Astr 513, Fall 2020
Homework #2
Due Date: Friday Oct 9, 5pm

Notes:

Please scan your solutions and submit them by email to `dpsaltis@email.arizona.edu`

The difficulty of each problem (according to the instructor!) is given in terms of the number of stars that follow the problem number. Being able to solve the 1-star problems is the minimum requirement for everyone to feel that they have understood the basic concepts. Completing the 3-star problems requires more effort and deeper understanding.

Note: It is critical to emphasize here that this is a very simplistic and old model of epidemiology and **should not be used to inform any personal or professional decisions for real-life situations.** Please use vetted sources of information that are based on detailed, advanced modeling for the spread of infections deceases, e.g., a good starting point is the CDC page:
`https://www.cdc.gov/csels/dsepd/ss1978/index.html`

Epidemiology

Differential equations are very useful in studying the outcome of contagious diseases. One simple model, the so-called SIR model, was introduced in 1927 by Kermack & McKendrick and uses differential equations to calculate the evolution of the fraction of Susceptible, Infected, and Recovered people in a population.

Let f_s the fraction of people in a population that are well but susceptible, f_i the fraction that are infected and currently sick, f_r the fraction that recovered and are immune, and f_d the fraction that have died. People that are not immune get ill at a rate that is proportional to the product $f_s f_i$, with a characteristic timescale for the spreading of the disease of τ_s. The characteristic timescale for the ill to either recover or pass away is τ_d, and the fraction of people that recover and become immune is X.

The evolution of the population is described by the set of equations

$$\frac{df_s}{dt} = -\frac{f_s f_i}{\tau_s} \tag{1}$$

$$\frac{df_i}{dt} = \frac{f_s f_i}{\tau_s} - \frac{f_i}{\tau_d} \tag{2}$$

$$\frac{df_r}{dt} = X\frac{f_i}{\tau_d} \tag{3}$$

$$\frac{df_d}{dt} = (1-X)\frac{f_i}{\tau_d} \tag{4}$$

(a)* Write a program that solves the above system of equations for a disease that spreads at $\tau_s = 1$ days, $\tau_d = 20$ days, $X = 0.2$. Consider a city of 100000 people and an initial situation of one sick person. Plot the fraction of healthy, ill, immune, and dead people as a function of time. What is the outcome of this outbreak after 5 years?

(b)* In a different city of the same population, the authorities imposed a quarantine such that the timescale for spreading was longer, i.e., $\tau_s = 15$ days. What is the outcome of the outbreak in that city after 5 years?

(c)** Using the above equations, you can prove that

$$\frac{d}{dt}\left(f_s + f_i + f_r + f_d\right) = 0 \; , \tag{5}$$

i.e., that the sum of the four fractions is a constant (and equal to unity). For how long can you integrate your model before numerical errors start introducing significant change in this constant?

(d)*** How would you update this model in order to introduce, e.g., immunity at birth because of vaccination or a delay time between the time a person gets infected and the time the person can become infectious?

(e)*** Download the COVID-19 data from the JHU github repository
`https://github.com/CSSEGISandData/COVID-19`
for the US or your favorite state or country. Can you estimate (not formal fitting) the values of the various characteristic timescales for this disease?