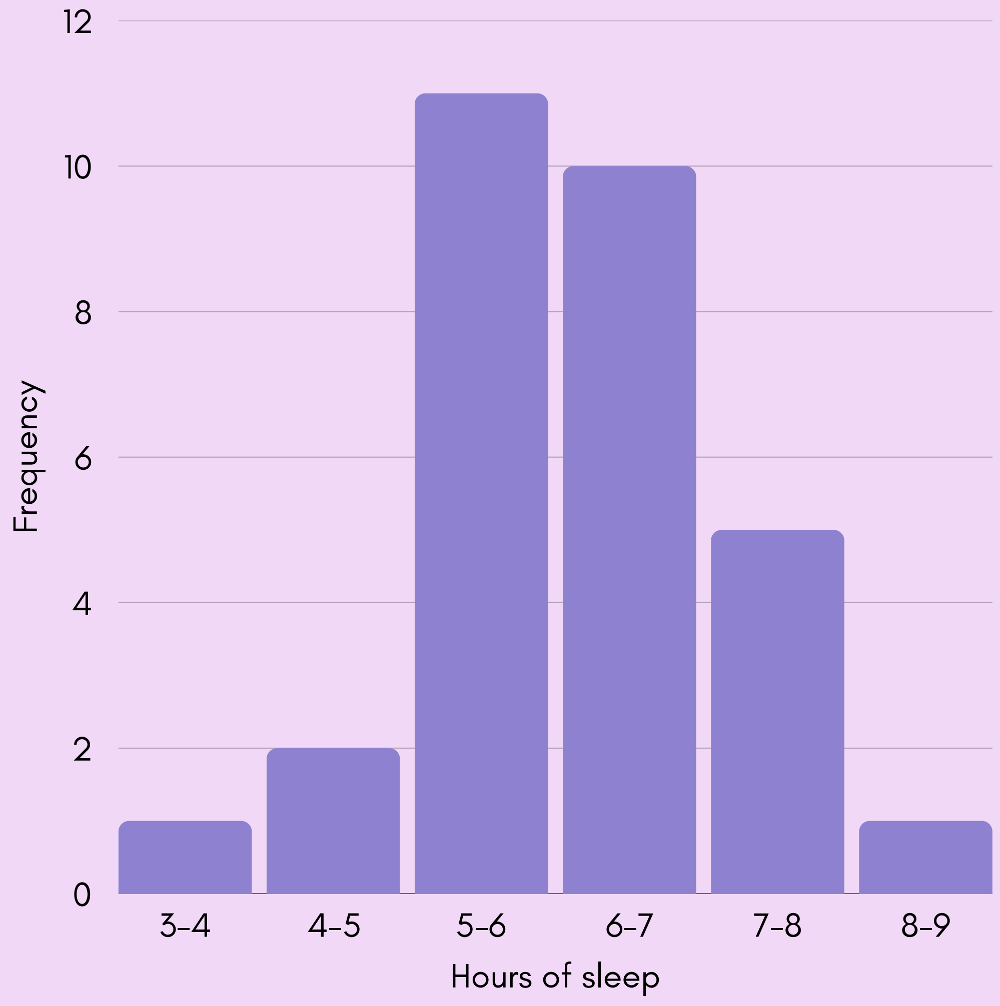
Introduction

What time do you go to sleep?



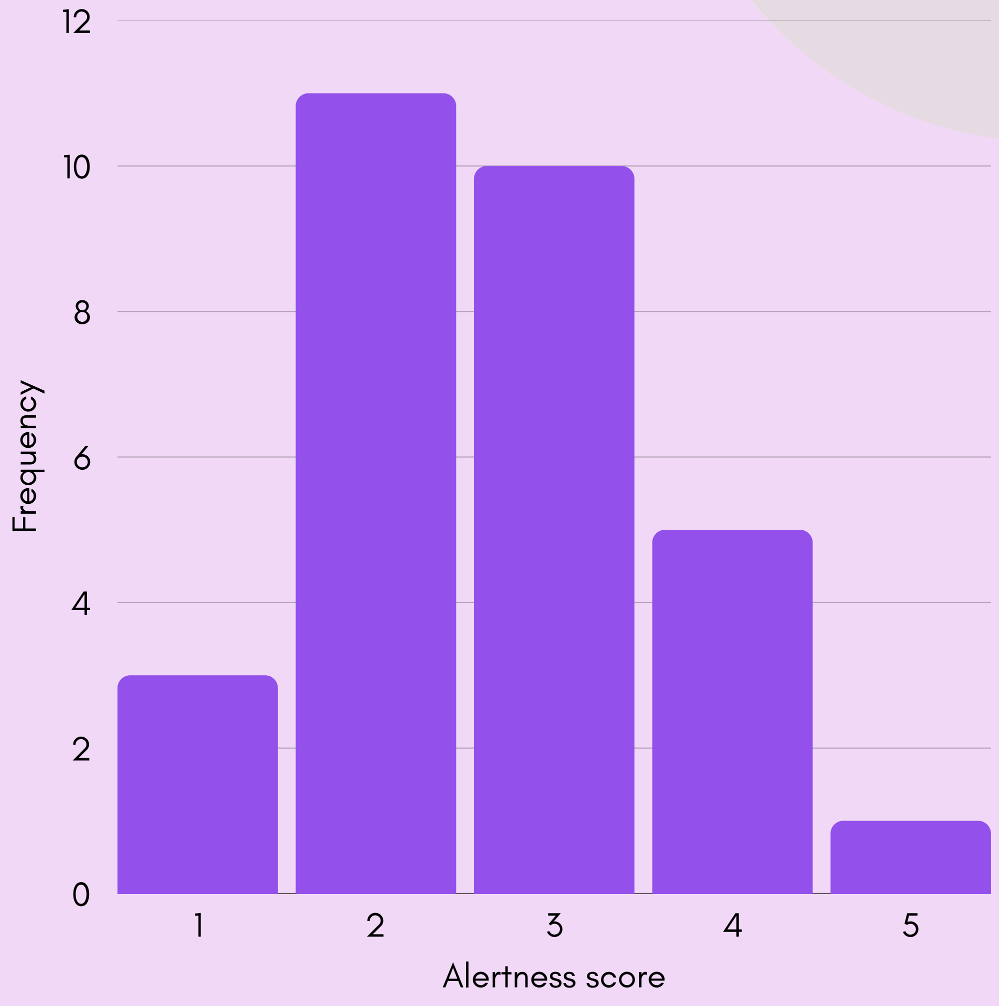
- Sleeping in alignment with the circadian rhythm improves sleep quality and daytime alertness.
- Misaligned sleep can lead to decreased alertness, reduced performance, and poorer mood.

How many hours of sleep do you get?



- Adults generally require 7-9 hours of sleep per night for optimal cognitive and physical function.
- Insufficient sleep accumulates sleep pressure (homeostatic drive), leading to impaired attention, memory, and decision-making.

How alert do you feel when you wake up?

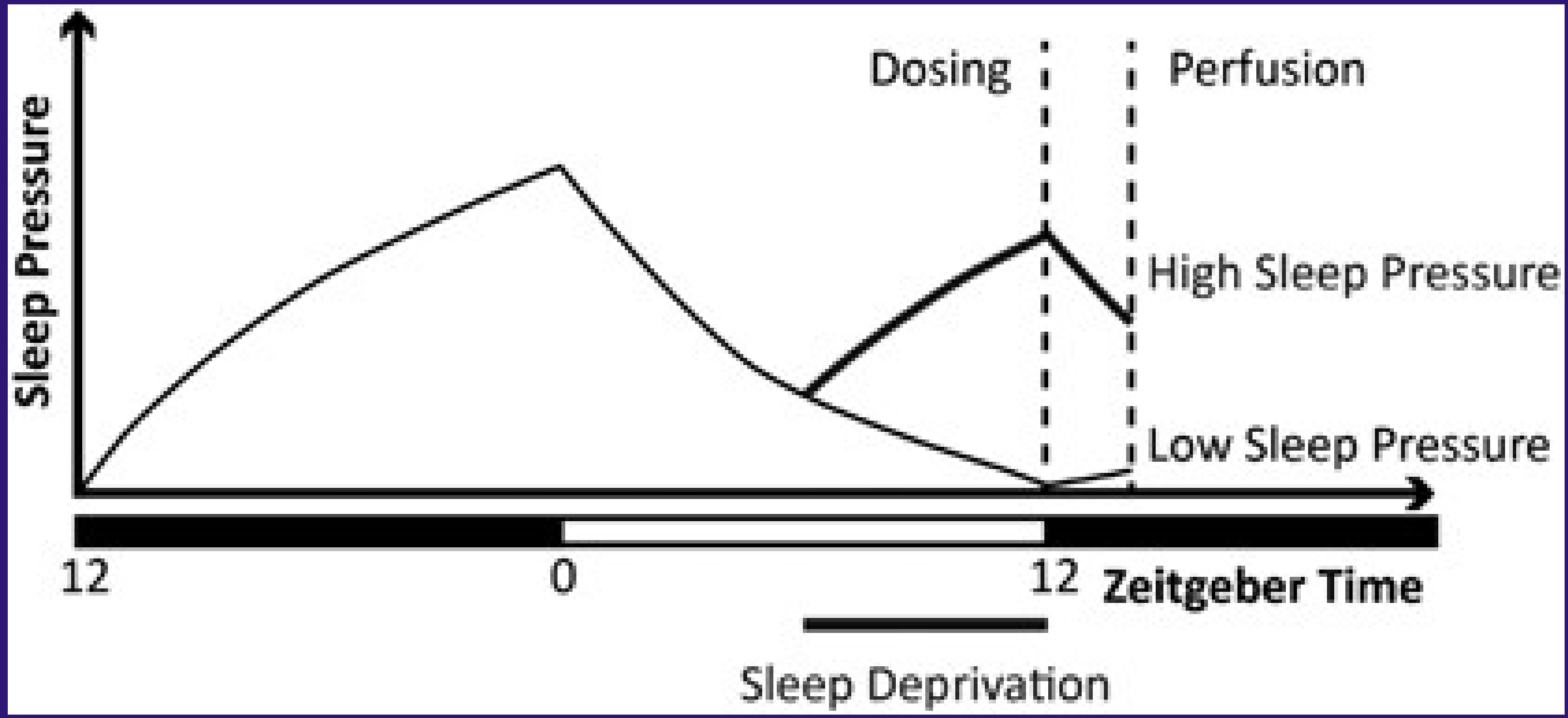


Alertness after waking up is affected by:

- Circadian phase
- Sleep stage at awakening
- Sleep continuity and quality: Fragmented or low-quality sleep

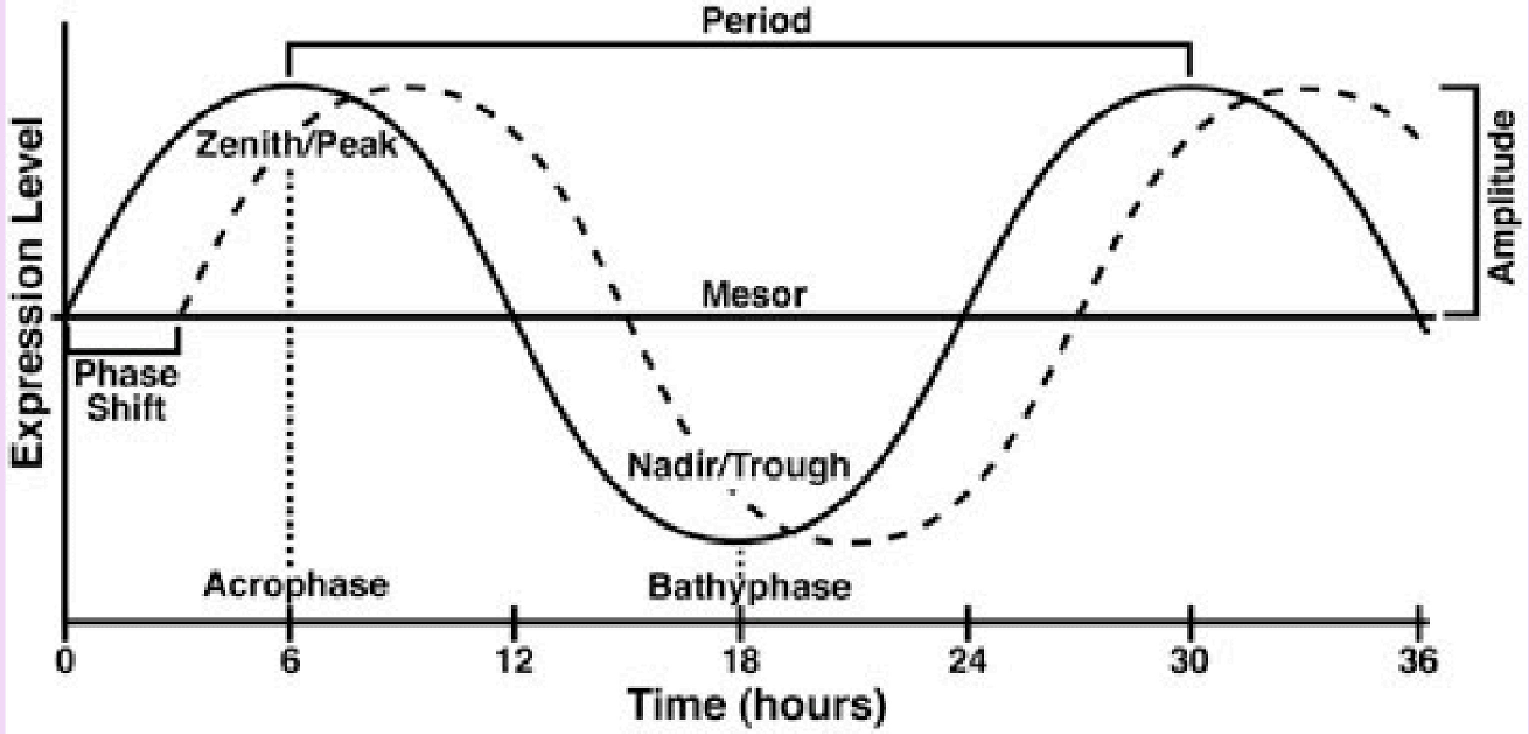
Two-process model of sleep regulation

PROCESS S



- Homeostatic sleep pressure
- Sleep-dependent process
- Builds up during wakefulness periods and dissipates during sleep

PROCESS C



- Circadian rhythm
- Sleep-independent process
- An internal 24-hour clock that regulates sleep-wake timing and alertness independently of prior sleep duration.

Alertness = Process S + Process C

Our Equation

Rate of Homeostatic Decay/Gain
Controls how quickly alertness moves back toward the baseline.

Baseline Alertness
The natural, “resting” alertness level when neither the homeostatic pressure nor circadian input is strongly acting.

Sine Wave for Circadian Rhythm
Models the natural daily rise and fall in alertness driven by the body's internal circadian rhythm.

$$\frac{dA}{dt} = -\alpha(A - A_{base}) + \beta \cdot sin(\frac{2\pi t}{T_{circadian}}) + S(t)$$

Alertness Level
Represents the person’s current level of alertness at any given point in time.

Amplitude of Circadian Influence
Determines how strongly the circadian rhythm pushes alertness up or down.

Sleep Suppression Term
Models the effect of being asleep on alertness.

Parameters	Value	Reasoning
A_{base}	0.5 (on a scale of 0.0-1.0)	• It is a mid-range baseline value • It represents the neutral alertness level when neither circadian nor sleep drive dominates
α	0.058	• Governs how quickly sleep pressure dissipates after waking • The half-life of sleep pressure is said to be between 10-15 hours so we chose a value that corresponds to a 12 hour half-life
β	0.9 (on a scale of 0.1-1.0)	• This determines the influence of circadian rhythm on alertness. We chose a higher value to make the contribution more prominent.
$T_{circadian}$	24	• Standard duration for the circadian rhythm
δ	0.4	• This value captures the balance between sleep and the underlying circadian influences
A_0	A_{base}	• The initial value for alertness is chosen as the base value
max_penalty	0.2 to 0.3	• Misaligned schedules can cause 20-30% decrease in cognitive performance

Our Model

Assumptions:

Circadian rhythm as a proxy for alertness

Alertness as an array of values from 0.2 to 0.3

Duration of sleep: 8 hours

Ideal time to go to sleep: 10:30PM

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Alertness reduced by 24.0 %
Enter your sleep time and wake time in 24-hour format (e.g., 04:00 for 4 AM):
Enter your sleep time (HH:MM):
Enter your wake time (HH:MM):
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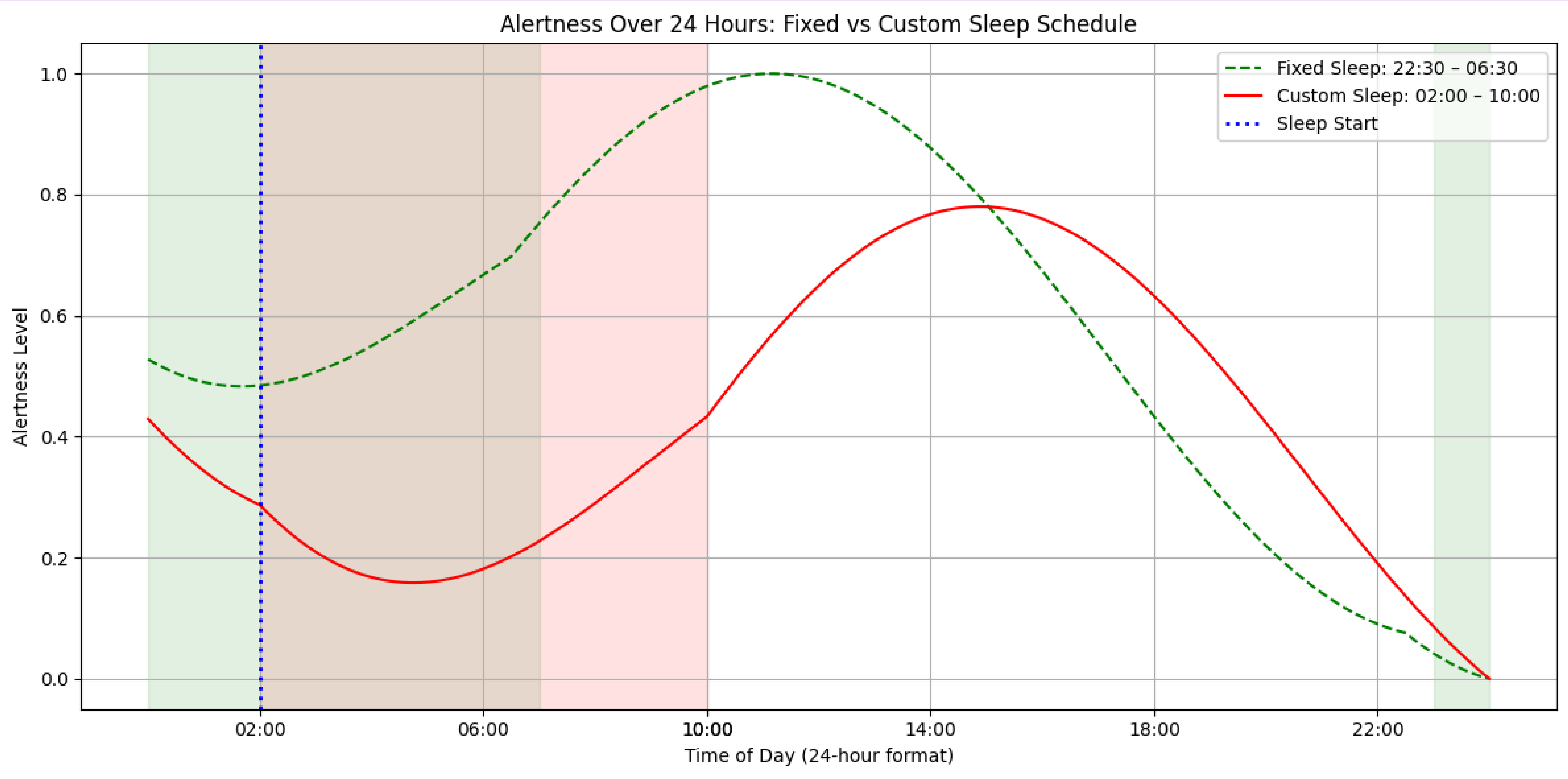
So, when did you sleep last night?

Results



Graph:

- Two lines: fixed (green dashed) and custom (red)
- Shaded regions show sleep periods
- Vertical dotted line = user's sleep start time

- Sleep timing affects alertness more than duration
- Reflects circadian overlap
- Morning dip = high risk for cognitive lapse

Limitations

FIXED CIRCADIAN RHYTHM

In reality, the rhythm is dynamic and light-sensitive.

BINARY SLEEP-WAKE STATE

The model switches sleep on/off sharply via $S(t)$, with no gradual transitions.

SINGLE-DAY SIMULATION

No cumulative fatigue or recovery tracking
Can't assess chronic sleep debt or adaptation

NO FEEDBACK LOOP BETWEEN SLEEP AND CIRCADIAN CLOCK

We only model a unidirectional relation.

NO ENVIRONMENTAL OR SOCIAL FACTORS

Doesn't account for:
Light-dark cycle shifts & Caffeine, stress, or blue light effects

NO INDIVIDUAL VARIATION

Parameters are fixed.
Ignores differences in chronotype, age, or individul routine.

Implications & Discussion

KEY INSIGHTS

- Simple models can capture complex physiological dynamics.
- Timing of sleep, not just duration, is crucial for daytime alertness.
- Misaligned sleep → delayed & diminished alertness peak.

MODEL ENHANCEMENTS

- Add light sensitivity and phase-response curves to make rhythms dynamic.
- Extend to multi-day simulations to model sleep debt or recovery.
- Introduce personalised parameters.
- Include sleep inertia effects post-wake.

REAL-WORLD INTEGRATION

- Could become a sleep optimisation tool: users input their sleep timing → get alertness predictions.
- Could evolve into tools used in chronotherapy, shift scheduling, or fatigue management systems.

Good Night & Sweet Dreams (at 10:30 pm)