Forecasting Techniques

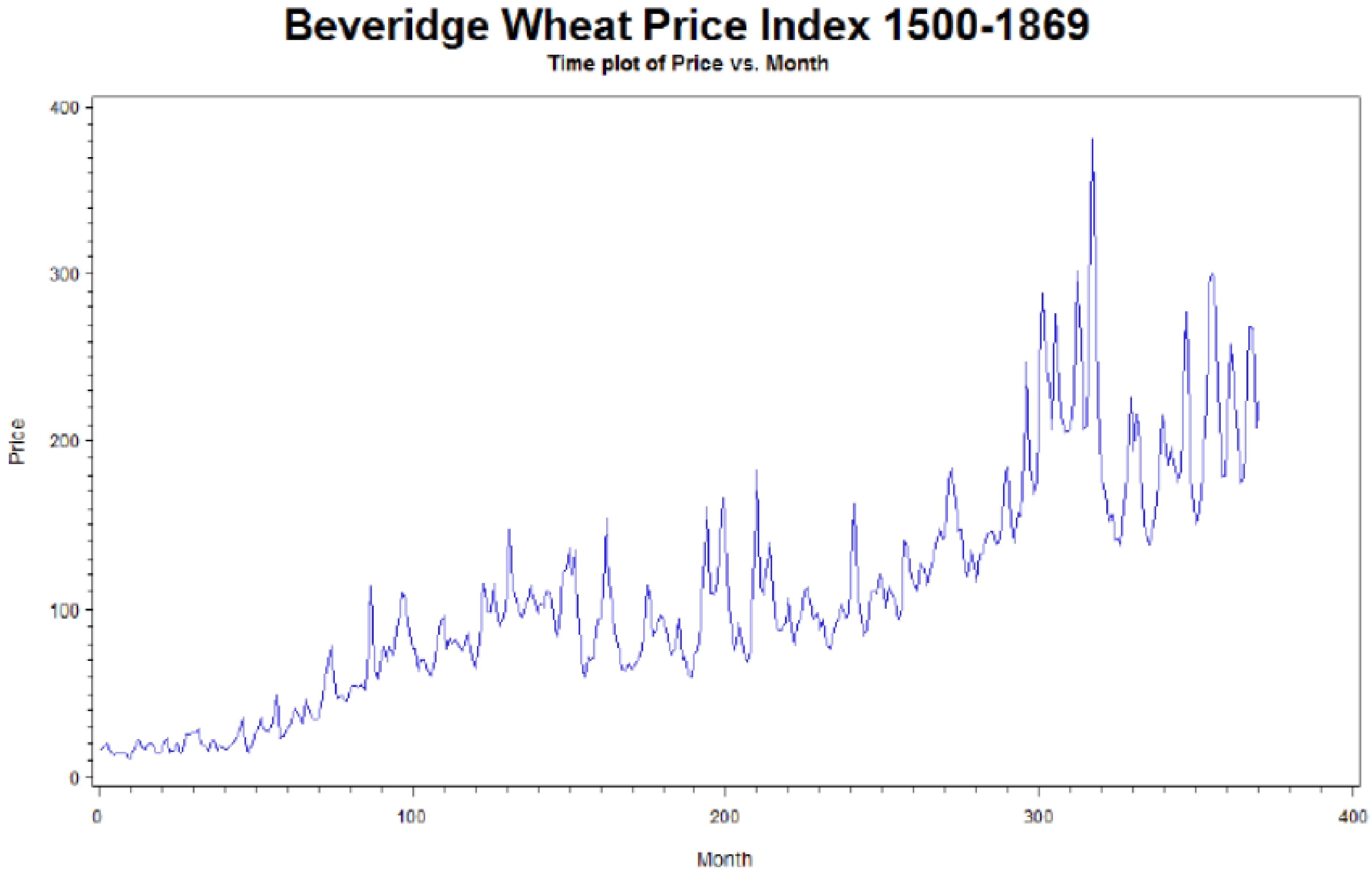
Outline

- Time Series Forecasting
- Naïve Forecasting
- Moving Averages
- Smoothing Methods
- Decomposition

What is a Time Series?

- A time series plot is a two-dimensional plot of time series data
- Vertical Axis – measures the variable of interest
- Horizontal Axis – corresponds to the time period

What is a Time Series?



What is a Time Series?

What is a Time Series?

What is NOT a Time Series?

- Point Processes – a little similar to time series data but really are not
- For example, dates of major railway disasters in the US
- Records of dates pertaining to machine/server downtimes

Date	Disaster (Typhoon)
November, 1984	Undang
November, 1985	Saling
November, 1987	Herming
November, 1990	Ruping
October, 1995	Rosing
December, 2006	Reming

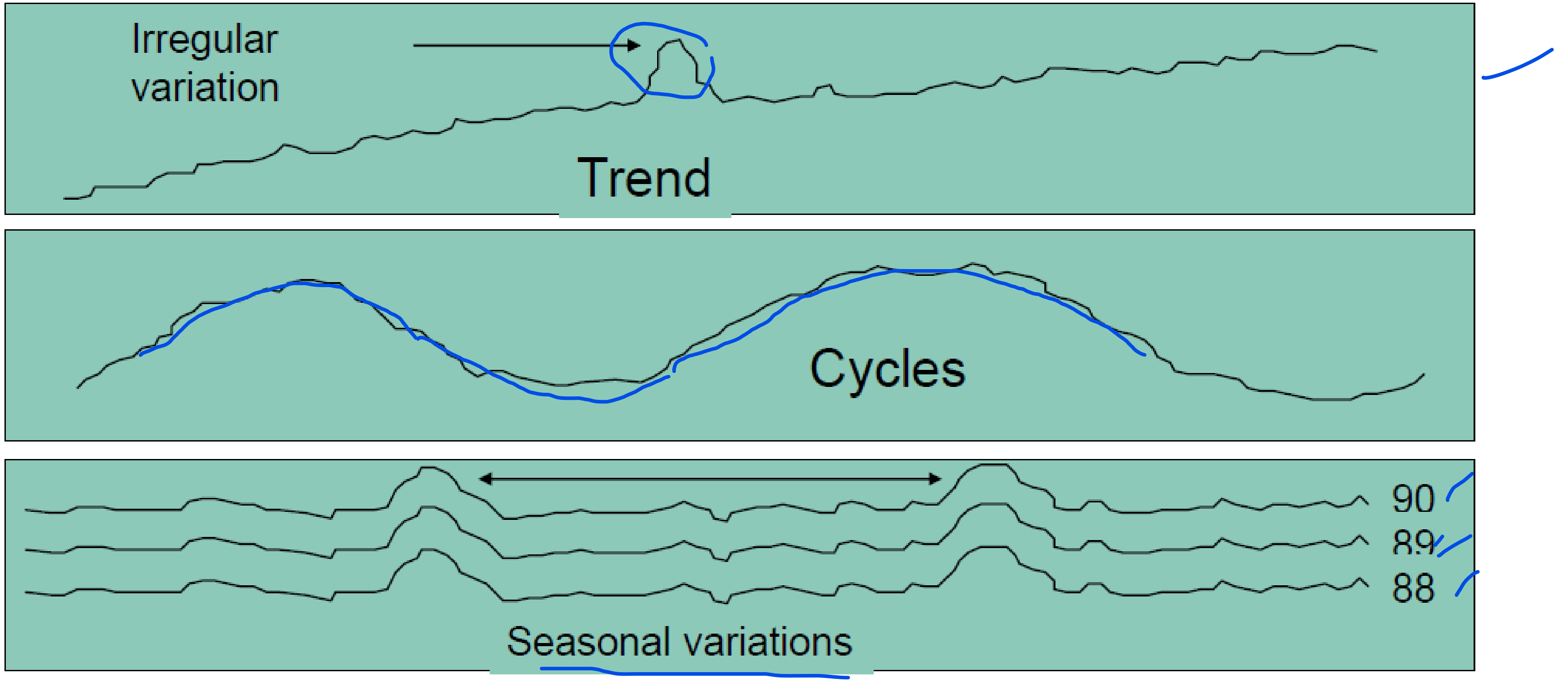
Time Series Forecasting

- Elements of a Demand Plot that are seen in a Time Series

1. *Trend*-long-term (upward or downward) movement in data
2. *Seasonality*-short-term regular (recurring) variations in data
3. *Cycle*–regular variation in data that are longer in term (usually beyond years) than seasons
4. *Irregular variations* -caused by unusual circumstances
5. *Random variations* -caused by chance

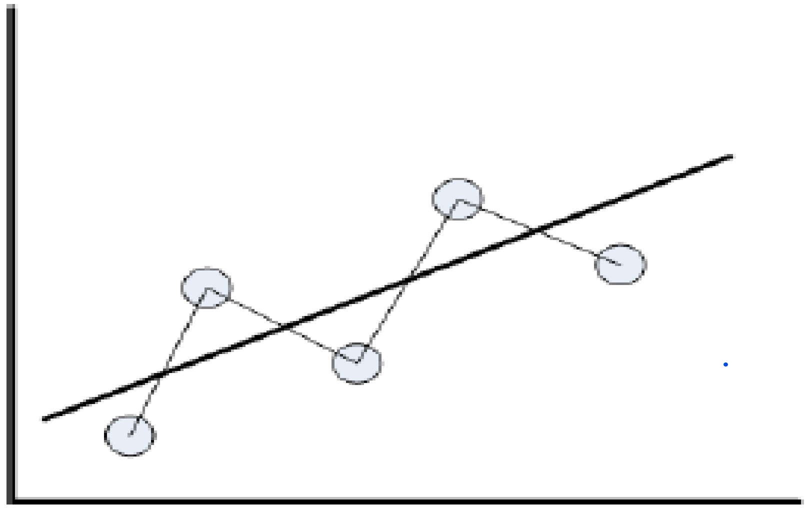
Time Series Forecasting

- Elements of a Demand Plot that are seen in a Time Series



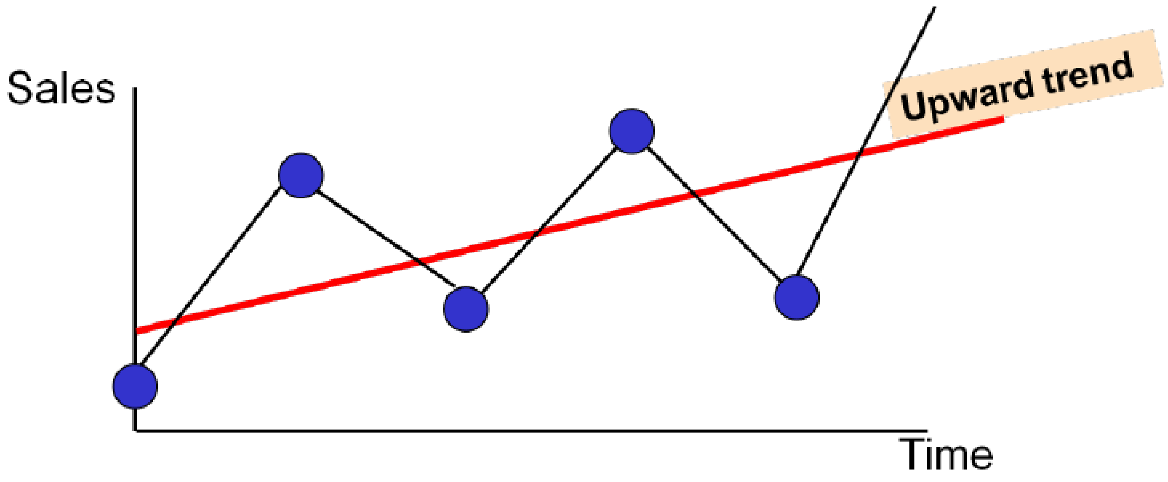
Trend Component

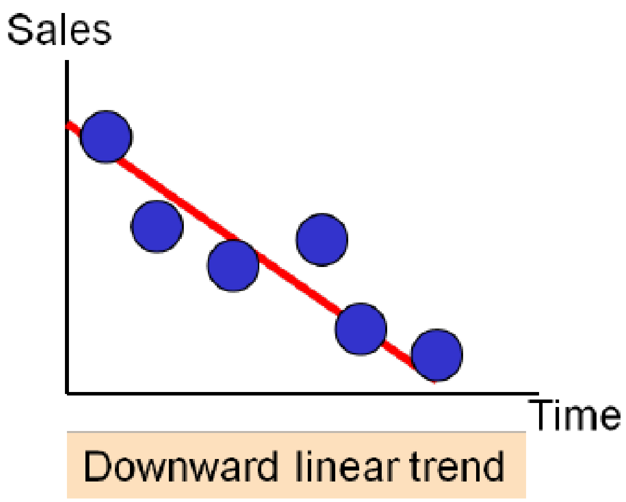
- Accounts for the gradual shifting of the time series to relatively higher or lower values over a long period of time.
- Trend is usually the result of long-term factors such as changes in the population, demographics, technology, or consumer preferences.
- Ex., climate variables exhibit cyclic variation over long periods

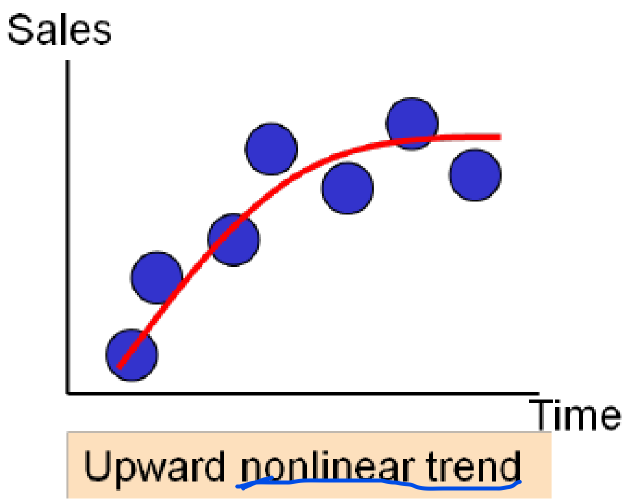


Trend Component

Trend Component

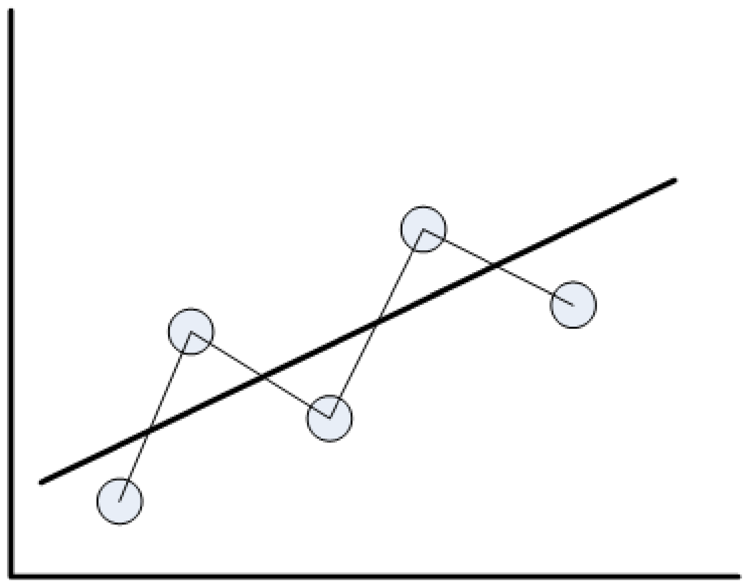






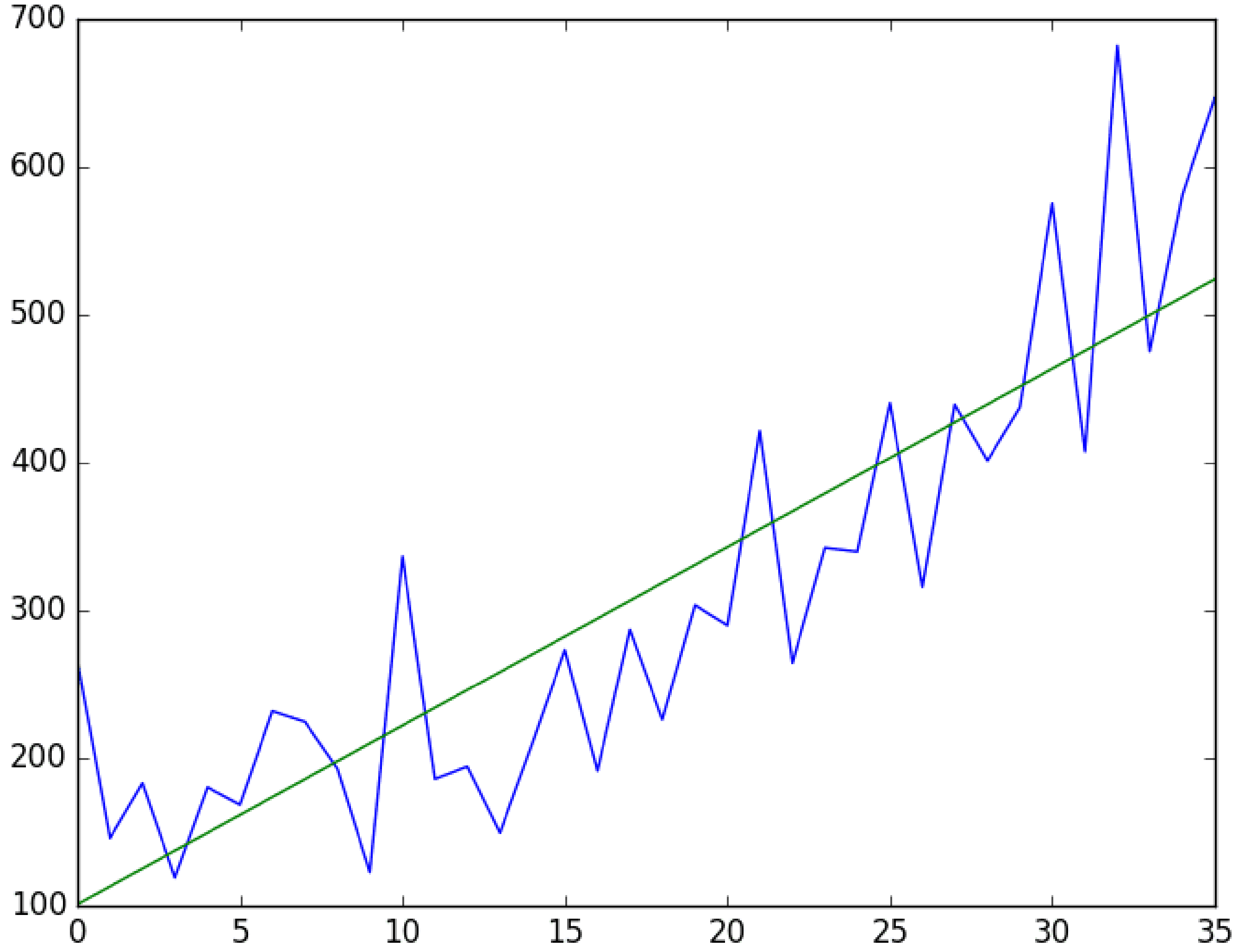
Trend Component

- Types of Trend
- Stochastic–forecast intervals grow over time
- Global
  - Polynomial –linear or quadratic
  - Exponential
  - Logistic
- Local(e.g., piecewise trend)



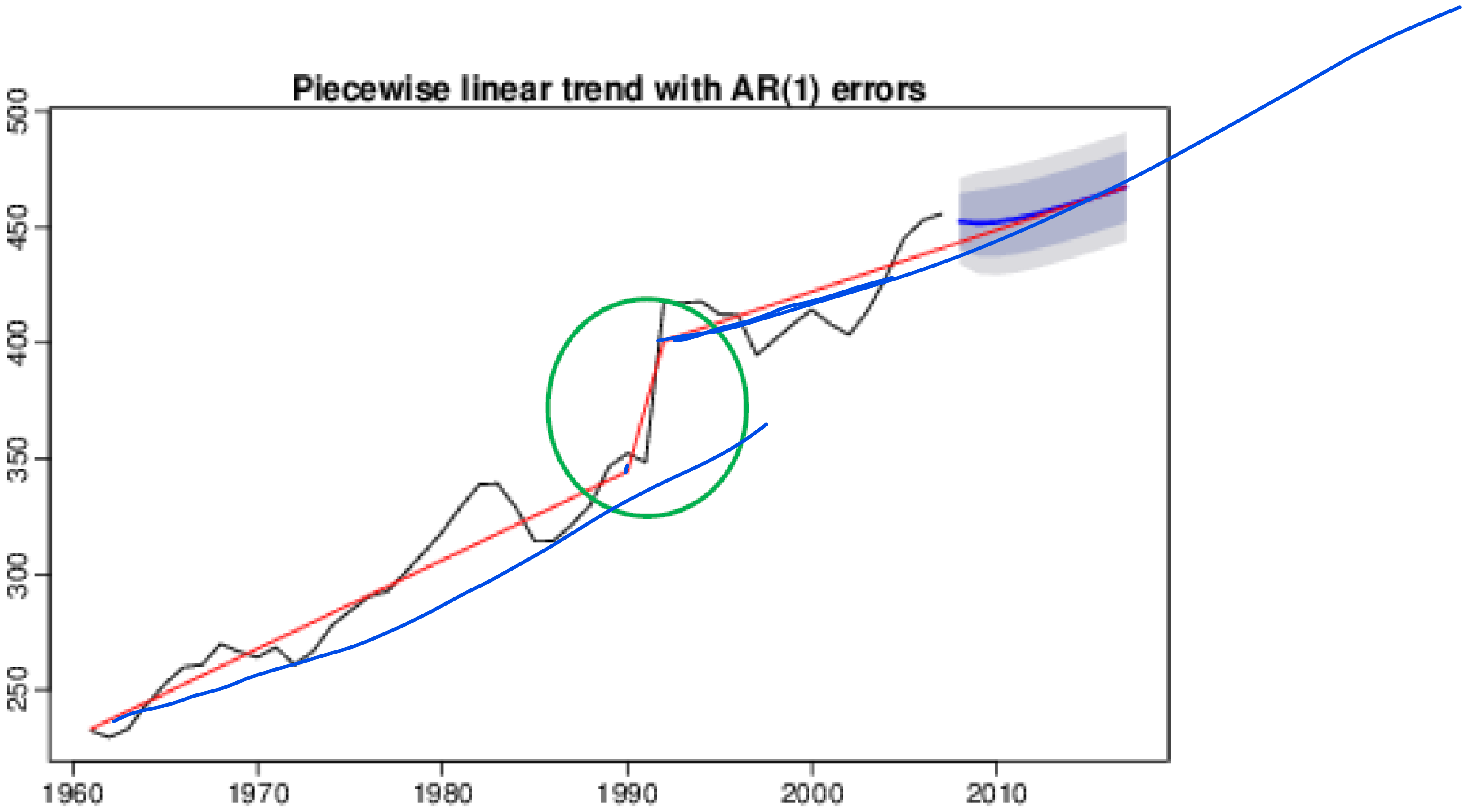
Trend Component

- Global



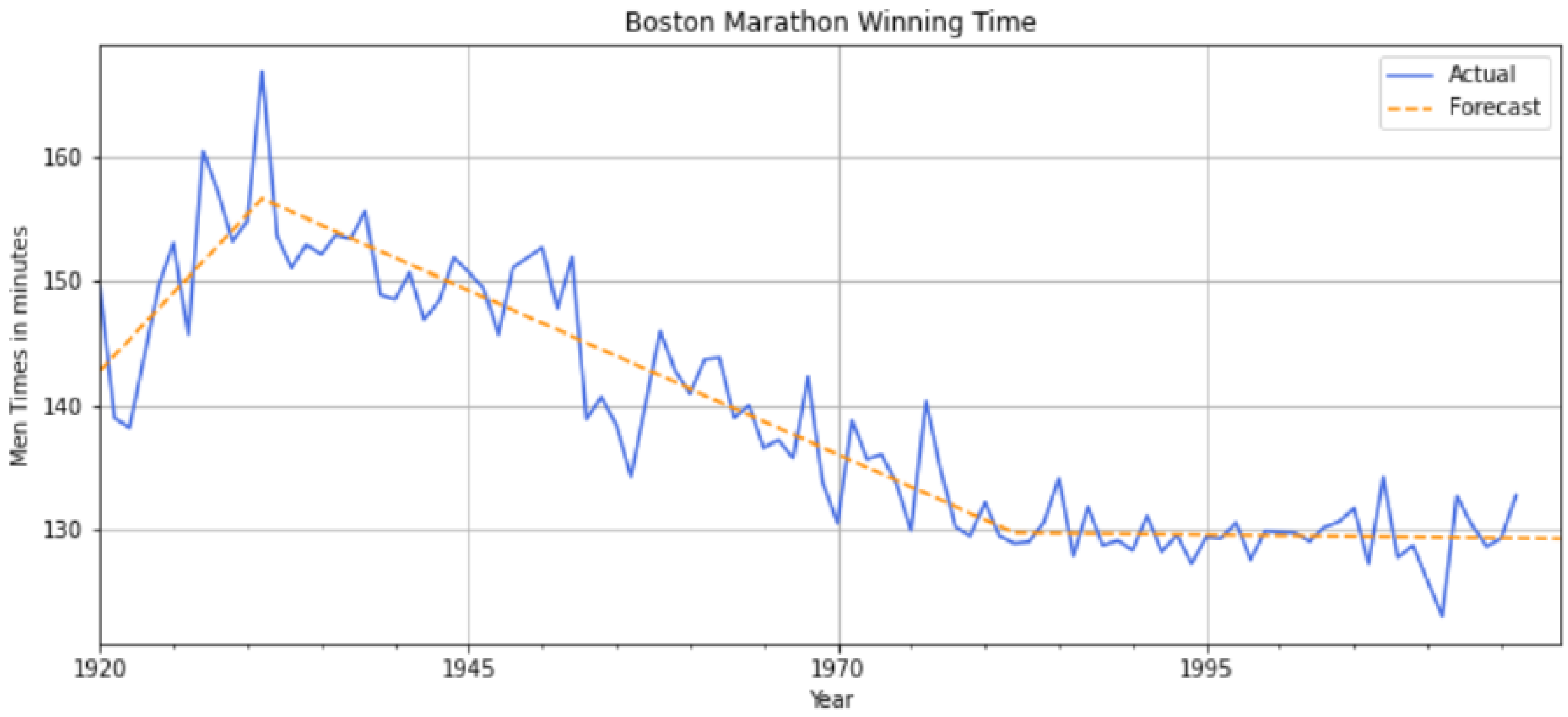
Trend Component

- Local (e.g., piecewise trend)



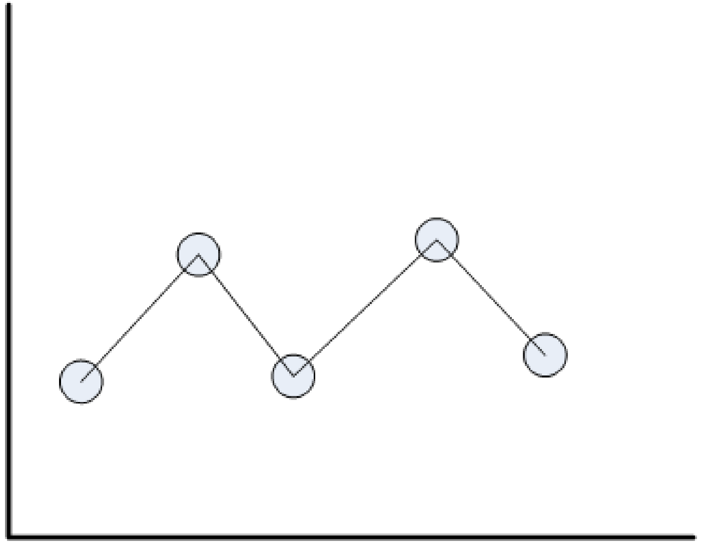
Trend Component

- Local (e.g., piecewise trend)

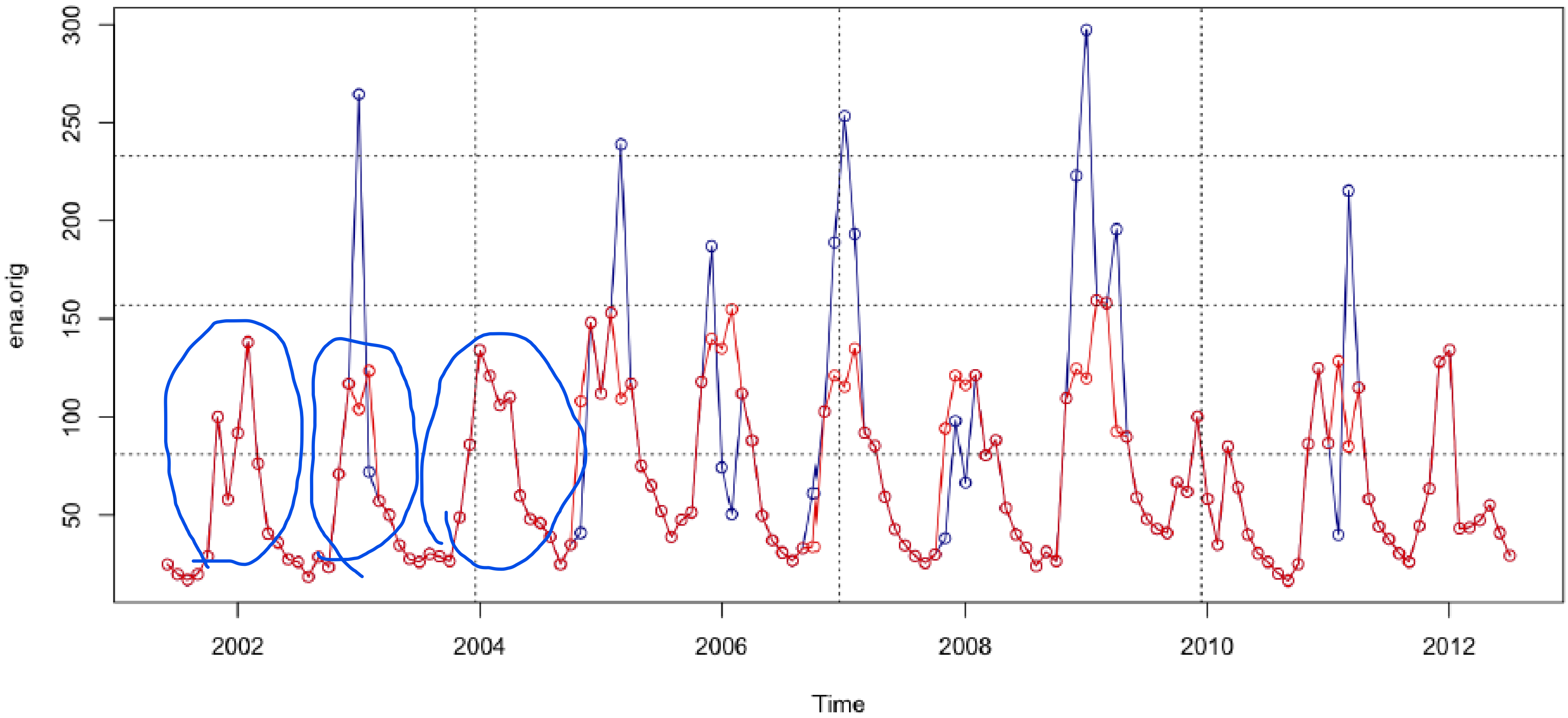


Seasonal Component

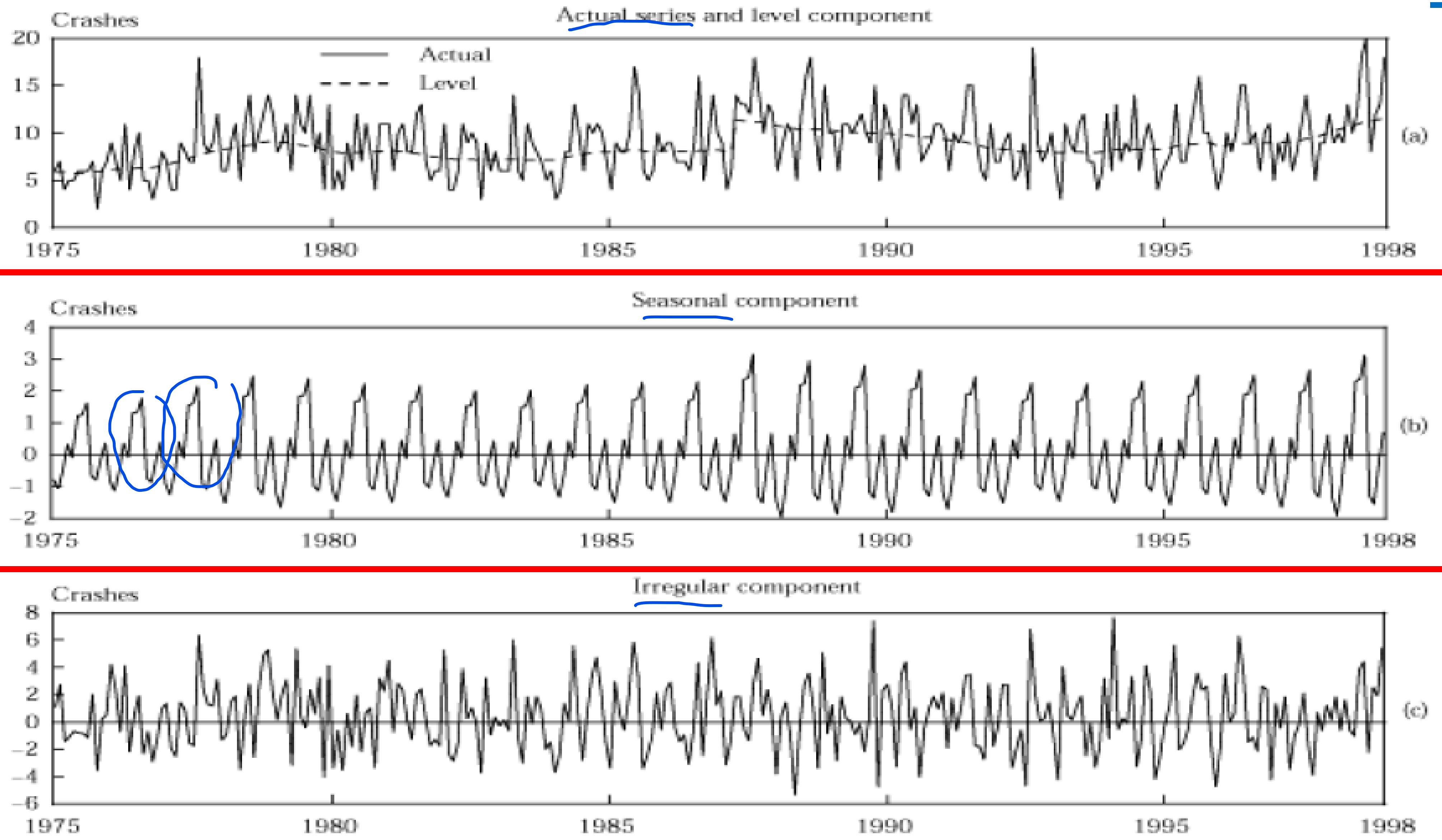
- Accounts for regular patterns of variability within certain time periods, such as a year, month, time of day, etc..
- The variability does not always correspond with the seasons of the year (i.e.winter, spring, summer, fall).
- There can be, for example, within-week or within-day "seasonal" or "periodic" behavior.
- Ex., Unemployment is typically "high" in winter and "lower" in summer



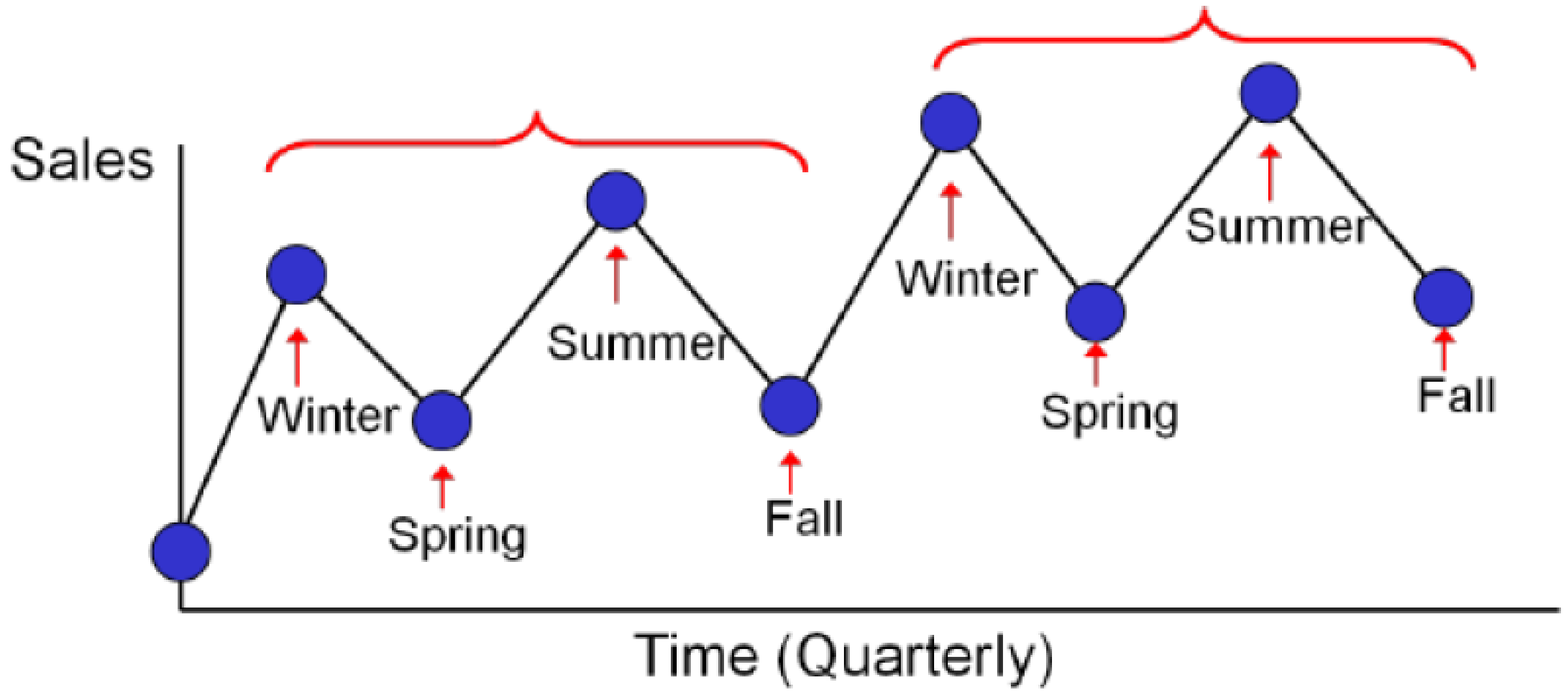
Seasonal Component



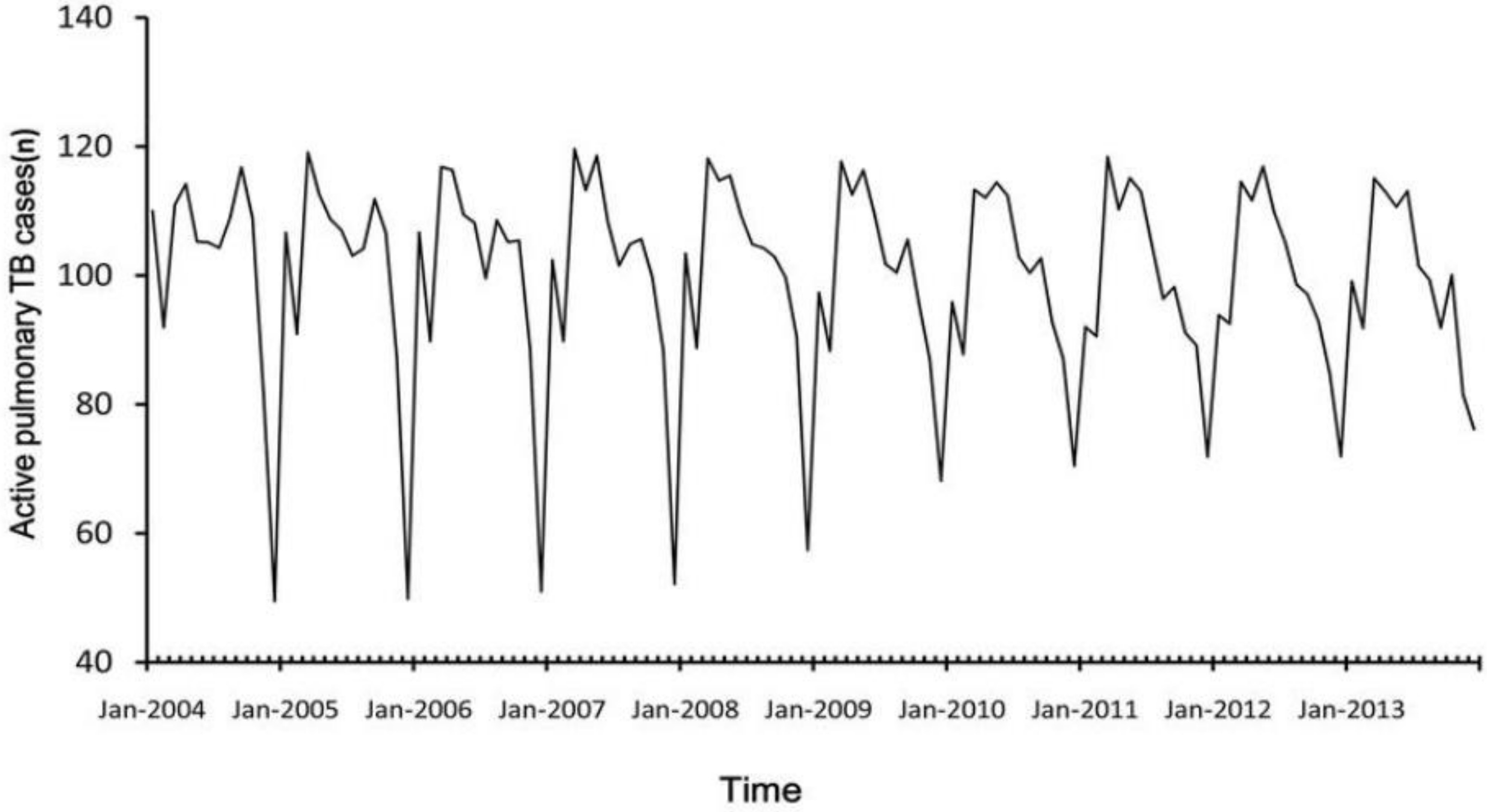
Seasonal Component



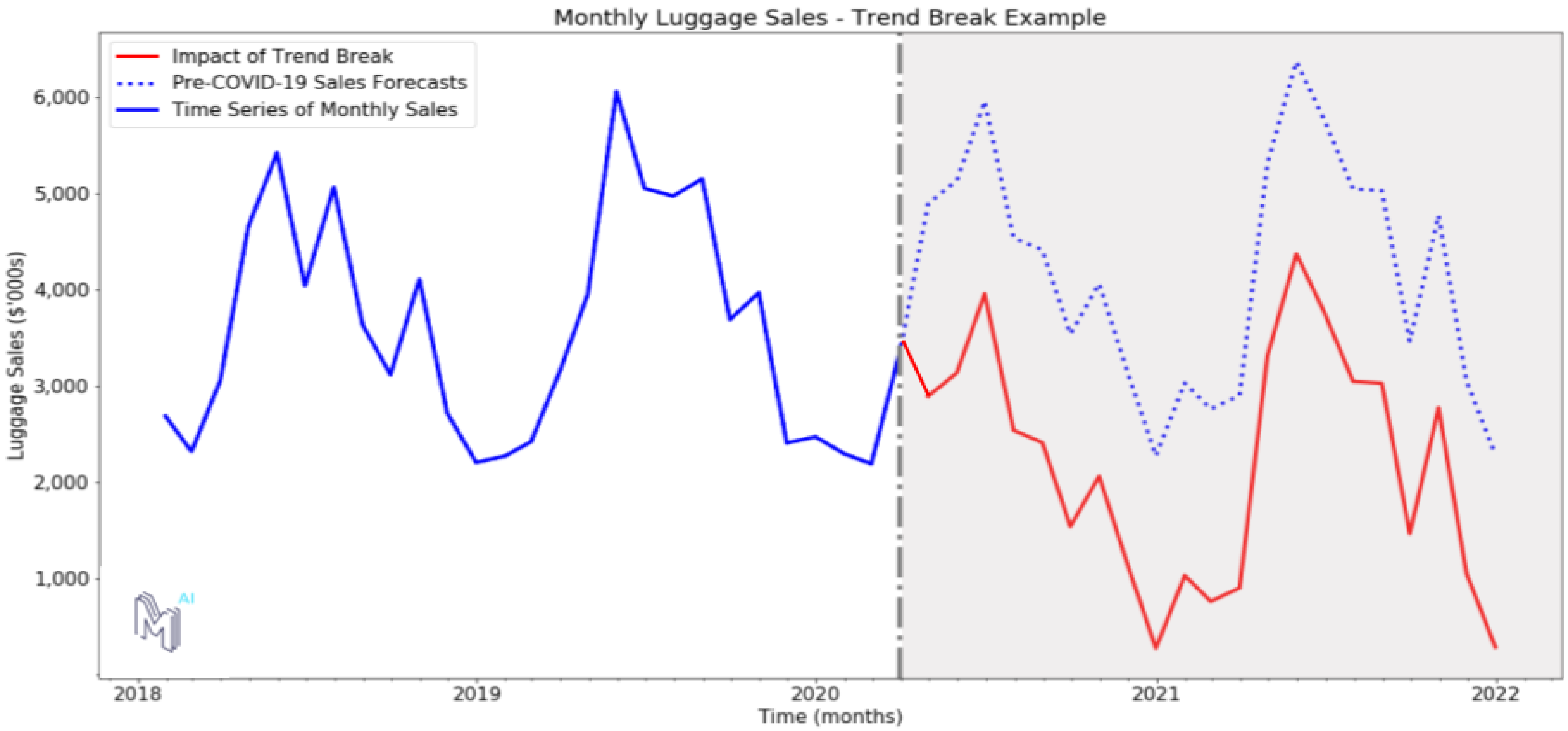
Seasonal Component



Seasonal Component

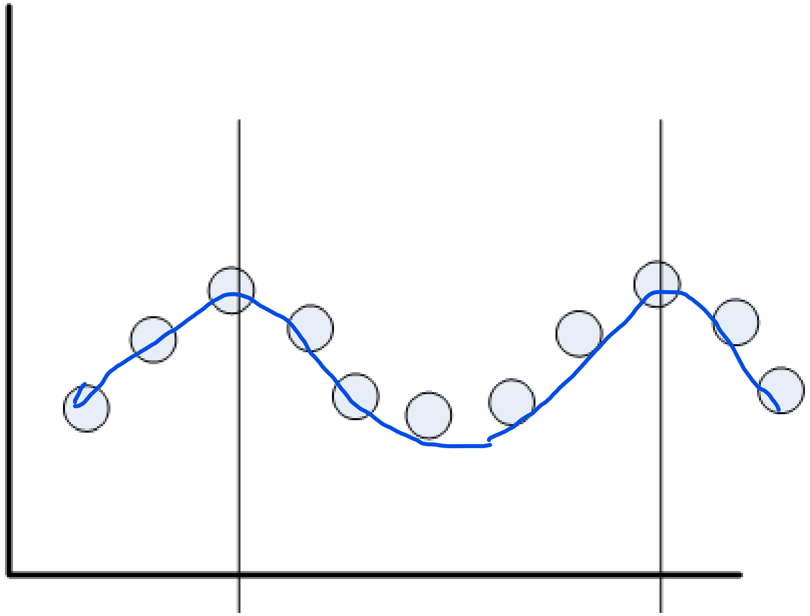


Seasonal Component

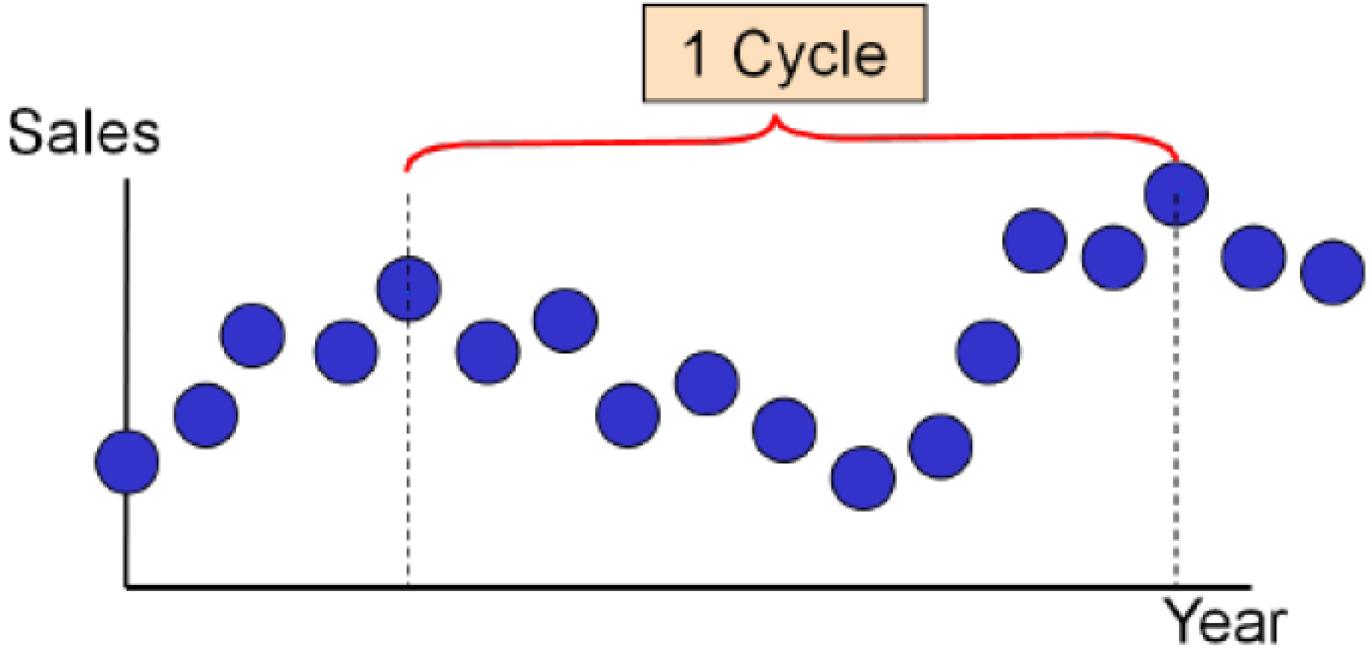


Cyclical Component

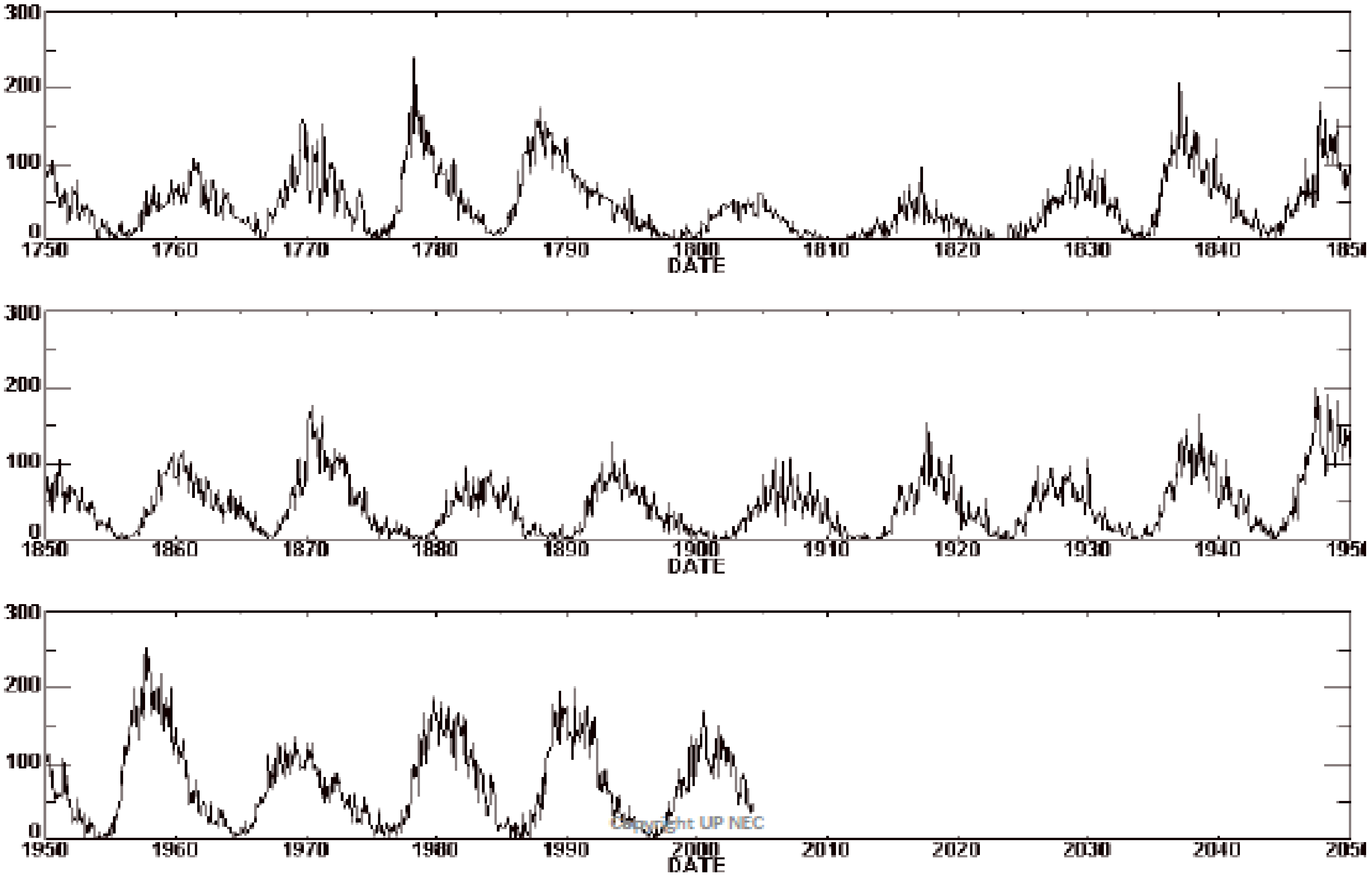
- Any regular pattern of sequences of values above and below the trend line lasting more than one year (or at 2 years) can be attributed to the cyclical component.
- Usually, this component is due to multiyear cyclical movements in the economy.



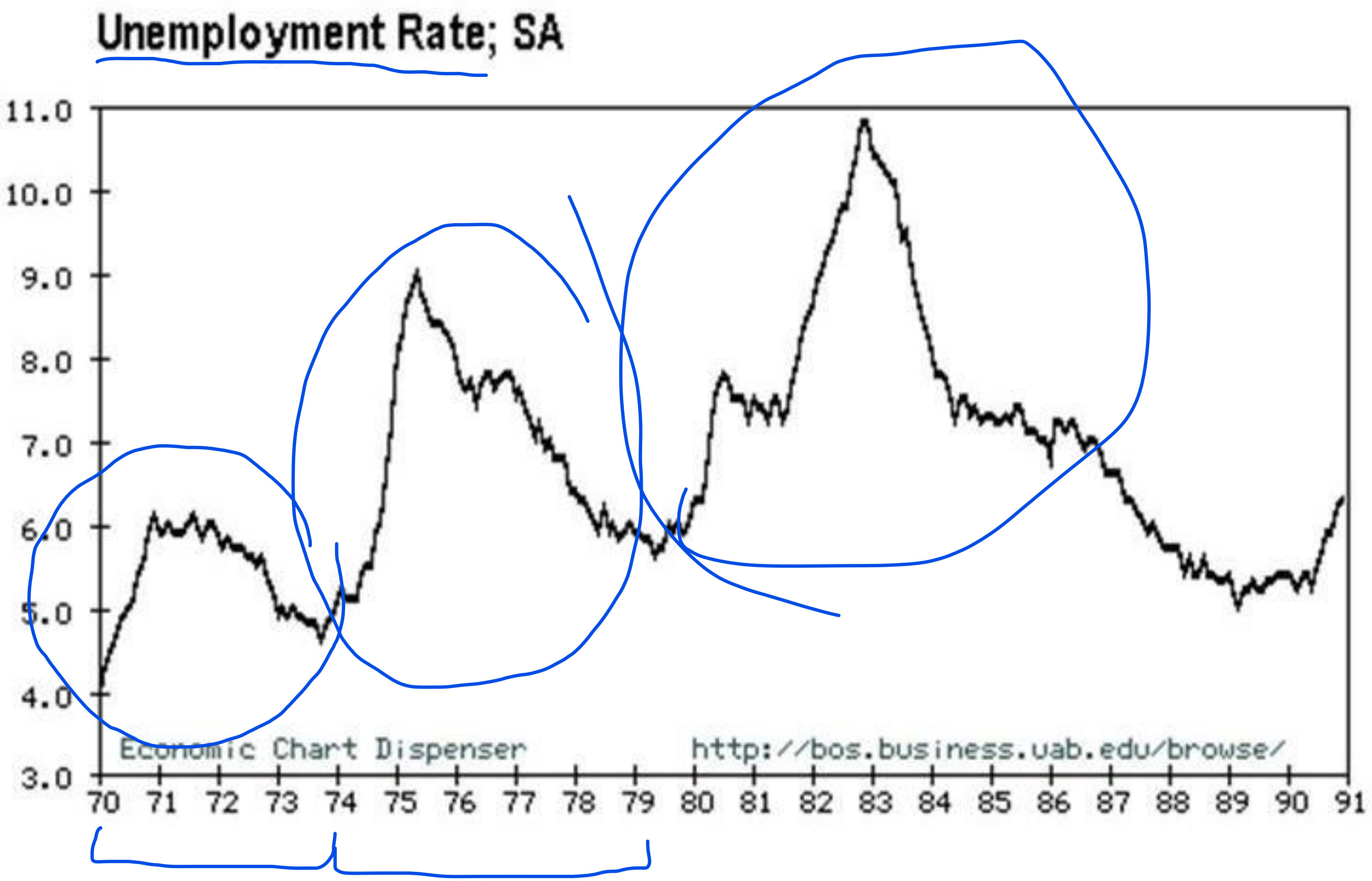
Cyclical Component



Cyclical Component



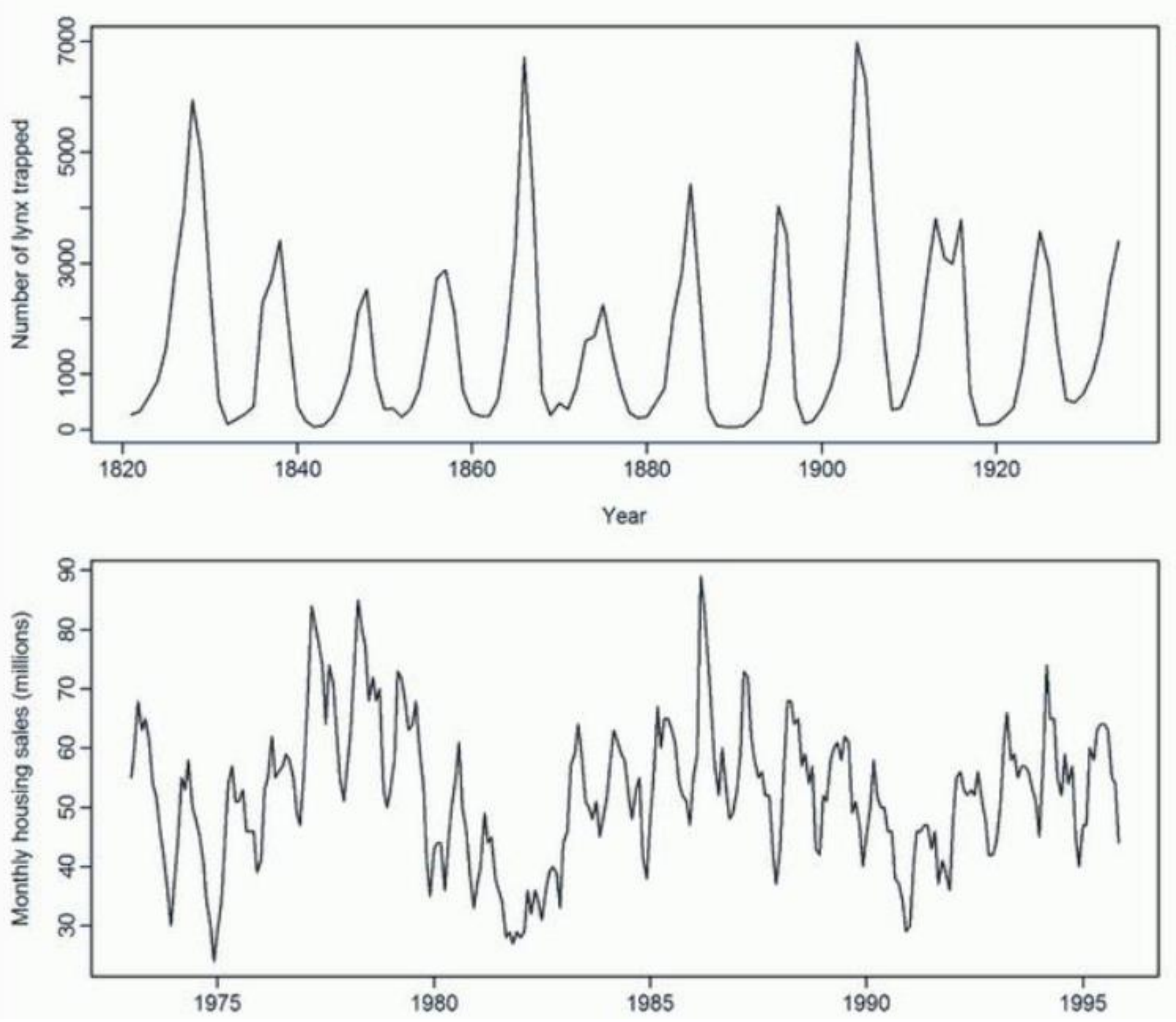
Cyclical Component



Seasonal vs. Cyclical Component

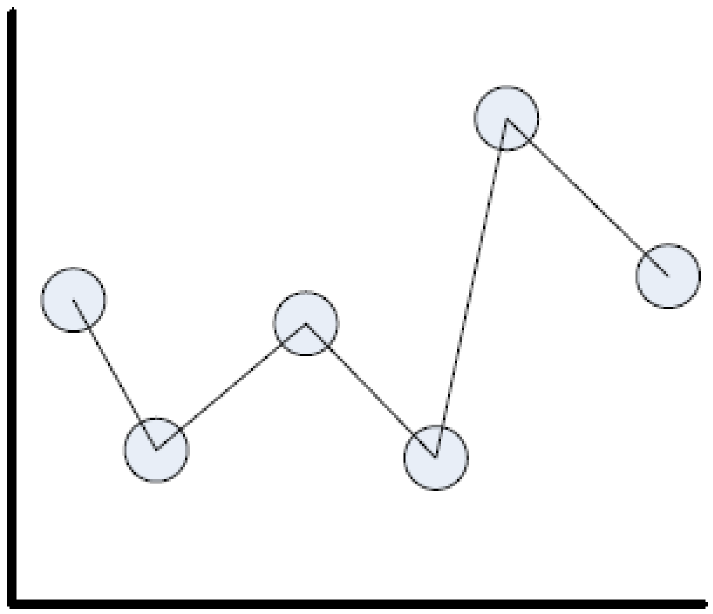
Cycles are not fixed at 8 to 10 years

Cycles are not fixed at 6 to 10 years;
Seasonality within each year

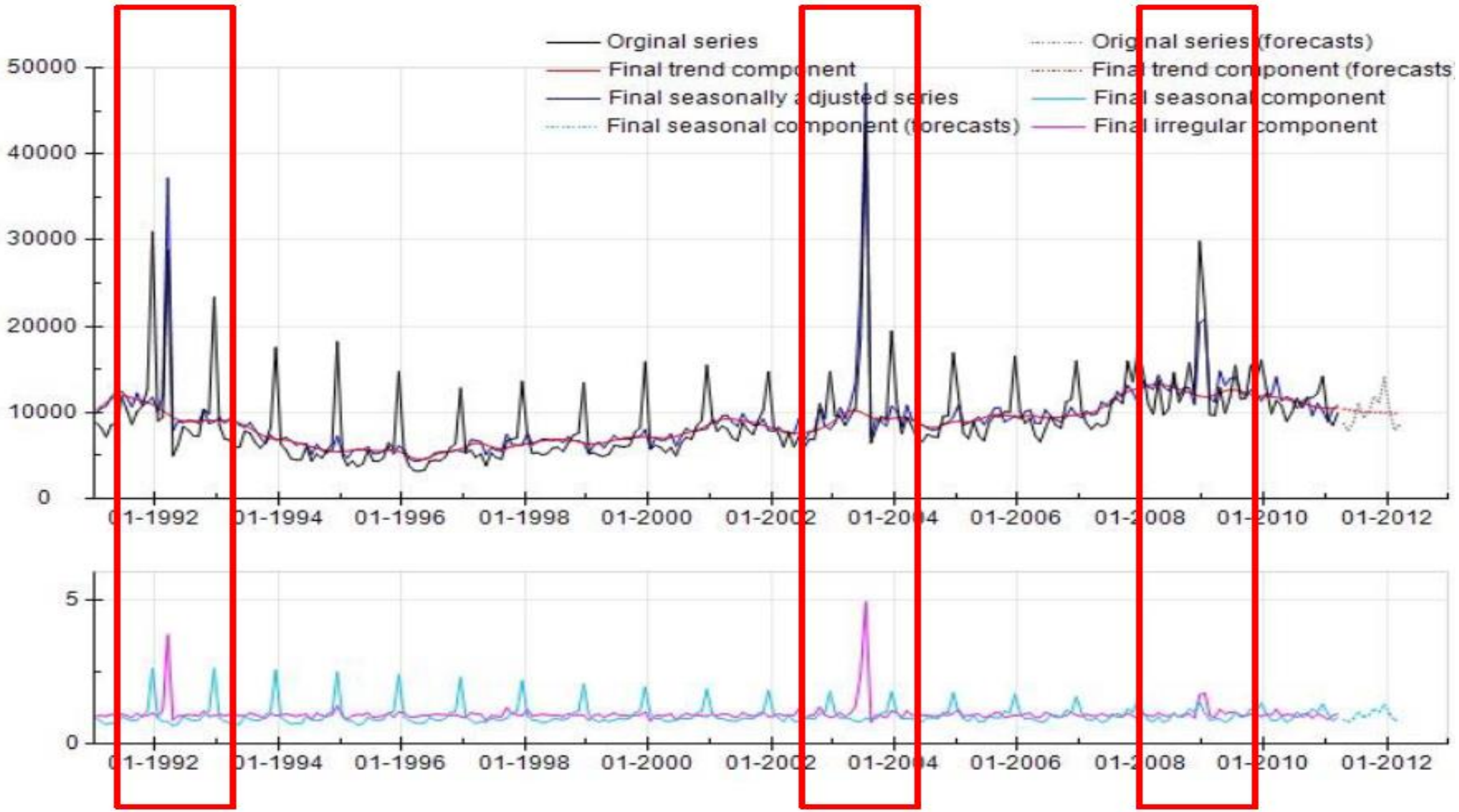


Irregular Component

- Component of a time series not accounted for by the other three components
- Random Error
- Usually ignored in analysis but forms the basis for model evaluation (regression)
- Unpredictable, random, “residual” fluctuations
- Noise in the time series
- Stochastic factors

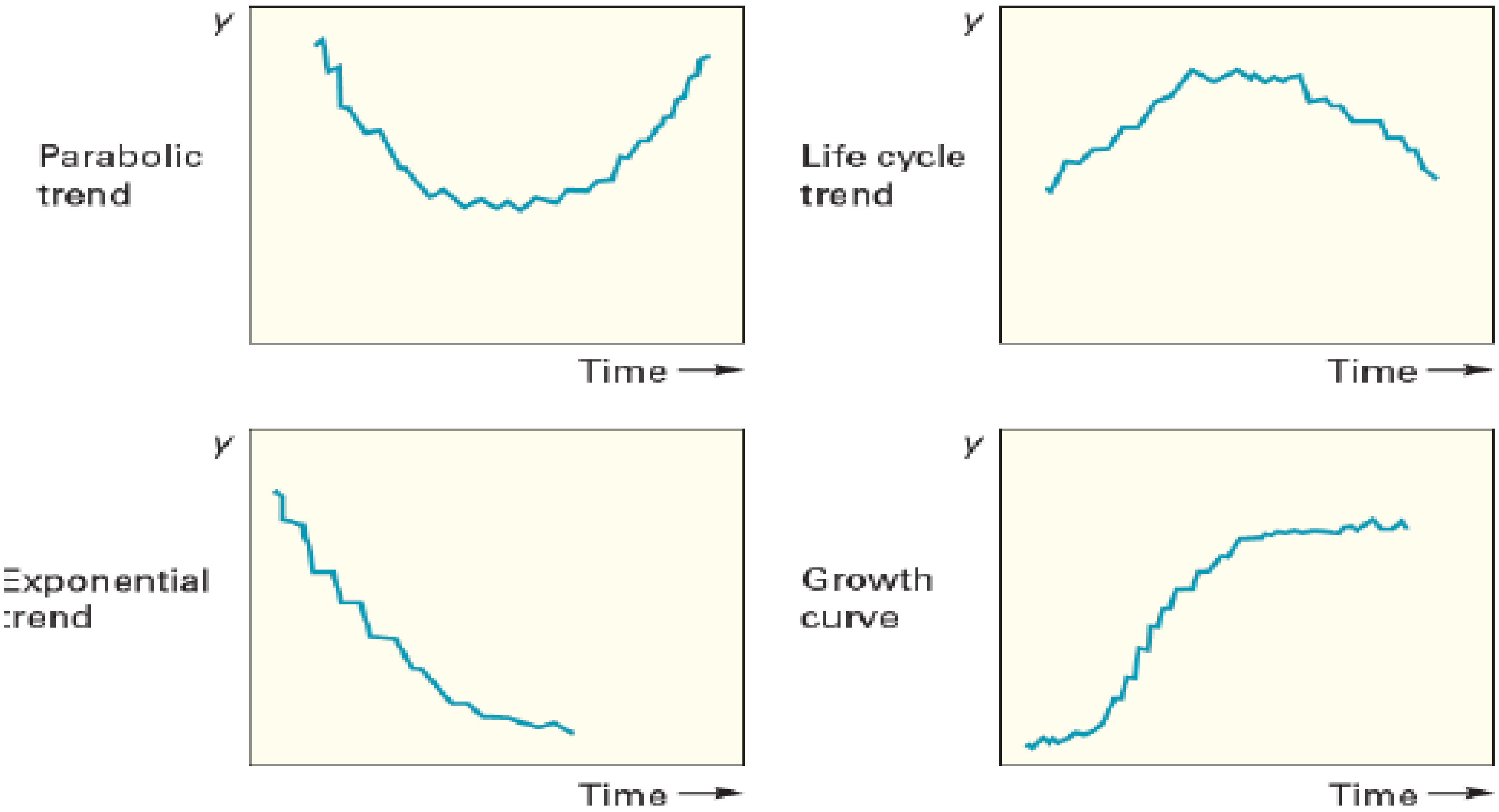


Irregular Component



Time Series Forecasting

Other Behaviors of a Demand Plot that seen on Time Series



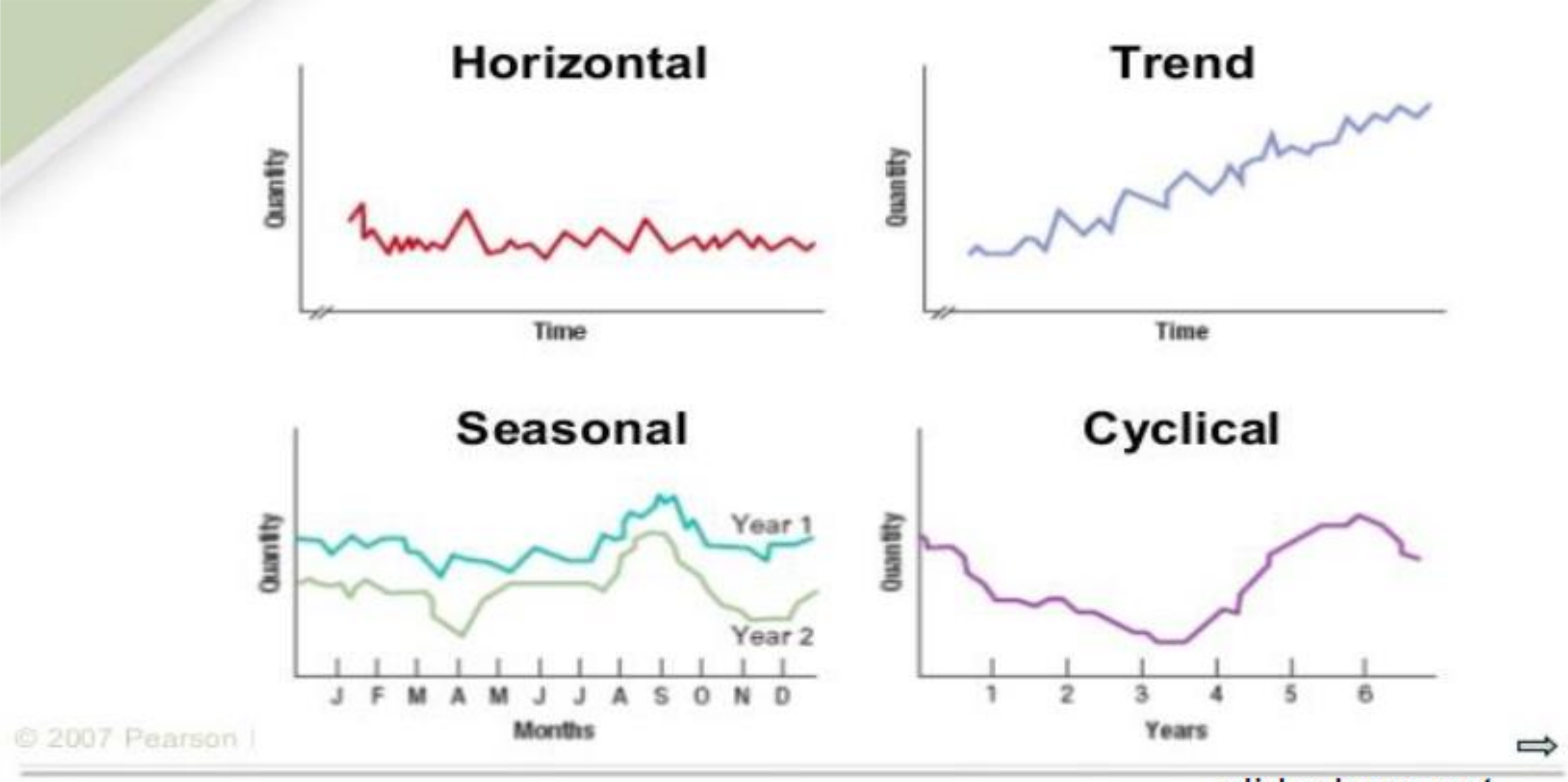
Stevenson, Operations Management

Time Series Forecasting
Notes on the Different Types of Forecasting Techniques

- Few companies use one technique only
- Most companies perform several forecasts using several techniques and combine results
- Combination of forecasts provide better accuracy

Appropriate Technique for Historical Behavior

- Historical Behaviors:

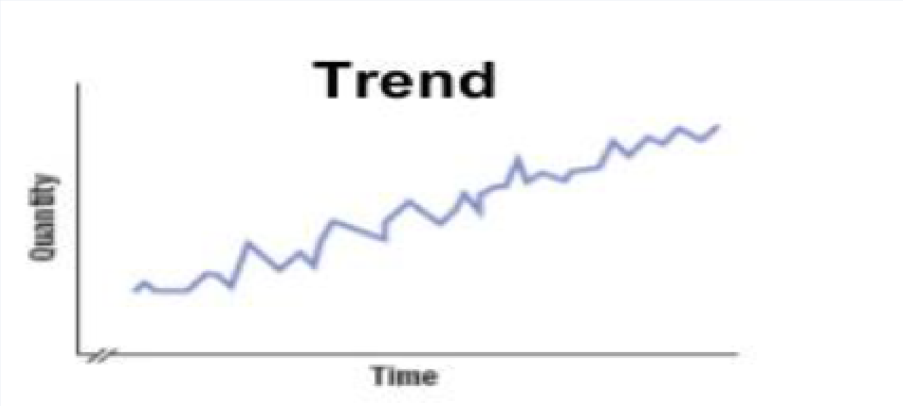


slideshare.net

Appropriate Technique for Historical Behavior

If the Historical Pattern is	Appropriate Technique(s)

	• naive
	• moving averages:
	  • simple moving average (SMA)
	  • weighted moving average (WMA)
	• smoothing
	  • single exponential smoothing

	• trend line (trend equation)
	  • regression (independent variable: time)
	• smoothing
	  • double exponential smoothing

Appropriate Technique for Historical Behavior

<table>
<tr><th>If the Historical Pattern is</th><th>Appropriate Technique(s)</th></tr>
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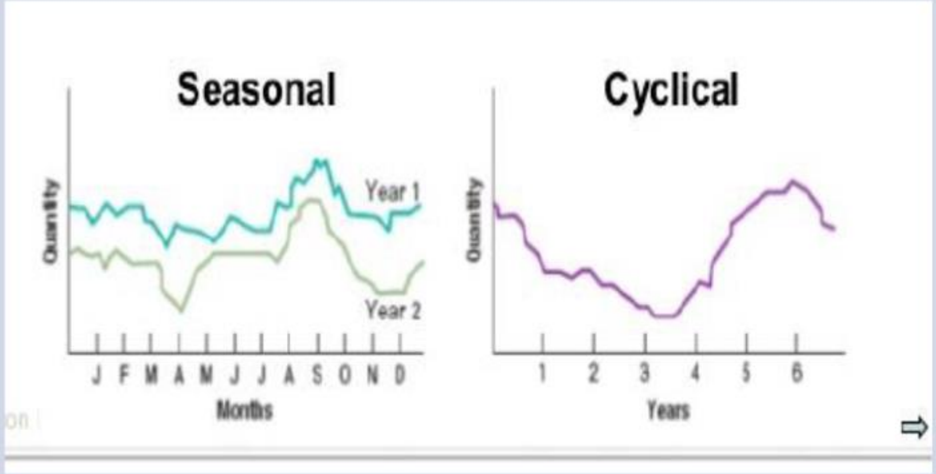

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• forecasting using seasonal index

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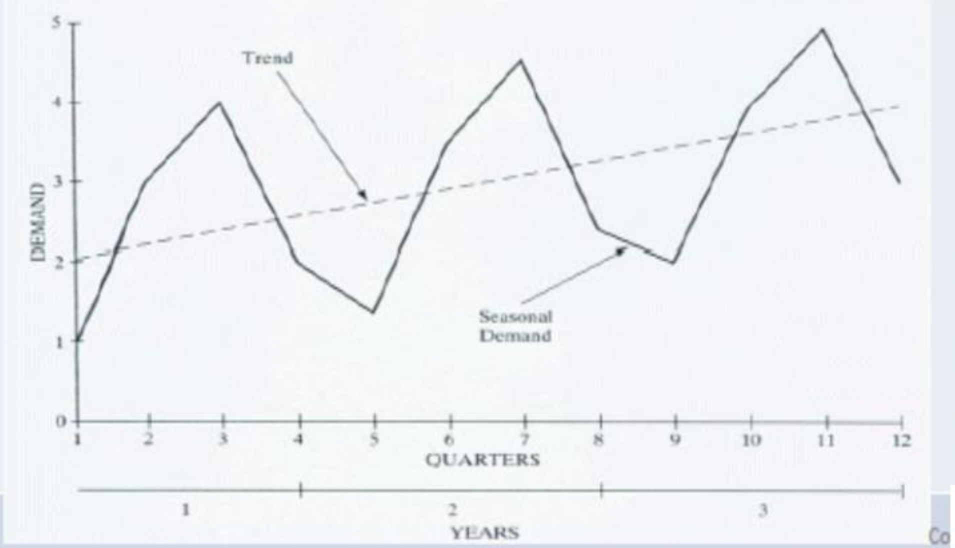

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• decomposition of the combination of pattern into individual patterns, forecasting, then combining forecasts of individual patterns

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- **Naïve Forecasting**
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- Smoothing Methods
- Trend Line Equation
- Decomposition

Naïve Method

"What happened today will happen tomorrow" .

Day	M	T	W	Th	F
Actual Demand (pieces)	75	90	100	80	85
Forecast (pieces)	-	75	90	100	80

Naïve Method

Period	1	2	3	4	5	6	7	8	Ave
Demand	74	90	100	60					
Base Level				60					
Forecast					60				

Naïve Method

- For plots with seasons (ex. daily seasons)

Day	M	T	W	Th	F	M
Demand	75	90	100	80	85	~~90~~
Forecast						75

***Then the next M will have a forecast of ~~90~~ 75

- For plots with trend

Day	M	T	W	Th	F	M
Demand	75	90	100			
Forecast		60	75			

***If you think there is trend of +15 between days, then forecast will have +15 between periods

Naïve Method

Advantages:

- Simple to use
- Virtually no cost
- Data analysis is non-existent
- Easily understandable

Disadvantage

- No smoothing of data (forecast values can be highly fluctuating)
- Cannot provide high accuracy

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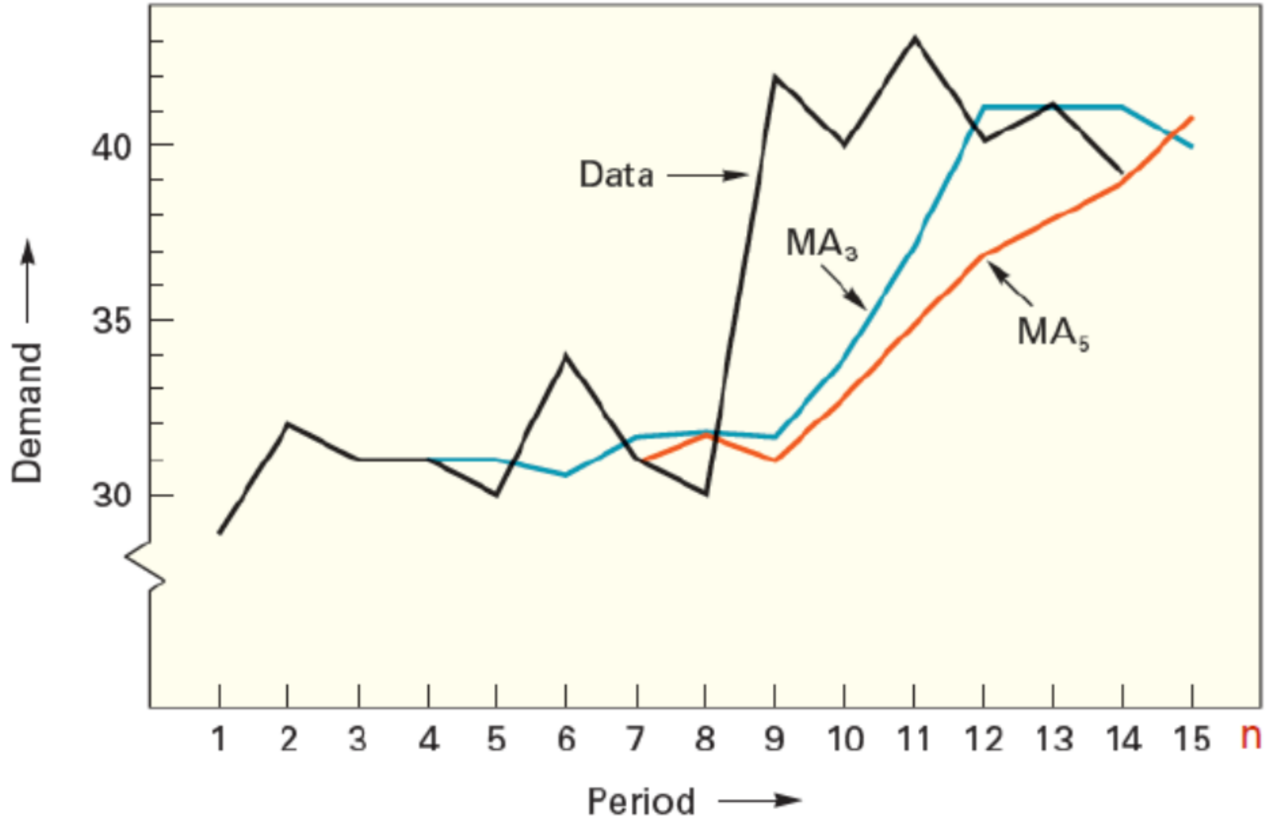
Moving Averages: Simple Moving Average



$$\text{Forecast } n+1 = MA_n = \frac{\sum_{i=1} A_i}{n}$$

$$F_{t+1} = \frac{\sum_{i=t-n+1}^{t} A_i}{n}$$

Moving Averages: Simple Moving Average

- **Smoothes out randomness**by averaging positive and negative random elements over several periods
- n --number of periods (*this example uses n = 4*)

	Day					
	M	T	W	Th	F	S
Actual Demand (pieces)	75	90	100	80	85	90
Forecast by SMA, n = 4 (pieces)					86.25	88.75

Forecast for F = (80 + 100 + 90 + 75)/4 = 86.25

Moving Averages: Weighted Moving Average

- Same idea as SMA, but less smoothing: more weight on recent demand data
- n --number of periods

α_i – weight applied to period t-i+1

Moving Averages: Weighted Moving Average

- Same idea as SMA, but less smoothing
- Typically, more weight is placed on recent demand data
- n --number of periods

	Day					
	M	T	W	Th	F	S
Actual Demand (pieces)	75	90	100	80	85	90
Forecast by SMA, n = 3 [0.6, 0.3, 0.1] (pieces)				94.5	87.0	85.0

Forecast for Th = (0.6*100)+(0.3*90) +(0.1*75) = 94.5

n-1 = 0.6

n-2 = 0.3

n-3 = 0.1

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Smoothing Methods: Exponential Smoothing

- Simpler equation, equivalent to WMA
- ***Forecast$_{t+1}$ = (α)(Actual) + (1-α)(Prev. Forecast)***

Prev

$F_{t+1} = \alpha A_t + (1-\alpha) F_t$

α – exponential smoothing parameter ($0 < \alpha < 1$)

Month	1	2	3	4	5	6
Demand	75	90	100	80	85	90
Forecast	80	79	82	87	85	85

Example. If α = 0.3,

*Forecast (2) = (0.3) x 75 + (0.7) x 80 = **79***

Forecast (3) = (0.3) x 90 + (0.7) x 79 = 82

Forecast (4) = (0.3) x 100 + (0.7) x 82 = 87

Smoothing Methods: Exponential Smoothing

Another way to look at it...

$$\textit{Forecast} = (\alpha)(\overset{\text{Prev}}{\textit{Actual}}) + (1-\alpha)(\textit{Prev. Forecast})$$

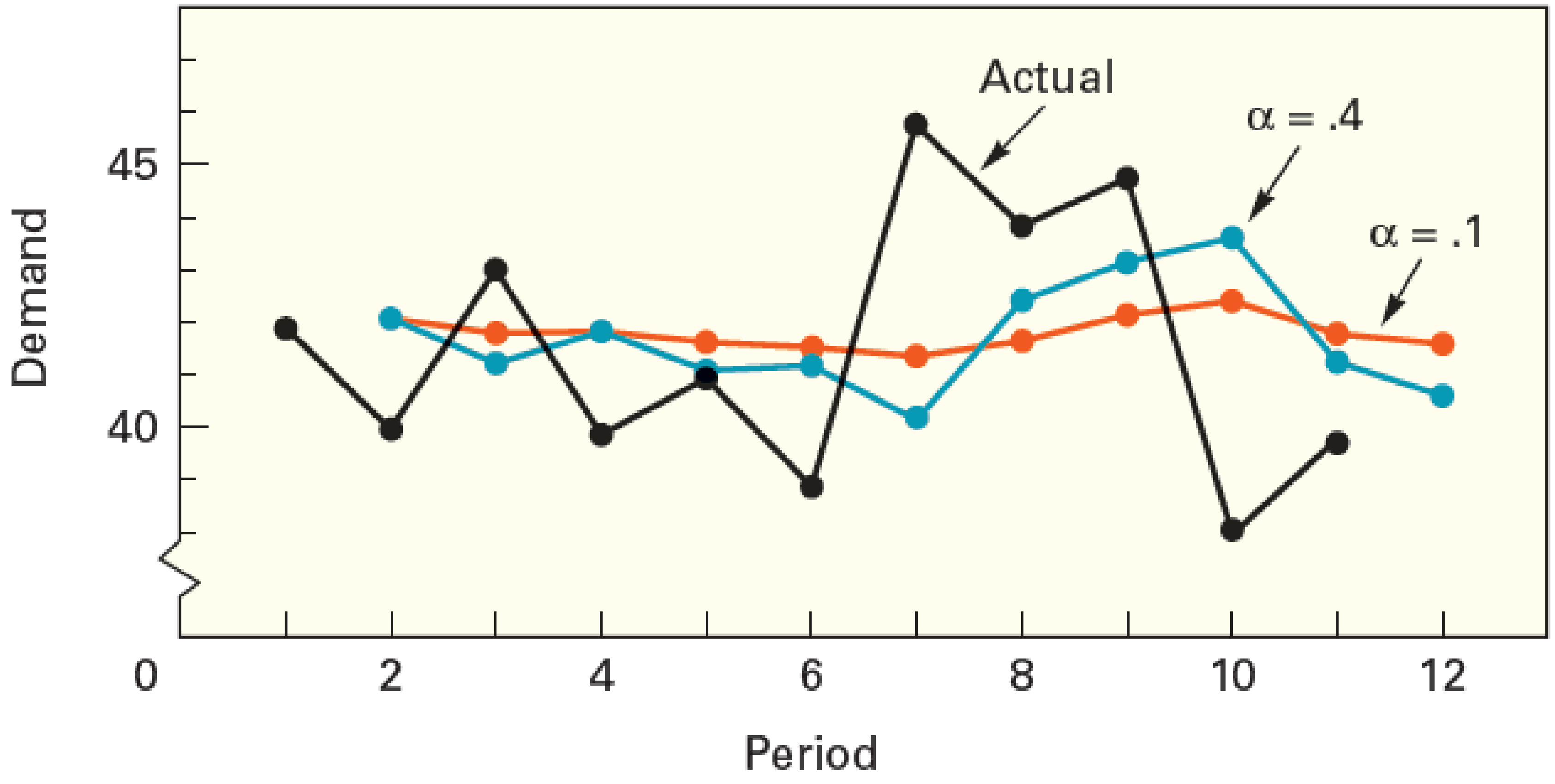
Rearranging Values...

$$\text{Forecast} = \alpha \overset{\text{Prev}}{\text{Actual}} + \underline{\text{Prev. Forecast}} - \alpha\text{Prev. Forecast}$$

$$= \text{Prev. }\underline{\text{Forecast}} + \underline{\alpha}(\overset{\text{Prev}}{\text{Actual}} - \text{Prev. Forecast})$$

$$\text{Forecast} \quad = \underline{\text{Prev. Forecast}} + \underline{\alpha}(\underline{\text{Error}})$$

Smoothing Methods: Exponential Smoothing



Practice 1 (Open Notes; Worksheet ok)

Create a 12-month forecast for Product A in card units. Starting with Nov 2010, up to Sept 2011. Assume that the forecast for Oct 2010 is 150 units. And α is 0.2

Product A	Oct 2010	Nov	Dec	Jan 2011	Feb	Mar
Actual Demand	163	132	73	89	128	104
	Apr	May	Jun	Jul	Aug	Sep
Actual Demand	115	102	127	73	69	129

Trend Adjusted Exponential Smoothing

Recall that:

Exponential Smoothing

Prev

$$Forecast = (\alpha)(Actual) + (1-\alpha)(Prev.\ Forecast)$$

Or

$$Prev.\ Forecast + \alpha(Error)$$

Trend Adjusted Exponential Smoothing

Smoothing Factor: A number between 0 and 1. This is the weight of the previous actual demand.

Example: Assuming $\alpha = 0.3$

Month	1	2	3	4	5	6
Demand	75	90	100	80	85	90
Forecast	80	**79**	**82**	**87**	85	85

$F_{t+1} = \alpha A_t + (1-\alpha) F_t$

*Forecast (2) = (0.3) x 75 + (0.7) x 80 = **79***

*Forecast (3) = (0.3) x 90 + (0.7) x 79 = **82***

*Forecast (4) = (0.3) x 100 + (0.7) x 82 = **87***

Trend Adjusted Exponential Smoothing

Smoothing Factor: A number between 0 and 1. This is the weight of the previous actual demand.

+NEW: *Trend Factor:* Also between 0 and 1. This is the weight of the change in trend.

Forecast = *Base* + *Trend*

Base: (α)(Actual) + (1-α)(Prev. Forecast)

Prev

Trend: (β)(Base – Prev.Base) + (1- β) (Prev.Trend)

$$F_{t+1} = B_{t+1} + T_{t+1}$$

$$B_{t+1} = \alpha A_t + (1-\alpha) F_t$$

$$T_{t+1} = \beta(B_{t+1} - B_t) + (1-\beta) T_t$$

Trend Adjusted Exponential Smoothing

Forecast = Base + Trend $\alpha = 0.3$, $\beta = 0.2$

Base: (α)(Prev. Actual) + (1-α)(Prev. Forecast)

Trend: (β)(Base – Prev.Base) + (1- β)(Prev.Trend)

Month	1	2	3	4	5	6
Demand	74	90	100	60	92	111
Base	74	74				
Trend	0	0				
Forecast	74	74				

Forecast (2) Base = (0.3)(74) + (0.7)(74) = ***74***

Forecast (2) Trend = (0.2)(74-74) + (0.8)(0) = ***0***

Forecast = ***74***

Trend Adjusted Exponential Smoothing

Forecast = Base + Trend $\alpha = 0.3$, $\beta = 0.2$

Base: (α)(Prev. Actual) + (1-α)(Prev. Forecast)

Trend: (β)(Base – Prev.Base) + (1- β)(Prev.Trend)

Month	1	2	3	4	5	6
Demand	74	90	100	60	92	111
Base	74	**74**	78.8			
Trend	0	**0**	0.96			
Forecast	74	**74**	79.8			

Forecast (3) Base = (0.3) x 90 + (0.7) x 74 = ***78.8***

Forecast (3) Trend = (0.2) x (78.8-74) + (0.8) x 0 = ***0.96***

Forecast (3) = ***79.8***

Trend Adjusted Exponential Smoothing

Forecast = Base + Trend $\alpha = 0.3$, $\beta = 0.2$

Base: (α)(Prev. Actual) + (1-α)(Prev. Forecast)

Trend: (β)(Base – Prev.Base) + (1- β)(Prev.Trend)

Month	1	2	3	4	5	6
Demand	74	90	100	60	92	111
Base	74	**74**	**78.8**	**85.8**	79.6	83.7
Trend	0	**0**	**0.96**	**2.17**	0.49	1.21
Forecast	74	**74**	**79.8**	**88**	80.1	84.9

Forecast (4) Base = (0.3)(100) + (0.7)(79.8) = ***85.8***

Forecast (4) Trend = (0.2)(85.8-78.8) + (0.8)(.96) = ***2.17***

Forecast (4) = ***88.0***

Practice 3

Use Double Exponential Smoothing for the following actual demand data for a 4-month period: 30, 35, 37, 40 corresponding to Month 1, 2, 3 and 4, respectively

$\alpha = 0.3$, $\beta = 0.2$

Initial Trend = 0; Initial Base = 30

Solve for the forecast for month 2, 3, and 4. Assume that the forecast for month 1 is forecasted using the naïve method.

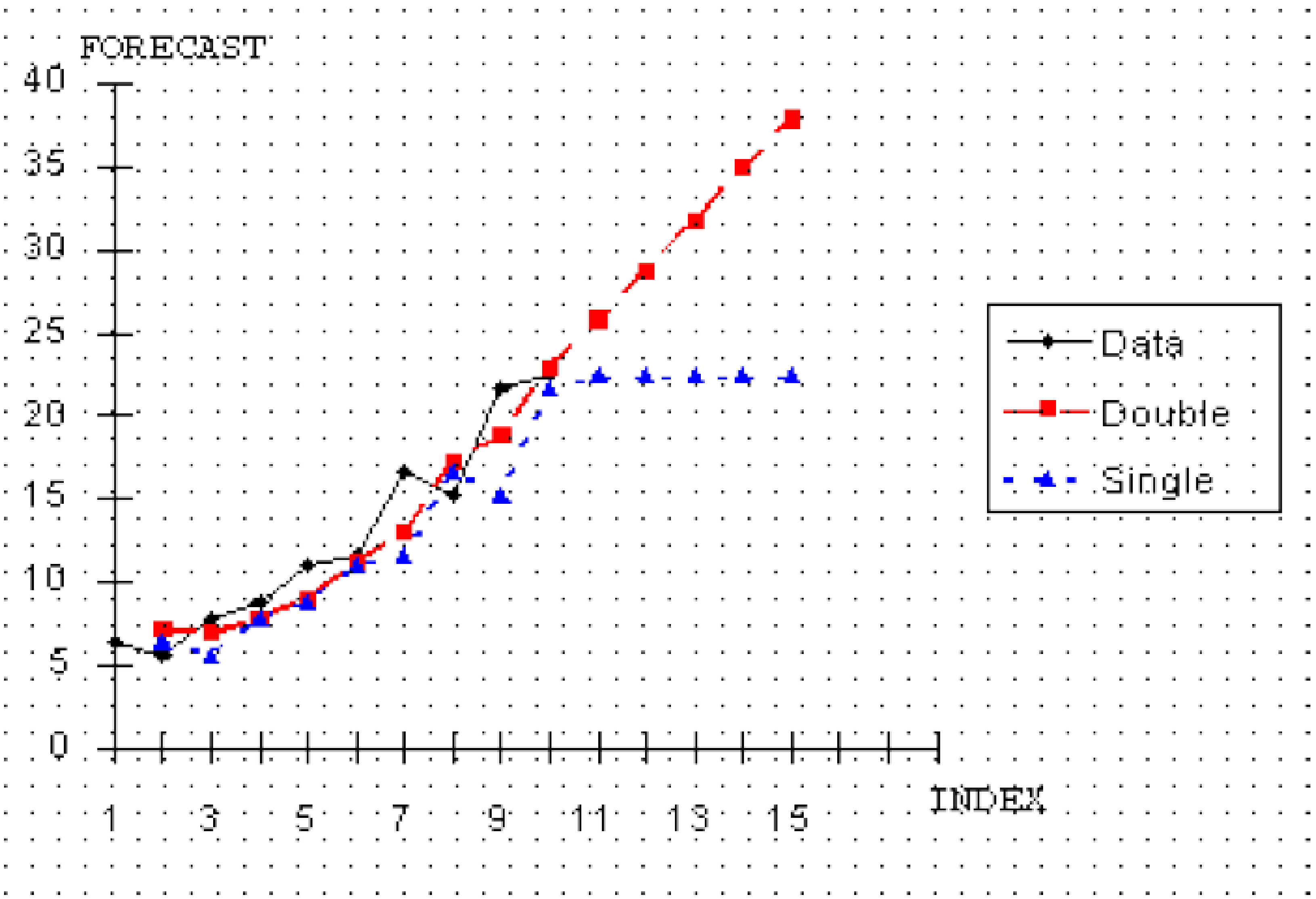
double

Forecast = Base + Trend $\alpha = 0.3$, $\beta = 0.2$

Base: (α)(Prev. Actual) + (1-α)(Prev. Forecast)

Trend: (β)(Base – Prev.Base) + (1- β)(Prev.Trend)

Trend Adjusted Exponential Smoothing



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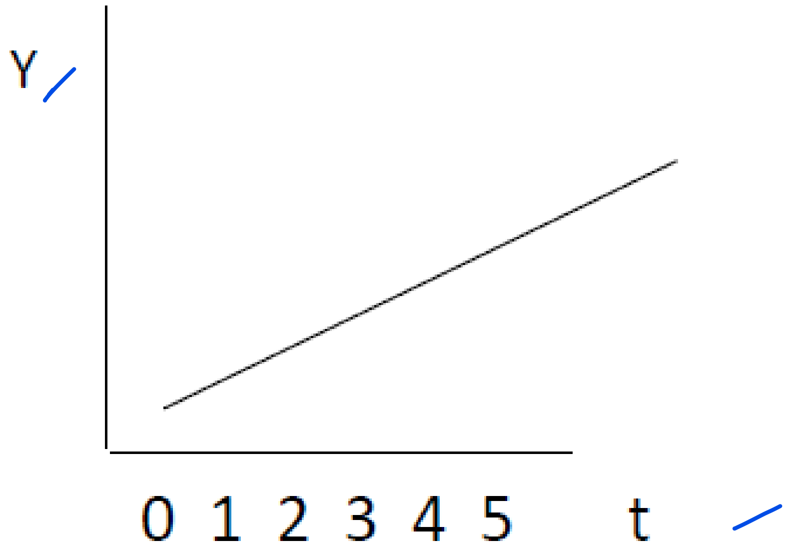
Linear Trend Equation

- *b* is similar to the slope.
- However, since it is calculated with the variability of the data in mind, its formulation is not as straight-forward as our usual notion of slope.

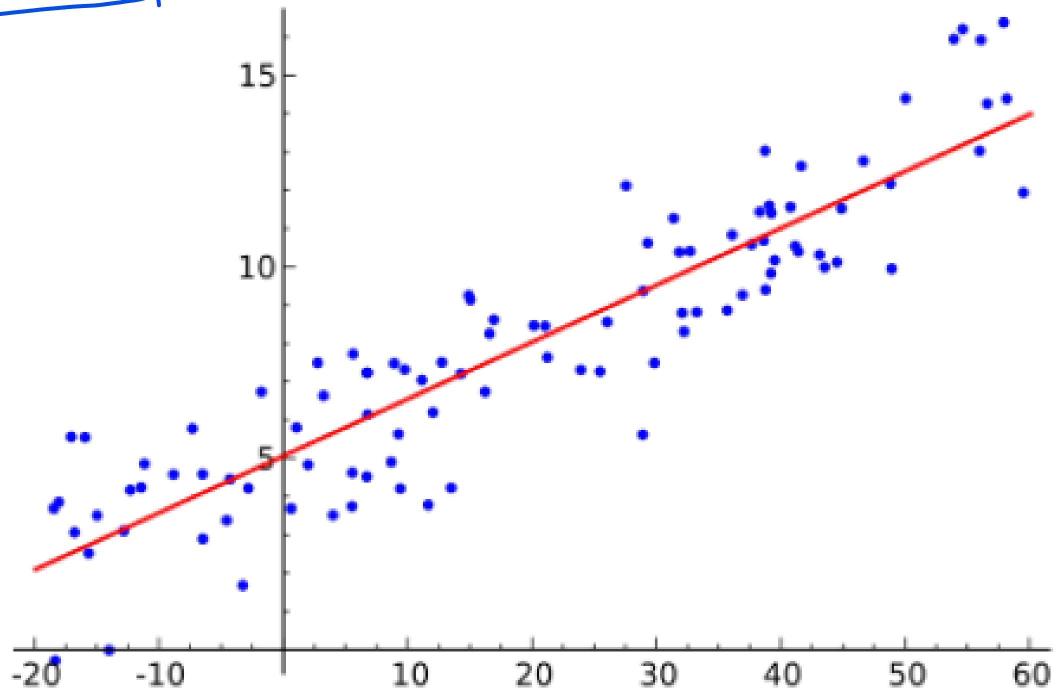
$$\boxed{Y_t = a + bt}$$



Linear Trend Equation

$$\boxed{b = \frac{n\sum(ty) - \sum t \sum y}{n\sum t^2 - (\sum t)^2}}$$

$$\boxed{a = \frac{\sum y - b\sum t}{n}}$$



Example

t, weeks	t^2	Y, sales	ty
1	1	150	150
2	4	157	314
3	9	162	486
4	16	166	664
5	25	177	885
$\sum t = 15$ $(\sum t)^2 = 225$	$\sum t^2 = 55$	$\sum y = 812$	$\sum ty = 2499$

Example

$$b = \frac{5\,(2499) \;-\; 15(812)}{5(55) \;-\; 225} = \frac{12495 \;-12180}{275 \;-225} = 6.3$$

$$a = \frac{812 \;-\; 6.3(15)}{5} = 143.5$$

$$\boxed{y = 143.5 + 6.3t}$$

at $t=6$

$y = 143.5 + 6.3(6) = 181.3$

> lm (y ~ t)

$y = 143.5 + 6.3t$

$F_{t+1} = 143.5 + 6.3(t+1)$

Practice 4

Sales have been declining since the release of an ineffective radio commercial. The sales (in hundred thousands) for the past 8 weeks are: 80, 74, 73, 68, 71, 65, 61, 60.

Estimate the forecast for the 12th week.

Hints:

1. Estimate a and b of the linear equation.
2. Find $Y = a + b(t)$ where $t = 12$.

Practice 5

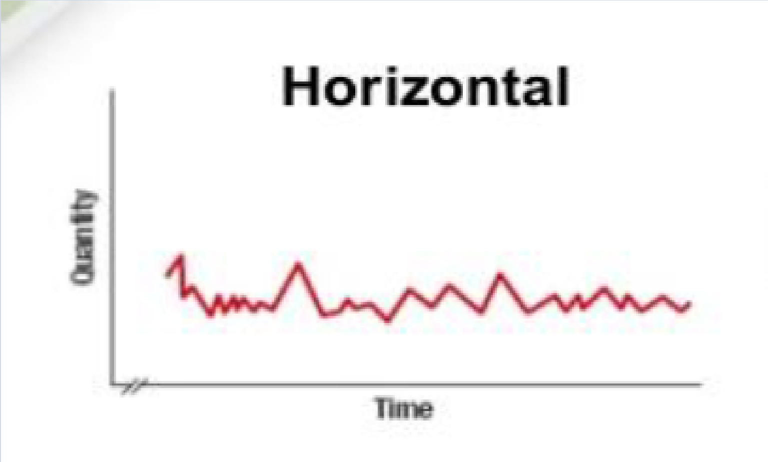
The sales of Jensen Foods, a small grocery chain, since 2010 are:

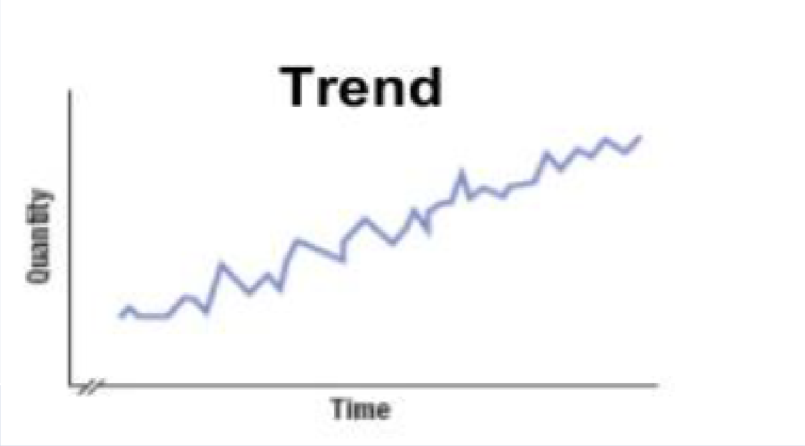
Year	2010	2011	2012	2013	2014
Sales ($ millions)	7	10	9	11	13

Determine the least squares trend-line equation.

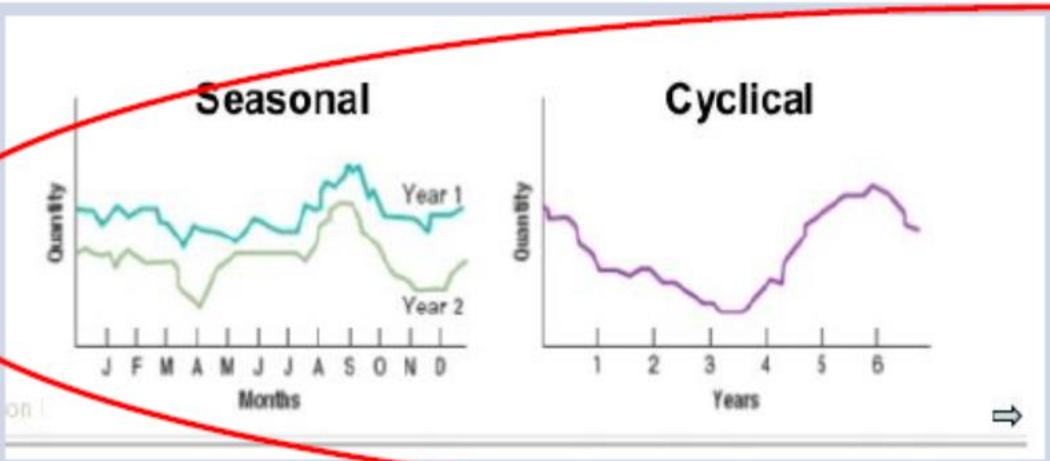
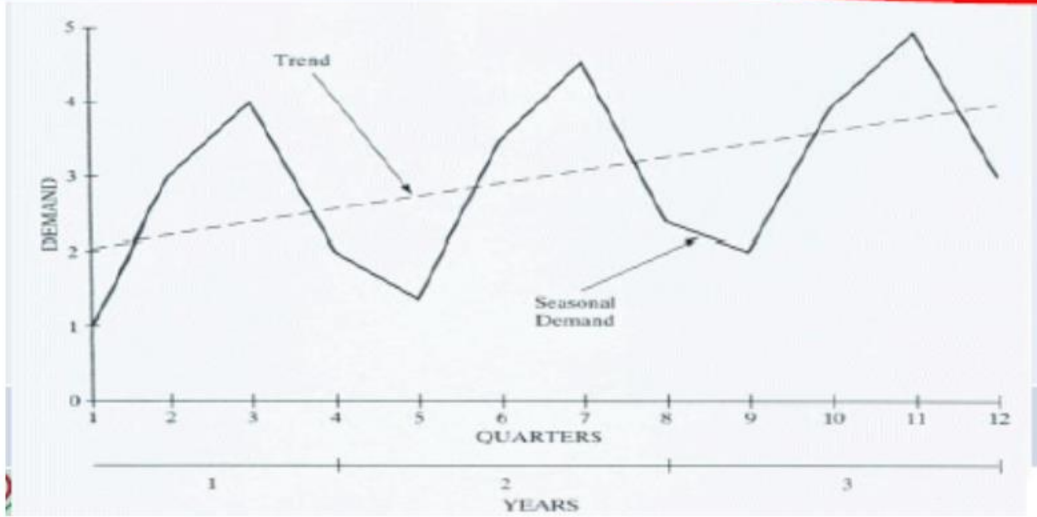
Appropriate Technique for the Historical Behavior

f the Historical Pattern is	Appropriate Technique(s)

	• naive
	• moving averages:
	    • simple moving average (SMA)
	    • weighted moving average (WMA)
	• smoothing
	    • single exponential smoothing

	• trend line (trend equation)
	    • regression (independent variable: time)
	• smoothing
	    • double exponential smoothing

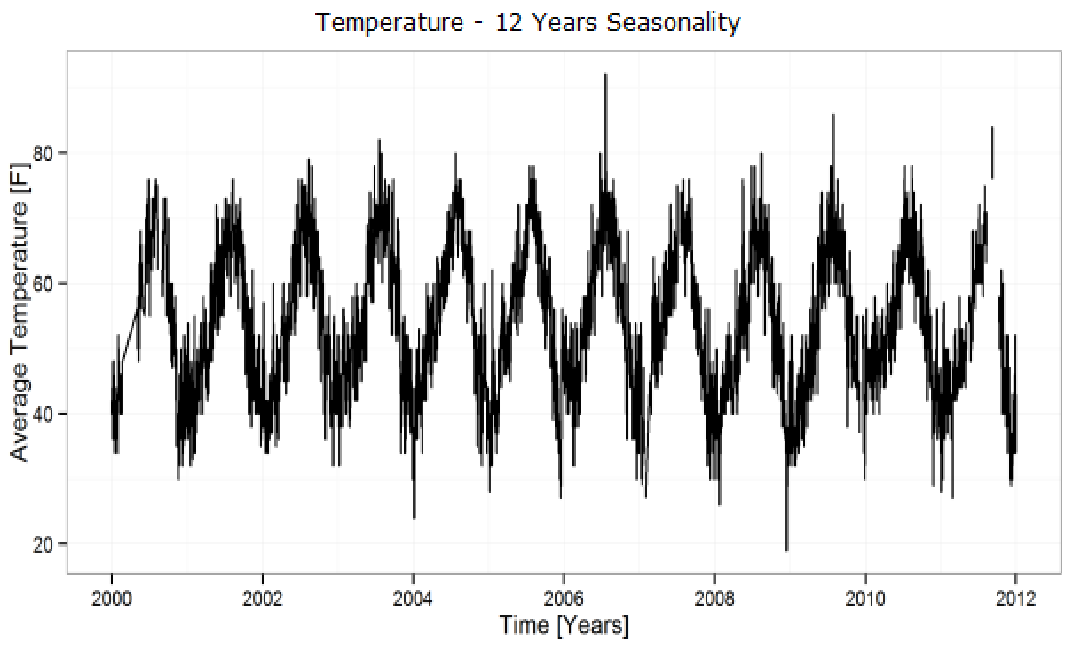
Appropriate Technique for the Historical Behavior

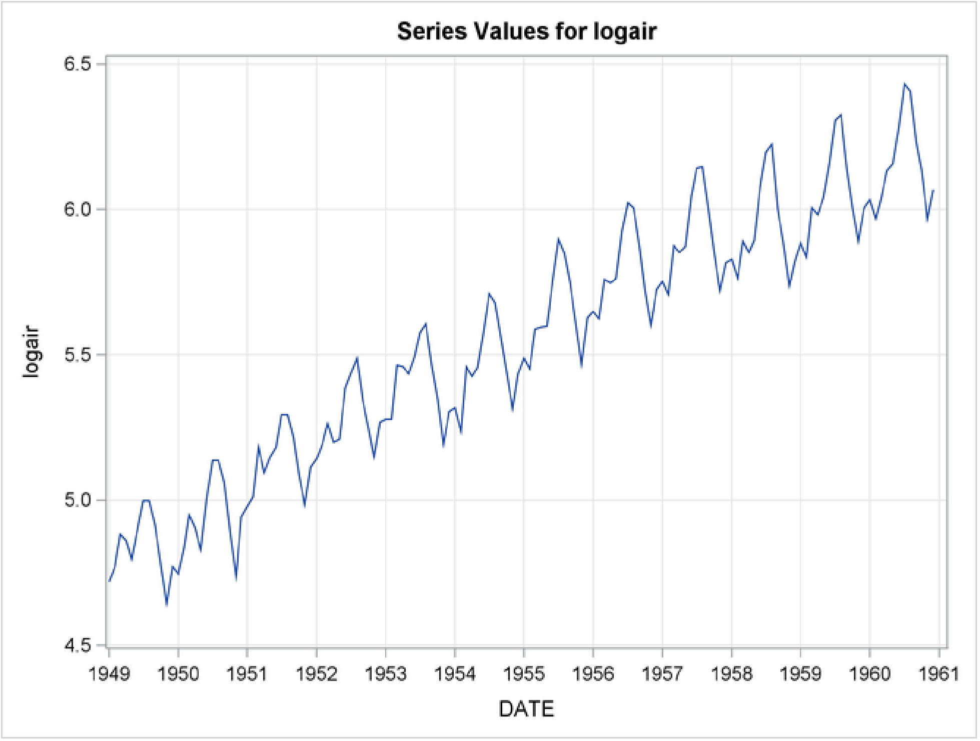
<table>
<tr><th>If the Historical Pattern is</th><th>Appropriate Technique(s)</th></tr>
<tr><td>
</td><td>• forecasting using seasonal index</td></tr>
<tr><td>
</td><td>• decomposition of the combination of pattern into individual patterns, forecasting, then combining forecasts of individual patterns</td></tr>
</table>

Forecasting using Seasonal Index

Linear Trends do not consider the effects of the "seasons" and its cycle.
Cycles may vary from: Time of the Day, Week, Month, Year, and so on…
for example:

- "At month end, truck utilization always doubles"
- 60% of our customers arrive during 1130am-130pm.





Forecasting using Seasonal Index

Usual Approach

1. Determine the length of the cycle.
2. Estimate the Seasonal Relatives
   – What is the effect of each "season"
3. Estimate the linear trend
4. Forecast using the linear trend
5. Adjust using the Seasonal Relatives

Forecasting using Seasonal Index

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Seasonal Indices

- Determining Seasonal Relatives
- The following shows the number of visitors of an online bulk sales site on a weekly basis.

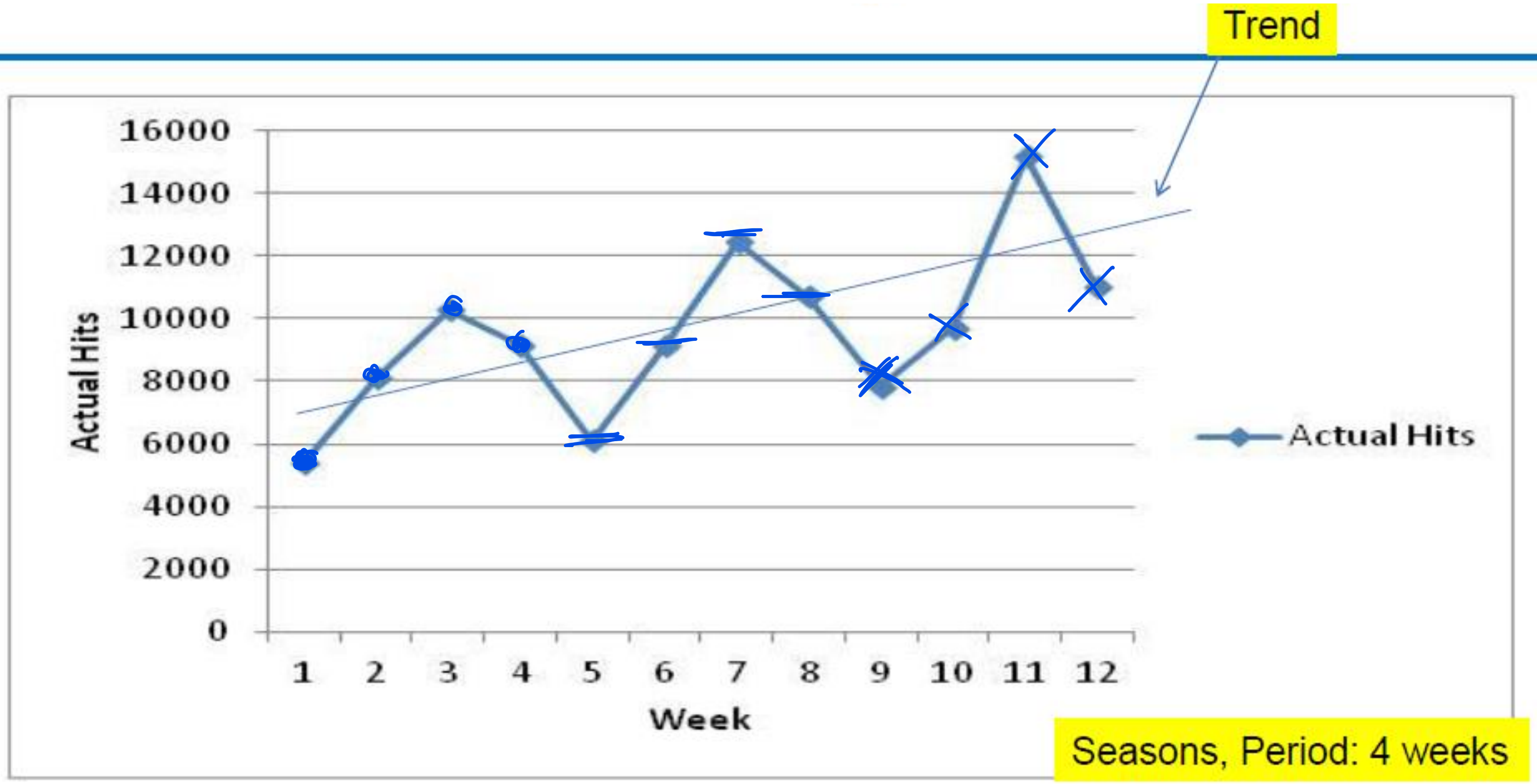
Week	Hits	Week	Hits
1	5,384	7	12,460
2	8,081	8	10,717
3	10,282	9	7,825
4	9,156	10	9,693
5	6,118	11	15,177
6	9,139	12	10,990

Every 3 weeks?

4 weeks?

6 weeks?

Period of Recurring Pattern



Seasonal Indices

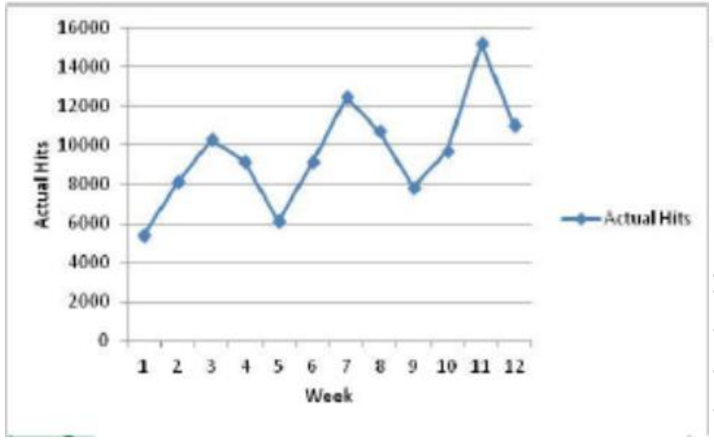
1. Split the data into 4 week - cycles.
2. Get the average per week per cycle..
3. Divide each period by the grand average.

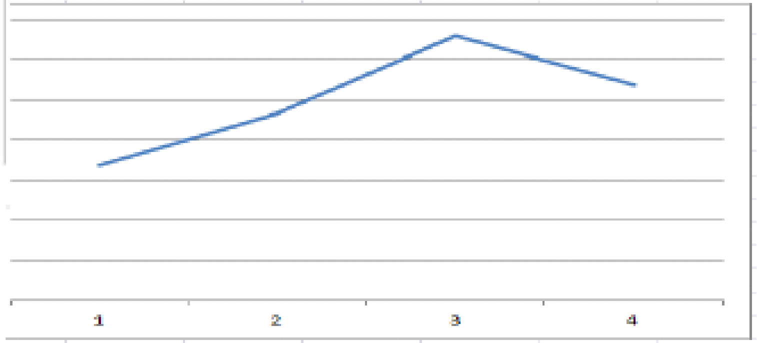
Week	Cycle 1	Cycle 2	Cycle 3	Average	*SR*	
1	5,384	6,118	7,825	***6,442***	**0.672*	= 6442/9585
2	8,081	9,139	9,693	***8,971***	***0.936***	= 8971 / 9585
3	10,282	12,460	15,177	***12,639***	***1.319***	= 12.639/9585
4	9,156	10,717	10,990	***10,287***	***1.073***	= 10,287/9.585
Grand Average				***9,585***		

**6,442 / 9,585 = 0.672*

Seasonal Indices

These seasonal relatives mean that every start of a cycle (1st,5th,9th,13th...week) the number of visitors are around 67% of the average number of visitors.
It also means that every 3rd week of the cycle, one can expect 31% more visitors (compared to the average) in the website.





Week	*Seasonal Relative*
1	**0.672*
2	*0.936*
3	*1.319*
4	*1.073*

Seasonal Indices

<u>Usual Approach (Decomposition Method)</u>

1. Determine the length of the cycle.
2. Estimate the Seasonal Relatives
   - What is the effect of each “season”
3. Estimate the linear trend
4. Forecast using the linear trend
5. Adjust using the Seasonal Relatives

Seasonal Indices

- De-seasonalize data: dividing each data point by its corresponding seasonal relative

- *5,384 / 0.672
  = 8,011.91

Week	De-Hits	Week	De-Hits
1	8,011.91	7	9,446.55
2	8,633.55	8	9,987.89
3	7,795.30	9	11,644.35
4	8,533.09	10	10,355.77
5	9,104.17	11	11,506.44
6	9,763.89	12	10,242.31

Seasonal Indices

- Estimate the Linear Trend using De-seasonalized data

$Y = a + bt$

$a = 7626.3$

$b = 301.41$

$Y = 7626.3 + 301.41t$

Week	De-Hits	Week	De-Hits
1	8,011.91	7	9,446.55
2	8,633.55	8	9,987.89
3	7,795.30	9	11,644.35
4	8,533.09	10	10,355.77
5	9,104.17	11	11,506.44
6	9,763.89	12	10,242.31

Seasonal Indices

- Forecast the number of expected visitors on the 15th week.

  $Y = a + bt$

  $a = 7626.3$

  $b = 301.41$

- 15th Week → $t = 15$

  $Y = 301.41(15) + 7626.3 =$ 12,147.45

- Seasonality → $SR_t = 1.319$

  12,147.45(1.319) = $\boxed{16{,}022.49}$ → forecast on 15th week

Seasonal Indices and Decomposition

Complete Approach for Decomposing the **Season Pattern** and **Trend Pattern**

1. Determine the length of the cycle
   (in this example, length = 4 weeks)
2. Estimate the Seasonal Relatives
   – What is the effect of each "season"
3. **Remove the seasonal component** from the data to get the "non-seasonal" or "de-seasonalized" component. (Divide each data point with the seasonal relative.)
4. Obtain **trend estimate** for the desired period using the trend equation.
5. **Add back** the seasonality by multiplying the seasonal relative.

Seasonal Indices and Decomposition

- A coffee shop owner wants to estimate demand for the next two quarters for hot chocolate.
- Sales data consist of **trend and seasonality**.

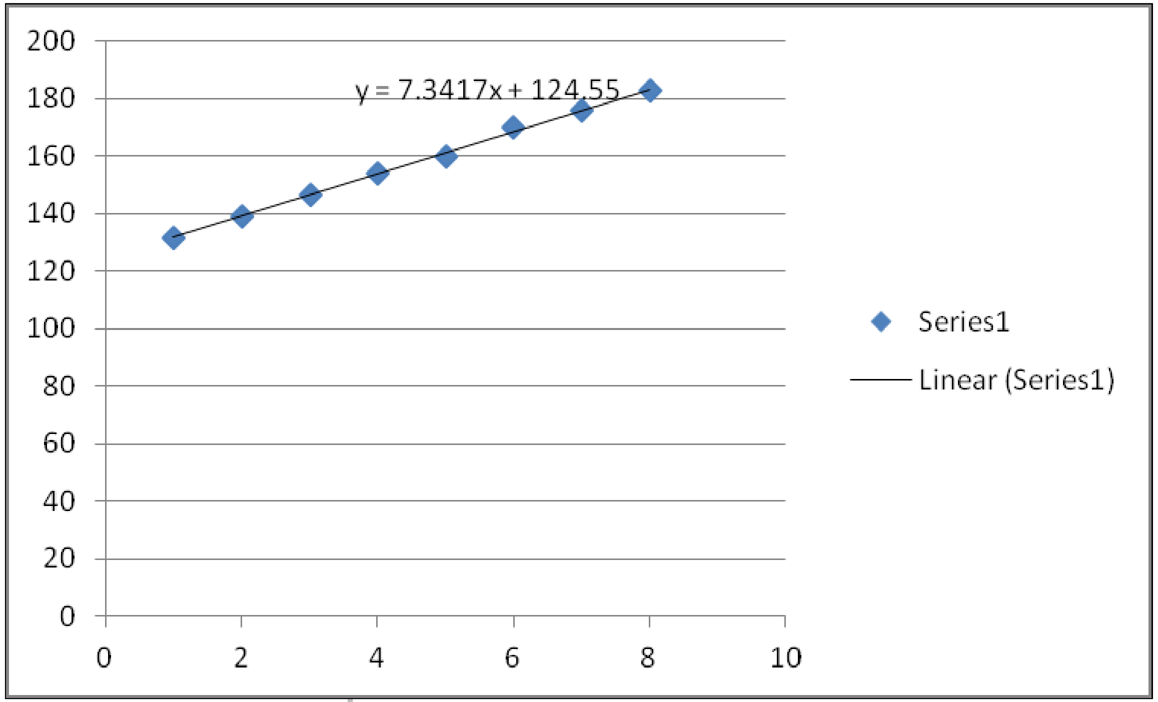
a) Quarter relatives are 1.20 for the first quarter, 1.10 for the second quarter, 0.75 for the third quarter, and 0.95 for the fourth quarter. Use this information to deseasonalize sales for quarters 1 through 8.

b) Using the appropriate values of quarter relatives and the equation $F_t = 124 + 7.5t$ for the trend component, estimate demand for periods 9 and 10.

Seasonal Indices and Decomposition

a.

Period	Quarter	Sales (gal.)	÷	Quarter Relative	=	Deseasonalized Sales
1	1	158.4	÷	1.20	=	132.0
2	2	153.0	÷	1.10	=	139.1
3	3	110.0	÷	0.75	=	146.7
4	4	146.3	÷	0.95	=	154.0
5	1	192.0	÷	1.20	=	160.0
6	2	187.0	÷	1.10	=	170.0
7	3	132.0	÷	0.75	=	176.0
8	4	173.8	÷	0.95	=	182.9



b. The trend values are:

Period 9: $F_t = 124 + 7.5(9) = 191.5$

Period 10: $F_t = 124 + 7.5(10) = 199.0$

Period 9 is a first quarter and period 10 is a second quarter. Multiplying each trend value by the appropriate quarter relative results in:

Period 9: $191.5(1.20) = 229.8$

Period 10: $199.0(1.10) = 218.9$

References

- Hugos, M. H. (2011). Essentials of supply chain management. Hoboken: Wiley.

- Stevenson, W. J. (2008). *Operations management*. New York, NY: McGraw-Hill Education.
  (various reading materials on forecasting and supply chain management)

Reference for each photo is cited with the photo.