Computer Graphics Lab Assignment 1

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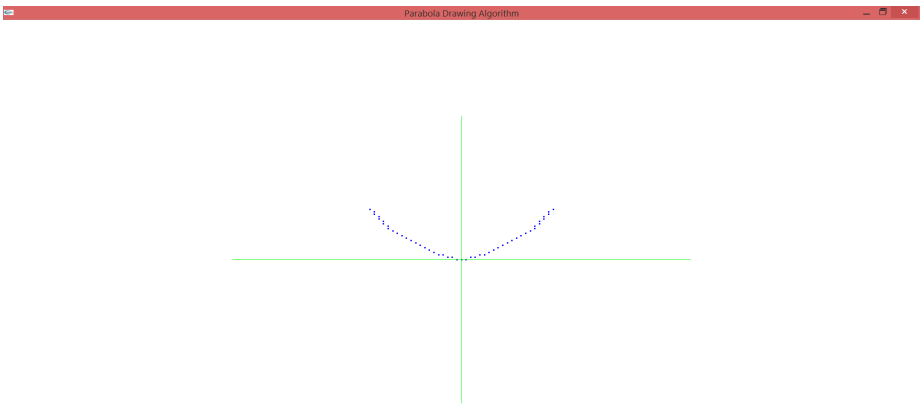
Abstract

An algorithm using the midpoint method for scan converting a parabola.

1 Brief

Design an algorithm using the midpoint method for scan converting the function $y = \frac{x^2}{20}$ in the range $-20 \leq x \leq 20$. Identify the regions where the algorithm would be invoked and how symmetry could be employed to improve its efficiency. Implement the same using OpenGL.

2 Output



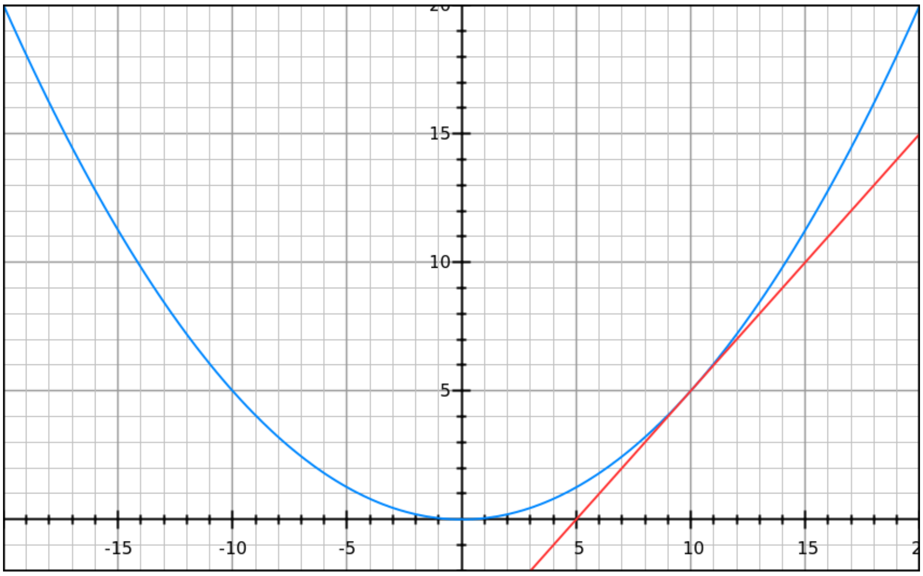
3 Code

```
scrPt paraPt; paraPt.x=0; paraPt.y=0;

GLint p=-9; //Initial value of midpoint parameter

//Plot intial parabola vertex
paraPlotPoints(paraPt);

//Calculate next points and plot in
while(paraPt.x<=10){
        paraPt.x++;
        if(p+2*paraPt.x+3<0){
                p+=2*paraPt.x+3;
            paraPt.y+=1/2;
        }
        else{
                paraPt.y+=3/2;
                p+=2*paraPt.x-17;
        }
        paraPlotPoints(paraPt);
}

p=paraPt.x*paraPt.x+paraPt.x-20*paraPt.y-79/4;

while(paraPt.y<=20){
        paraPt.y++;
        if(p+2*paraPt.x-18<0){
                p+=2*paraPt.x-18;
            paraPt.x+=3/2;
        }
        else{
                paraPt.x+=1/2;
                p-=20;
        }
        paraPlotPoints(paraPt);
}
```

```
void paraPlotPoints(scrPt paraPt)
{
        setPixel(paraPt.x, paraPt.y);
        setPixel(-paraPt.x, paraPt.y);
}
```

4 Preliminaries

Given $f(x) = \frac{x^2}{20}$
We have $f'(x) = \frac{x}{10} \Rightarrow f'(10) = 1$
Let $F(x) = x^2 - 20y$



We deduce that for x=0 to x=10 the pixel plotted is either to the right or to the top-right and for y=5 to y=20 the pixel plotted is either to the top or to the the top-right.

In other words for x=0 to x=10 the x coordinate is incremented uniformly by 1 unit and the y coordinate by 0.5 or 1.5 units. For y=5 to y=20 the vice versa is true.

We have effectively divided the 1st quadrant into two regions. Note that we shall only plot the 1st quadrant points and let the y axis symmetry take care of the rest. Now consider the midpoint parameter 'd'.

For the first region

$d_{old} = F(x+1, y+0.5) = x^2 + 2x - 20y - 9$

$d_{right} = F(x+2, y+0.5) = x^2 + 4x - 20y - 6 = d_{old} + 2x + 3$

$d_{top-right} = F(x+2, y+1.5) = x^2 + 4x - 20y - 26 = d_{old} + 2x - 17$

For the second region

$d_{old} = F(x+0.5, y+1) = x^2 + x - 20y - 19.75$

$d_{top} = F(x+0.5, y+2) = x^2 + x - 20y - 39.75 = d_{old} - 20$

$d_{top-right} = F(x+1.5, y+2) = x^2 + 3x - 20y - 37.75 = d_{old} + 2x - 18$

The first point plotted is the vertex of the given parabola (0,0) followed by the points of region 1.
Hence, the first value of the midpoint parameter is calculated as

$d_{old,(0,0)} = F(x+1, y+0.5)|_{(0,0)} = (x^2 + 2x - 20y - 9)|_{(0,0)} = 0 + 0 - 0 - 9 = -9$

5 Algorithm

1. Plot vertex point (0,0).
2. Calculate first value of the midpoint parameter d_{old}.
3. Plot points of first region by looping from x=0 to x=10 incrementing x by one each time.

   i. At each value of x calculate parameters d_{right}, $d_{top-right}$. If $d_{right} < 0$ then (x+2, y+0.5) is inside parabola and is the pixel to be choosen. Else choose (x+2, y+1.5). Update decision parameter d.

   ii. Plot choosen point (X, Y) and the symmetrical point (-X, Y).
4. Recalculate midpoint parameter for region 2.
5. Plot points of second region by looping from y=5 to y=20 incrementing y by one each time.

   i. At each value of y calculate parameters $d_{top-righy}$, d_{top}. If $d_{top-right} < 0$ then (x+1.5, y+2) is inside parabola and is the pixel to be choosen. Else choose (x+0.5, y+2). Update decision parameter d.

   ii. Plot choosen point (X, Y) and the symmetrical point (-X, Y).
6. End.

6 Misc.

There are two regions in which the algorithm has been invoked. These are the two regions bounded by the x-axis, the y-axis and the line y=x-5. These has been described above.

Symmetry has been employed to effectively half the work. We only calculate points in the first quadrant (X, Y). We plot their image about the y-axis (-X, Y) without having to calculate these points. It can be proved mathematically that for every (X, Y) that satisfies the function $y = \frac{x^2}{20}$, (-X, Y) also satisfies the function.