Problem 5:

1. Show that the formula for the altitude, h, of a satellite above the Earth's surface is

$$h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$$

$$a = v^2/r = \frac{GM}{r^2} = \frac{4\pi^2 r}{T^2} \quad ; \; r = \text{orbit radius} = R+h$$

$$r^2 \cdot \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} \cdot r^2$$

$$\frac{4\pi^2 r^3}{T^2} = GM \rightarrow r^3 = \frac{GMT^2}{4\pi^2} \rightarrow r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$* \; r = R+h$

$$R+h = \sqrt[3]{\frac{GMT^2}{4\pi^2}} \rightarrow \boxed{h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R}$$

2. Script is saved on bitbucket under HW1_5.py

a) @ $T = 1$ day $= 86{,}400$ seconds
$h = 3.59 \times 10^7$ m //

b) @ $T = 90$ min $= 5{,}400$ seconds
$h = 2.79 \times 10^5$ m //

c) @ $T = 45$ min $= 2{,}700$ seconds
$h = -2.18 \times 10^6$ m //

Conclusion: ~~The~~ As the period decrease so does the altitude.

3.

@ $T = 23.93$ hrs $= 86{,}148$ seconds, $h = 2.02 \times 10^7$ m

- It is roughly a 43.73 % difference from a solar day period
- Smaller altitude than T = 1 solar day