Congressional Apportionment in the United States of America

To what extent does the apportioned representation equally represent states in the United States Congress?

World Studies: Mathematics and Global Politics

Equality and Inequality

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Introduction

The Founding Fathers of the United States sought to create a balanced government that would fairly represent both citizens and states in the nation. However, what does "fair" actually mean? Despite efforts to establish a democracy for the American people, not every state is equally represented in the various aspects of government.

The United States government consists of three different branches, one of which is the Legislative Branch, known as Congress. As outlined in Article 1 of the United States Constitution, it "grants Congress the sole authority to enact legislation and declare war, the right to confirm or reject many Presidential appointments, and substantial investigative powers."[1] Due to the conflicting desires of large and small states for representation, a solution called the Great Compromise was formed.[2] It was decided that Congress would be composed of two different houses, the Senate with equal state representation, and the House of Representatives where state representation was based on population. In the Senate, each state receives two representatives, for a total of 100 senators. On the other hand, the House of Representatives bases the distribution of representatives on population, with a total of 435 seats. Every state is guaranteed at least one House seat, and the remaining seats are apportioned through a mathematical method according to population.[3]

Congress is vital to both American and global politics, for the United States is very involved in international organizations and relationships. Given Congress's instrumental role in both creating laws and deciding wars, the method used to apportion seats becomes vital to the

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[1] "The Legislative Branch." *The White House*, The United States Government, www.whitehouse.gov/about- the-white-house/the-legislative-branch/.
[2] "A Great Compromise." *U.S. Senate: A Great Compromise*, 12 Dec. 2019, www.senate.gov/artandhistory/ history/minute/A_Great_Compromise.htm.
[3] *United States Constitution*. Art. I, Sec. 2, Cl. 3.

role each state plays. Math plays a role in calculating both the apportionment of Congressional seats and the degree of "fairness" in state representation, as measured in bias towards less populated, or "small," states and heavily populated, "large," states. This investigation will discuss and answer the question: To what extent does the apportioned representation equally represent states in the United States Congress?

Senate

The Senate is one half of the United States Congress and, per the Constitution, is made up of "two Senators from each State, chosen by the Legislature thereof, for six Years; and each Senator shall have one Vote,"[4] and with the fifty states total results in a total of 100 senators. Aside from the general responsibilities of Congress, the Senate is specifically responsible for trying all impeachment trials[5] and is able to "propose or concur with Amendments as on other Bills."[6] As a member of the Senate, senators "have more options to slow the progress of a bill" as compared to the House, and this is "intended to encourage deliberation [...] of issues."[7] As a result, majority party leaders "must work with minority party leaders - and often all senators - to determine the floor agenda."[8] Reflected in both the Senate's procedures and makeup, each state has equal representation and individual states have the ability to enact change.

Every state, regardless of size or population, is awarded with the same number of seats. This presents an initial problem of skewed representation as larger states hold the same amount of influence as smaller states. Mathematically, this is seen in the disparity between my

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[4] *United States Constitution*. Art. I, Sec. 3, Cl. 1.
[5] *United States Constitution*. Art. I, Sec. 3, Cl. 6.
[6] *United States Constitution*. Art. I, Sec. 7, Cl. 1.
[7] "Difference Between House and Senate: American Government." *Maryville Online*, Maryville University, 4 Dec. 2020, https://online.maryville.edu/blog/difference-between-house-and-senate/.
[8] Ibid.

calculation of California's and Wyoming's, the most and least populated states, percentage of the total national population from the 2010 census data.[9]

$$\frac{CA\ pop}{USA\ pop} = \frac{37341989}{309183463} = 0.1208$$

$$\frac{WY\ pop}{USA\ pop} = \frac{568300}{309183463} = 0.001838$$

$$\frac{0.1208}{0.001838} = 65.71$$

This indicates that California makes up roughly 12% of the national population, while Wyoming comprises under two-tenths of a percent. When these numbers are compared, California has about 66 times more people. Although, in the Senate, both of these states receive equal representation with the same number of Senators.

This leads to a clear bias towards small states, given they hold the same amount of political say, while maintaining a fraction of the population. However, there is a reason why the Constitution outlines the government this way. If the Legislative Branch only concerned itself with the wants of the majority, or the popular vote, small states would be insignificant in political matters, because of the larger states' overwhelming majority. In the Senate, less-populated states are able to express their opinions as smaller states and balance opinions that may differ in larger states. Less populated states are often more rural, while densely populated states frequently indicate urban environments. In the initial construction of the government, the Founding Fathers sought to include the instrumental agricultural communities and not allow them to be neglected by the populated urban cities. The Senate's method of seat apportionment ensures that each

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[9] "Congressional Apportionment." *2010 Census Briefs*, United States Census Bureau, 2019, https://www.census.gov/content/dam/Census/library/publications/2011/dec/c2010br-08.pdf.

aspect of the United States' diverse population is accounted for, in at least one house of the Legislative Branch.

This bias towards smaller states in the Senate is why the Constitution stipulates a bicameral legislature, with the House of Representatives divvying up seats respective to population. However, even the House has its drawbacks as no method is immune to bias.

House of Representatives

The House of Representatives makes up the other half of the United States Congress, with 435 voting members. The House also participates in general legislature responsibilities, as well as proposing bills for raising revenue.[10] Additionally, the House "shall have the sole Power of Impeachment"[11] which suggests the removal of an individual from the "offices of president, vice president, federal judges, and other federal officers."[12] Congress's capability for impeachment is at the core of the procedure of 'checks and balances,' a vital aspect of American government. This separation of powers is found in many other democratic governments today, for it ensures that one branch does not reach unregulated power over the others.

The process that distributes all of the House of Representative seats amongst the states is called "apportionment." The Constitution mandates that "Representatives [...] shall be apportioned among the several States... According to their respective Numbers […] but each State shall have at Least one Representative."[13] This method of representation, based on states' population, counteracts the favor to small states in the Senate's set distribution of seats. In terms

[10] *United States Constitution*. Art. I, Sec. 7, Cl. 1.
[11] *United States Constitution*. Art. I, Sec. 2, Cl. 5.
[12] "Difference Between House and Senate: American Government." *Maryville Online*, Maryville University, 4 Dec. 2020, https://online.maryville.edu/blog/difference-between-house-and-senate/.
[13] *United States Constitution*. Art. I, Sec. 2, Cl. 3.

of bias towards states, the House of Representatives is significantly more equitable when considering an individual citizen given that larger states receive more delegates. However, a crucial part of the Constitution's clause for apportionment is that every state must receive at least one delegate. If the 435 seats were doled out based on purely mathematical percentages, some states would not receive any representation at all. Their fraction of the population does not result in receiving a full delegate, so their state would, otherwise, be absent from the House. To prevent this, the Constitution stipulates that every state must have at least one seat, for they are a functioning aspect of the nation and deserve a voice. Despite efforts to create a house that takes population into account, this criterion introduces some unequal representation against larger states.

$$
\begin{aligned}
WY &= \frac{568,300}{1} = 568300 \\
MT &= \frac{994,416}{1} = 994416 \\
RI &= \frac{1,055,247}{2} = 527623.5
\end{aligned}
$$

The issue of constituency, or "the number of people a congressperson represents,"[14] also must be considered because of the impact that one seat can have on a smaller state. In Wyoming, the least populated state, their one delegate represents only 568,300 people; however, Montana just falls short of earning a second seat through apportionment and has a constituency of roughly a million people. Conversely, Rhode Island, the next most populated state after Montana, has just enough population to swing an additional delegate. This gives Rhode Island the lowest constituency, while Montana has the greatest, despite the two states having a population

[14] Guerrero, Shannon and Biles, Charles M., "The History of the Congressional Apportionment Problem through a Mathematical Lens" (2017). Congressional Apportionment. 27. http://digitalcommons.humboldt.edu/apportionment/27.

difference of about 60 thousand people. And while the national population continues to grow, "the House has remained at 435 [seats] for virtually the past century."[15] Some apportionment critics have argued that the best solution for this problem would be adding more seats to the House, as it would lessen the dramatic difference of a state gaining or losing a single seat.[16]

Apportionment Methods

For mathematically apportioning seats, there has been a lot of controversy throughout the country's history when deciding on the optimal method. Dating back to Alexander Hamilton and Thomas Jefferson, multiple apportionment methods have been proposed, all claiming to be the optimal solution. Below is a list of the proposed methods in more detail and the outlined procedure to illustrate how the methods are applied.

Largest Fractions (LF)

The first of these methods is "Largest Fractions," and is one of the most commonly proposed solutions for apportionment. It is a straightforward implementation where states receive seats based on a rounded proportion of their fraction of total population. The term for this is quota, defined as "the fraction that the state's population represents of the total population, multiplied by the total number of seats."[17]

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[15] "Difference Between House and Senate: American Government." *Maryville Online*, Maryville University, 4 Dec. 2020, https://online.maryville.edu/blog/difference-between-house-and-senate/.
[16] Wilson, Chris. "How to Fix the House: Add About 500 Seats." *Time*, Time, 15 Oct. 2018, https://time.com/ 5423623/house-representatives-number-seats/.
[17] Young, H. Peyton. "Fairness in Apportionment." *The United States Census Bureau*, Johns Hopkins University/The Brookings Institution, Jan. 2004, https://www.census.gov/history/pdf/Fairness_in_ Apportionment_Young.pdf.

$$ideal\ district\ size\ =\ \frac{national\ population}{number\ of\ house\ seats}$$

$$quota\ =\ \frac{state\ population}{ideal\ district\ size}$$ [18]

In the Largest Fraction algorithm, this quota is then truncated, for the floor of the quota, and that many seats are distributed amongst the states. The total of the truncated fractions will leave a portion of seats not yet distributed, so the algorithm then assigns the excess seats one at a time, in order of the largest truncated fraction remaining.

To illustrate how this LF method is calculated, consider the following fictional 7 state nation, with one very large state, and a total population of 50,000. I calculated the following with a 100 seat House of Representatives.

$$district\ size\ =\ \frac{50000}{100}\ =\ 500$$

State	A	B	C	D	E	F	G	**total**
Population	45745	830	730	725	720	700	550	50000
District Size	500	500	500	500	500	500	500	
Quota	91.49	1.66	1.46	1.45	1.44	1.4	1.1	
Quota Floor	91	1	1	1	1	1	1	97
Truncated Fraction	.49	.66	.46	.45	.44	.4	.1	
LF Apportionment	92	2	2	1	1	1	1	100

Table 1: Largest Fractions Apportionment Example.

As seen in Table 1, after the initial quotas were determined, summing the truncated values assigned only 97 of the 100 seats. The remaining 3 seats were then apportioned based on

[18] Bennett, S., et al. *The Apportionment Problem: The Search for the Perfect Democracy*. Teacher's Manual ed., mod. 8, HiMAP, 1986.

the states with the largest truncated fraction, so first to state B with largest fraction of 0.66, then state A and state C, respectively.

While this LF method is clear and gives a seemingly reasonable apportionment, there are drawbacks to this approach, based on the sensitivity to change of the fractional remainders. One such issue was named the “Alabama Paradox,” found in 1880 when LF was in use.[19] In this situation, when the total number of House seats increased, even though all state populations remained the same, Alabama actually lost a seat due to the expanded House size. From then on, only divisor methods were applied to Congressional Apportionment.

Divisor Methods

Given the limitations of the LF method, other algorithmic approaches were proposed, all falling under the category of “divisor methods.” They avoid the issue of LF fractional sensitivity and assigning left over seats, by instead determining one divisor that will result in adjusted quotas for each state that sum to the total House seats. For divisor methods, “the number of seats assigned to a state is a function of its population, *P*, and a divisor, λ.”[20]

$$P_i = population\ of\ an\ individual\ state$$
$$\lambda = divisor$$

$$adjusted\ quota = q = \frac{P_i}{\lambda}$$

$$\lfloor q \rfloor = a$$

$$\lceil q \rceil = a + 1$$

[19] Ernst, Lawrence R. *Apportionment Methods for the House of Representatives and the Court Challenges*. Bureau of the Census Statistical Research Division, 1992.
[20] Ibid.

Each state's adjusted quota will then be rounded to a whole number of delegates, which, in turn, totals to the exact number of available seats. The divisor is divided by every state's population, and results in a definite way to apportion seats. While there are countless variations of divisor methods, the following five consist of ones that have been proposed as actual apportionment methods for the House of Representatives.

The state will receive either the floor or ceiling of the adjusted quota, a or $a + 1$, respectively, according to the particular method's choice of rounding function, d.[21] For each of the following examples of the apportionment methods, I determined an appropriate divisor by inspection until the sum of rounded adjusted quotas resulted in the overall amount of House seats.

Smallest Divisors (SD)

The "Smallest Divisors" method involves finding a divisor that, when all of the quotients are rounded up to the ceiling of the quota, their sum adds to the total number of seats available.

$$d(a) = a$$

State	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**total**
Population (P)	45745	830	730	725	720	700	550	50000
Divisor (λ)	525	525	525	525	525	525	525	
Quota	87.1333	1.5810	1.3905	1.3810	1.3714	1.3333	1.0476	95.2381
SD Apportionment	88	2	2	2	2	2	2	100

Table 2: Smallest Divisors Apportionment Example.

21 Ibid.

Greatest Divisors (GD)

The “Greatest Divisors” method is the opposite of the SD method, for all of the quotients are always rounded down to the floor of the quota, essentially truncating the number. A new common divisor is found that results in an exact sum, totalling to the whole number of representatives within the house.

$$d(a) = a + 1$$

State	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**total**
Population (P)	45745	830	730	725	720	700	550	50000
Divisor (λ)	485	485	485	485	485	485	485	
Quota	94.3196	1.7113	1.5052	1.4948	1.4845	1.4433	1.1340	103.0928
GD Apportionment	94	1	1	1	1	1	1	100

Table 3: Greatest Divisors Apportionment Example.

Major Fractions (MF)

“Major Fractions” splits the difference between SD and GD by rounding arithmetically, with any remainder 0.5 and above rounded up, and anything below 0.5 is rounded down. With this method, a different common divisor can be found that results in the rounded quotients adding to the total number of seats.

$$d(a) = a + 0.5$$

State	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**total**
Population (P)	45745	830	730	725	720	700	550	50000
Divisor (λ)	490	490	490	490	490	490	490	
Quota	93.3571	1.6939	1.4898	1.4796	1.4694	1.4286	1.1224	102.0408
MF Apportionment	93	2	1	1	1	1	1	100

Table 4: Major Fractions Apportionment Example.

Harmonic Means (HM)

The "Harmonic Means" method sets the rounding tipping-point as the harmonic mean of the lower and upper limits of the quotient. If the adjusted quota is greater than the harmonic mean, then the number is rounded up to the nearest integer; conversely, if the quotient is lower, then it will be rounded down.

$$d(a) = \frac{a(a+1)}{a+0.5}$$

State	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**total**
Population (P)	45745	830	730	725	720	700	550	50000
Divisor (λ)	515	515	515	515	515	515	515	
Quota	88.8252	1.6117	1.4175	1.4078	1.3981	1.3592	1.0680	97.0874
HM Apportionment	89	2	2	2	2	2	1	100

Table 5: Harmonic Means Apportionment Example.

Equal Proportions (EP)

This last method is similar to the last divisor algorithm, HM, but the "Equal Proportions" method instead relies on the geometric mean of the floor and ceiling of the quotient. Each state's population is divided by the new common divisor and is rounded according to its geometric mean, to then be added up for distributing all of the House seats.

$$d(a) = \sqrt{a(a+1)}$$

State	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**total**
Population (P)	45745	830	730	725	720	700	550	50000
Divisor (λ)	508	508	508	508	508	508	508	
Quota	90.0492	1.6339	1.4370	1.4272	1.4173	1.3780	1.0827	98.4252
EP Apportionment	90	2	2	2	2	1	1	100

Table 6: Equal Proportions Apportionment Example.

Comparison of House Apportionment Methods

To directly compare the difference of apportionment results between these various methods, I compiled the results into Table 7, below.

State	A	B	C	D	E	F	G	total
Population	45745	830	730	725	720	700	550	50000
LF	92	2	2	1	1	1	1	100
SD	88	2	2	2	2	2	2	100
HM	89	2	2	2	2	2	1	100
EP	90	2	2	2	2	1	1	100
MF	93	2	1	1	1	1	1	100
GD	94	1	1	1	1	1	1	100

Table 7: A direct comparison of various example apportionment methods.

This gives some insight into how each of these methods compare to one another. State A's population is appreciably larger when compared to other state populations, in particular State G. The largest disparity of seat distribution is seen in between the SD and GD methods. By changing only the chosen method and keeping state populations constant, the apportionment resulted in a 6 seat difference. This discrepancy indicates the importance behind choosing a method, for it can lead to very different apportionments, and, therefore, representation in government.

After comparing the various apportionment methods available, mathematicians outlined three axioms to describe what an optimal method would satisfy. The first of these was that the method would give each state either their quota floor or one higher, and the second being that if the number of seats increased, then no state would lose a seat if populations remained the same. Finally, the third axiom stated that if the population increased in just one state, then that state would not lose a seat. While LF fulfils the first axiom quota requirement, it fails to consistently guarantee the second and third axioms. This led to the adoption of the different divisor methods, which can guarantee the second and third axioms, but, ironically, cannot guarantee the first quota axiom. It wasn't until 1892 that Michael Balinski and H. Peyton Young proved no methods could ensure all three axioms at the same time. Their Apportionment Impossibility Theorem found that "no apportionment method can avoid all types of inequalities."[22] This powerful discovery means that there is no perfect apportionment method, so the question becomes which one is closest to achieving a "fair" apportionment.

Current Implementation

The current algorithm in place for apportionment, as of the 2010 census, is the Equal Proportions method. To determine the impact that EP has on the US's current apportionment, I also applied the other listed methods to the 2010 census data and calculated the different seat distributions. The current methodology to determining the overall apportionment is an iterative algorithm that uses the inverse of the rounding function *d* as a type of ranking function which produces priority values. The seats are then apportioned one by one to the state with the highest priority value in each step.[23]

For example, when apportioning seats according to the EP method, a state's priority value to move from *a* seats to $a + 1$seats would be equal to the state's population divided by the rounding function for EP method.

$$priority\ value\ =\ P_i \times \frac{1}{\sqrt{a(a+1)}}$$

[22] Bennett, S., et al. *The Apportionment Problem: The Search for the Perfect Democracy*. Teacher's Manual ed., mod. 8, HiMAP, 1986.
[23] "Computing Apportionment." *The United States Census Bureau*, 10 Nov. 2020, www.census.gov/topics/public-sector/congressional-apportionment/about/computing.html.

Since each state needs to receive at least one seat to begin with, the algorithm begins with all states with $a = 1$ as their initial seat value, and the method proceeds with which state has the highest priority value to receive the 51st seat. The highest priority value would receive that next 51st seat, and that particular state would recalculate its priority value where $a = 2$, because of the seat it gained. This determines which state has the highest "priority" for receiving the next seat, and is continued iteratively until all 435 seats are dispersed.

	Apportionment Method					
	LF	**SD**	**HM**	**EP**	**MF**	**GD**
WY	2	1	1	1	1	1
VT	2	1	1	1	1	1
ND	2	1	1	1	1	1
AK	2	1	1	1	1	1
SD	2	2	1	1	1	1
DE	2	2	1	1	1	1
MT	2	2	2	1	1	1
RI	2	2	2	2	1	1
MI	13	14	14	14	14	14
OH	15	16	16	16	16	17
PA	17	17	18	18	18	18
IL	17	18	18	18	18	19
FL	25	26	27	27	27	27
NY	25	26	27	27	27	28
TX	32	34	36	36	36	37
CA	48	50	52	53	53	55

Table 8: A comparison of resulting House seats given to the 8 smallest states versus the 8 largest states based on the 2010 census by the various apportionment methods.

I chose to only display the seat apportionments of the least and most populated states in Table 8 (although a full calculation for all fifty states was completed, see Appendix 1 and 2), to display the differences and bias of the individual apportionments. Given the varying results produced, this indicates the importance of the mathematical process behind representation and political decisions. When comparing apportionment for the smaller states, it is clear that Largest Fractions greatly favors these states, as it gives the smallest four states twice the representation as compared to the other methods. And when only focusing on divisor methods, the Smallest Divisors leans more towards the smaller states, for SD chooses to round up to the ceiling, therefore increasing the less populated states' odds of earning another seat. This is starkly compared to Greatest Divisors on the other end of the spectrum, as it heavily favors the larger states. When looking at California's distribution, the GD method gives five more seats than SD would, with the other three divisor methods falling in the middle of the two extremes. With a less apparent bias towards large or smaller states, it is harder to visually discern the differences between the three middle divisor methods.

Balinski and Young also determined a way to mathematically calculate the bias of these methods for larger and smaller states.[24] Using census data dating back to the start of the nation, they found the accumulated overall percent bias present in each of the methods. Their procedure included dividing states into their respective "categories: large, middle and small," but leaving out any states with a natural quota of less than 0.5, because, mathematically, they would not receive any representation and pull the bias towards the smallest states.[25] The next step was to "compute the per capita representation in the large states as a group and in the small states as a group."[26] Finally, they then calculated "the percentage difference between the two" to find "the method's relative bias toward small states in that year."[27]

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[24] Young, H. Peyton. "The Mathematics of One Person, One Vote." *APS News*, vol. 10, no. 4, Apr. 2001, p. 8., www.aps.org/publications/apsnews/200104/upload/apr01.pdf.
[25] Ibid.
[26] Ibid.
[27] Ibid.

	SD	HM	EP	MF	GD
Average Bias	18.3%	5.2%	3.4%	0.3%	-15.7%

Table 9: Balinski's and Young's calculation of average percentage bias from censuses 1790 to 1970.

Surprisingly, the Equal Proportions method, currently in use, resulted in about a 3.4% bias towards smaller states, while Major Fractions was the only one with a negligible percent bias. This led Balinski and Young to increase their support for the MF method, as its arithmetic rounding method, at 0.5, prevents any partiality towards smaller or larger states.

This made me wonder why EP is still the present method in use, despite Balinski and Young releasing their data decades ago. I found that one reason is Edward Huntington's research into pairwise comparisons. He assessed whether "switching a seat from one state to another [would] diminish the fairness of the apportionment as given by some measure of fairness."[28] By looking at the comparison from a relative perspective, this approach took into consideration that seats mean different things to more or less populated states. From this perspective, "if one is concerned with relative differences, all the criteria formulas are optimal only for Huntington-Hill."[29] On the other hand, Balinski and Young's calculation of percent bias took an objective absolute approach, which champions "measuring the absolute difference in representatives per person."[30]

Another aspect to consider in EP's selection was political motivations. Beyond mathematical reasoning, "the switch from Webster's to Hill's method in 1941 gave one more seat to Arkansas and one less to Michigan, which essentially guaranteed one more seat for the

[28] Malkevitch, Joseph. "Apportionment: Fairness and Apportionment." *American Mathematical Society*, 2021, www.ams.org/publicoutreach/feature-column/fcarc-apportionii2.
[29] Ibid.
[30] Ibid.

Democrats," the majority party at the time.[31] Aside from the mathematical reasoning behind method selection, politics, itself, plays an active role. Legislative decisions in American government are democratically based on voting, and the wants of the majority do not always align with objective reasoning.

While the bias towards smaller states is far less than in the Senate, even the House of Representatives is not perfect. Despite being designed to take population into consideration and allocate seats accordingly, the bias towards small states still persists.

Conclusion

In the US Constitution, Congress has the ability to both declare war and "regulate Commerce with foreign Nations."[32] This gives tremendous power to Congress as they can drastically alter international relations and US involvement in many global organizations and alliances.

Each state's number of congresspersons reflects their relative power and influence in these decisions. Additionally, the states' total representatives are combined from each House, including three additional seats from the District of Columbia, to comprise the U.S. Electoral College process for determining the President of the United States.[33] Therefore, the representation in an individual state's apportionment is of tremendous significance.

Several apportionment methods have been proposed and implemented throughout US history in attempts to best distribute representation among the fifty states. While a theoretically

[31] Ibid.
[32] *United States Constitution*. Art. I, Sec. 8, Cl. 3.
[33] "What Is the Electoral College?" *National Archives and Records Administration*, Dec. 2019, www.archives.gov/electoral-college/about.

perfect solution does not exist mathematically, some methods show far more bias than others towards large or small states.

Additionally, political motives also play a role and can outweigh mathematical reasoning when revising apportionment methods. Apportionment cannot be viewed from purely a mathematical standpoint, but also must consider the political ramifications that can influence and override decision making. As a result, mathematically, both the Senate and House of Representatives are biased toward small states.

While both houses are mathematically proven to be skewed towards small states, it brings up the issue that "equal" apportionment is not necessarily the same as "fair" apportionment. The two houses of the US Legislative Branch apportion seats seek to create a balance between representation based on state or population. Although, apportionment is not without flaws.

While there are a multitude of factors to consider when choosing an apportionment method, I propose that the United States Congress considers reverting back to the Major Fractions divisor method. While it does not take relative comparison into account, seats can be allocated based solely on absolute difference to give larger states more proportional representation. Given that small states are well represented in the Senate already, seat distribution in the House of Representatives should be considered objectively.

Works Cited

"A Great Compromise." *U.S. Senate: A Great Compromise*, 12 Dec. 2019, www.senate.gov/artandhistory/history/minute/A_Great_Compromise.htm.

Bennett, S., et al. *The Apportionment Problem: The Search for the Perfect Democracy*. Teacher's Manual ed., mod. 8, HiMAP, 1986.

"Computing Apportionment." *The United States Census Bureau*, 10 Nov. 2020, www.census.gov/topics/public-sector/congressional-apportionment/about/computing.html.

"Congressional Apportionment." *2010 Census Briefs*, United States Census Bureau, 2019, www.census.gov/content/dam/Census/library/publications/2011/dec/c2010br-08.pdf.

"Difference Between House and Senate: American Government." *Maryville Online*, Maryville University, Dec. 2020, online.maryville.edu/blog/difference-between-house-and-senate/.

Ernst, Lawrence R. *Apportionment Methods for the House of Representatives and the Court Challenges*. Bureau of the Census Statistical Research Division, 1992.

Guerrero, Shannon and Biles, Charles M., "The History of the Congressional Apportionment Problem through a Mathematical Lens" (2017). Congressional Apportionment. 27. digitalcommons.humboldt.edu/apportionment/27.

Malkevitch, Joseph. "Apportionment: Fairness and Apportionment." *American Mathematical Society*, 2021, www.ams.org/publicoutreach/feature-column/fcarc-apportionii2.

"The Legislative Branch." *The White House*, The United States Government, www.whitehouse.gov/about-the-white-house/the-legislative-branch/.

"What Is the Electoral College?" *National Archives and Records Administration*, Dec. 2019, www.archives.gov/electoral-college/about.

Wilson, Chris. "How to Fix the House: Add About 500 Seats." *Time*, Time, 15 Oct. 2018, time.com/5423623/house-representatives-number-seats/.

Young, H. Peyton. "Fairness in Apportionment." *The United States Census Bureau*, Johns Hopkins University/The Brookings Institution, Jan. 2004, www.census.gov/history/pdf/Fairness_in_Apportionment_Young.pdf.

Young, H. Peyton. "The Mathematics of One Person, One Vote." *APS News*, vol. 10, no. 4, Apr. 2001, p. 8., www.aps.org/publications/apsnews/200104/upload/apr01.pdf.

Appendices

Appendix 1: Apportionment Method Calculations (Divisor Methods)

Priority Value (PV) Method

	EP		MF		GD		SD		HM	
	Final PV	Seats	Final PV	Seats	Final PV	Seats	Final PV	Seats	Final PV	Seats
WY	401848.78	1	378866.67	1	284150.00	1	568300.00	1	426225.00	1
VT	445715.57	1	420224.67	1	315168.50	1	630337.00	1	472753.00	1
ND	477937.01	1	450603.33	1	337952.50	1	675905.00	1	506929.00	1
AK	510193.81	1	481015.33	1	360761.50	1	721523.00	1	541142.00	1
SD	579658.56	1	546507.33	1	409880.50	1	409880.50	2	614821.00	1
DE	637016.24	1	600584.67	1	450438.50	1	450438.50	2	675658.00	1
MT	703158.30	1	662944.00	1	497208.00	1	497208.00	2	414340.00	2
RI	430802.78	2	703498.00	1	527623.50	1	527623.50	2	439686.00	2
NH	539477.66	2	528578.00	2	660722.50	1	660722.50	2	550602.00	2
ME	544225.18	2	533229.60	2	666537.00	1	666537.00	2	555448.00	2
HI	558019.07	2	546744.80	2	455620.67	2	683431.00	2	569526.00	2
ID	642378.28	2	629399.60	2	524499.67	2	524499.67	3	655625.00	2
NE	528802.33	3	523378.57	3	610608.33	2	610608.33	3	534282.00	3
WV	536882.35	3	531375.71	3	619938.33	2	619938.33	3	542446.00	3
NM	596770.31	3	590649.43	3	516818.25	3	689091.00	3	602955.00	3
NV	605847.41	4	602096.00	4	541886.40	4	677358.00	4	609622.00	4
UT	619561.89	4	615725.56	4	554153.00	4	692691.25	4	623422.00	4
KS	640368.05	4	636402.89	4	572762.60	4	715953.25	4	644358.00	4
AR	654324.70	4	650273.11	4	585245.80	4	731557.25	4	658402.00	4
MS	665954.71	4	661831.11	4	595648.00	4	744560.00	4	670104.00	4
IA	682847.53	4	678619.33	4	610757.40	4	610757.40	5	687102.00	4
CT	653912.82	5	651205.09	5	596938.00	5	716325.60	5	656632.00	5
OK	687370.27	5	684524.00	5	627480.33	5	627480.33	6	690228.00	5
OR	702656.11	5	699746.55	5	641434.33	5	641434.33	6	705578.00	5
KY	671313.08	6	669324.00	6	621515.14	6	725101.00	6	673308.00	6
LA	702691.59	6	700609.54	6	650566.00	6	650566.00	7	704780.00	6
SC	620844.52	7	619463.33	7	663710.71	6	663710.71	7	622229.00	7

AL	641825.47	7	640397.60	7	600372.75	7	686140.29	7	643257.00	7
CO	674157.13	7	672657.33	7	630616.25	7	720704.29	7	675660.00	7
MN	710230.58	8	625279.88	8	664359.88	7	664359.88	8	711814.00	8
WI	671542.85	8	670380.00	8	633136.67	8	712278.75	8	672708.00	8
MD	682349.68	8	681168.12	8	643325.44	8	723741.13	8	683533.00	8
MO	708459.48	8	707232.71	8	667942.00	8	667942.00	9	709688.00	8
TN	672029.43	9	671098.00	9	637543.10	9	708381.22	9	672962.00	9
AZ	675957.93	9	675021.05	9	641270.00	9	712522.22	9	676896.00	9
IN	685326.92	9	684377.05	9	650158.20	9	722398.00	9	686278.00	9
MA	691447.19	9	690488.84	9	655964.40	9	728849.33	9	692407.00	9
WA	643908.47	10	643178.00	10	675336.90	9	750374.33	10	644640.00	10
VA	699595.12	11	698933.57	11	669811.33	11	730703.27	11	700257.00	11
NJ	705164.44	12	704600.08	12	629107.21	13	733958.42	12	705729.00	12
NC	709062.86	13	708576.37	14	637718.73	14	735829.31	13	709550.00	13
GA	671265.83	14	670866.62	14	648504.40	14	748274.31	13	671665.00	14
MI	683967.17	14	683560.41	14	660775.07	14	707973.29	14	684374.00	14
OH	701443.04	16	701120.91	16	642694.17	17	723030.94	16	701765.00	16
PA	688624.80	18	688373.24	18	670258.16	18	749112.06	17	688876.00	18
IL	695626.00	18	695371.89	18	677072.63	19	714687.78	18	695880.00	18
FL	687414.47	27	687300.84	27	675027.61	27	726952.81	26	687528.00	27
NY	706336.94	27	706220.18	27	669691.55	28	746963.65	26	706454.00	27
TX	692350.39	36	692285.42	36	664958.37	37	743188.76	34	692415.00	36
CA	698011.59	53	697981.10	53	666821.23	55	746839.78	50	711341.00	52

Appendix 2: Apportionment Method Calculations (Largest Fractions)

Every state must receive at least one seat per the stipulation of the Constitution, so the third column represents the floor of the state's quota plus the mandatory first seat. Highlighted values represent states who received an extra seat because of their high remainder value.

	Quota	$\lfloor$Quota$\rfloor$+1	Remainder	Final Seat Allocation
WY	0.7077	1	0.7077	2
VT	0.7849	1	0.7849	2

ND	0.8416	1	0.8416	2
AK	0.8985	1	0.8985	2
SD	1.0208	2	0.0208	2
DE	1.1218	2	0.1218	2
MT	1.2383	2	0.2383	2
RI	1.314	2	0.314	2
NH	1.6455	2	0.6455	3
ME	1.66	2	0.66	3
HI	1.702	2	0.702	3
ID	1.9593	2	0.9593	3
NE	2.281	3	0.281	3
WV	2.3159	3	0.3159	3
NM	2.5742	3	0.5742	4
NV	3.3738	4	0.3738	4
UT	3.4502	4	0.4502	4
KS	3.5661	4	0.5661	5
AR	3.6438	4	0.6438	5
MS	3.7086	4	0.7086	5
IA	3.8026	4	0.8026	5
CT	4.4599	5	0.4599	5
OK	4.6881	5	0.6881	6
OR	4.7923	5	0.7923	6
KY	5.4174	6	0.4174	6
LA	5.6707	6	0.6707	7
SC	5.7852	6	0.7852	7
AL	5.9807	6	0.9807	7
CO	6.282	7	0.282	7
MN	6.6182	7	0.6182	8
WI	7.0955	8	0.0955	8
MD	7.2097	8	0.2097	8
MO	7.4856	8	0.4856	8
TN	7.9388	8	0.9388	9
AZ	7.9852	8	0.9852	9
IN	8.0959	9	0.0959	9
MA	8.1682	9	0.1682	9
WA	8.4094	9	0.4094	9
VA	10.0087	11	0.0087	11
NJ	10.9672	11	0.9672	12

NC	11.9115	12	0.9115	13
GA	12.1129	13	0.1129	13
MI	12.3421	13	0.3421	13
OH	14.4053	15	0.4053	15
PA	15.8577	16	0.8577	17
IL	16.0189	17	0.0189	17
FL	23.5355	24	0.5355	25
NY	24.1834	25	0.1834	25
TX	31.4646	32	0.4646	32
CA	46.4988	47	0.4988	48
	total	409	+26	435