CS152 Final Project
Viet Hoang Tran Duong
Fall 2019

Problem description:

Connect 4 is a two-player connection game. One player is X, and the other is O. The player first chooses a column and then take turns dropping one symbol from the top into a seven-column, six-row vertically suspended grid. The pieces fall straight down, occupying the lowest available space within the column.

The nature of this game is maximizing the benefits from our current state configuration (by constructing lines of 3, etc.) while minimizing others' interests (blocking their tracks). This approach has the same assumptions as an algorithm called Minimax.

This problem is suitable for artificial intelligence to solve because it shares the same techniques as human (Minimax approach). However, humans have limited brain capacity, while we can process tremendously more information with computers. We can estimate the result a few steps ahead to better plan the move using computers and artificial intelligence.

Solution description:

For this project, I apply the minimax algorithm, alpha-beta-pruning (abPrune), and iterative deepening DFS (iter_DFS). Minimax algorithm with big depth can be very costly, so abPrune helps cut the search space, whereas iter_DFS helps search the game tree faster. I conduct three use cases: one is having AI depth-10 playing with AI depth-5 (depth-10 wins). Another is a pairwise match between an ai and another ai, with depth from 1 to 7. The final component is having a human player playing against an AI, with selection for difficulty (~ tree depth from 1-10).

I use an evaluation function (heuristics) to estimate the result at each stage: each stage has a 7x6 grid. I query all the lines with more than four elements, evaluate each line, and sum all these scores.

For abPrune and iter_DFS, because searching through the state space would be very time consuming, especially when the depth increases. This project is a game ai, which if we make the users wait too long, they will hate us. Hence, I set the max time deepening in the game tree to be 10 seconds, with additional backtracking will lead to approximately 12 seconds on average for a large depth tree.

Analysis

Win/lose results:

	Depth 1	Depth 2	Depth 3	Depth 4	Depth 5	Depth 6	Depth 7
Depth 1	1	1	1	-1	-1	1	0
Depth 2	1	1	1	-1	-1	1	0
Depth 3	1	1	1	1	1	1	1
Depth 4	1	1	1	-1	1	1	1
Depth 5	1	1	-1	1	1	1	1
Depth 6	-1	-1	-1	-1	-1	1	-1
Depth 7	1	1	1	1	1	-1	1

Table 1: Here is the table representing the result between ai with different depths.
Item at row i-th column j-th is the result of ai i-depth vs ai j-depth, with ai i-depth plays first.

Runtime results:

	Depth 1	Depth 2	Depth 3	Depth 4	Depth 5	Depth 6	Depth 7
Depth 1	0.00457552	0.00408533	0.00454516	0.00395346	0.00356839	0.00435801	0.00346317
Depth 2	0.00398466	0.00434899	0.00431299	0.00392859	0.00349739	0.00438201	0.00345622
Depth 3	0.0158956	0.015683	0.0139088	0.0163708	0.0156534	0.0161437	0.0175535
Depth 4	0.0596554	0.0599392	0.0607045	0.048482	0.0660112	0.061606	0.0541162
Depth 5	0.258132	0.260889	0.167018	0.21099	0.174693	0.274595	0.18067
Depth 6	0.604029	0.648862	0.629228	0.649323	0.626568	0.763951	0.87026
Depth 7	2.54892	2.62112	2.84403	2.1874	1.82776	2.20892	1.70822

Table 2: Here is the table representing the running time between ai with different depths.
Item at row i-th column j-th is the time running of the ai i-depth in the match between ai i-depth vs ai j-depth, with ai i-depth plays first.

	Depth 1	Depth 2	Depth 3	Depth 4	Depth 5	Depth 6	Depth 7
Depth 1	0.00479283	0.00417258	0.014122	0.0453128	0.160414	0.865195	2.29724
Depth 2	0.00399697	0.00405291	0.0146003	0.0452693	0.160301	0.870066	2.28603
Depth 3	0.00470075	0.00501165	0.0147515	0.0512394	0.164016	0.677507	2.39015
Depth 4	0.00404251	0.0040203	0.0130035	0.0456644	0.188725	0.682678	1.23407
Depth 5	0.00408171	0.00409657	0.0111873	0.0619916	0.190149	0.991128	2.48088
Depth 6	0.0029817	0.00303408	0.00985815	0.0418365	0.131977	0.669208	2.53293
Depth 7	0.0039162	0.00383435	0.0172389	0.0683354	0.160653	0.580855	2.03162

Table 3: Here is the table representing the running time between ai with different depths.
Item at row i-th column j-th is the time running of the ai j-depth in the match between ai i-depth vs ai j-depth, with ai i-depth plays first.

Figure analysis:

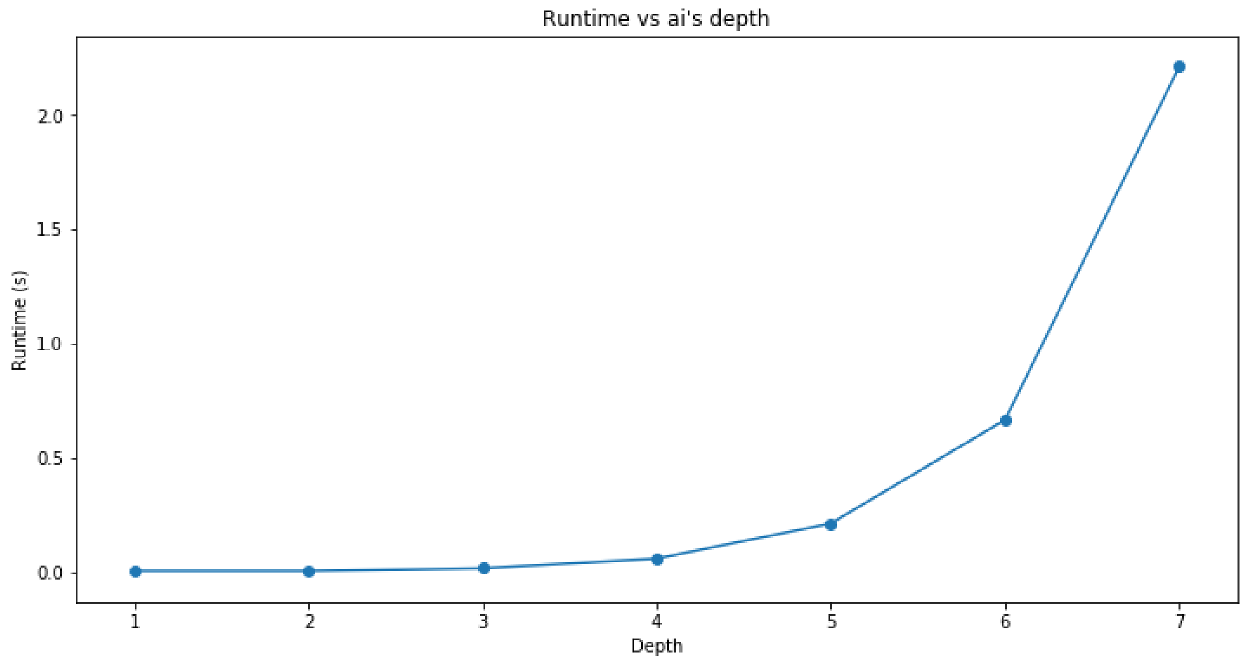


Figure 1: The runtime with respect to the depth of the algorithm. As we can see, the time is getting exponentially larger, which is reasonable because the number of nodes at each stage is getting exponentially larger concerning the increase of the depth.

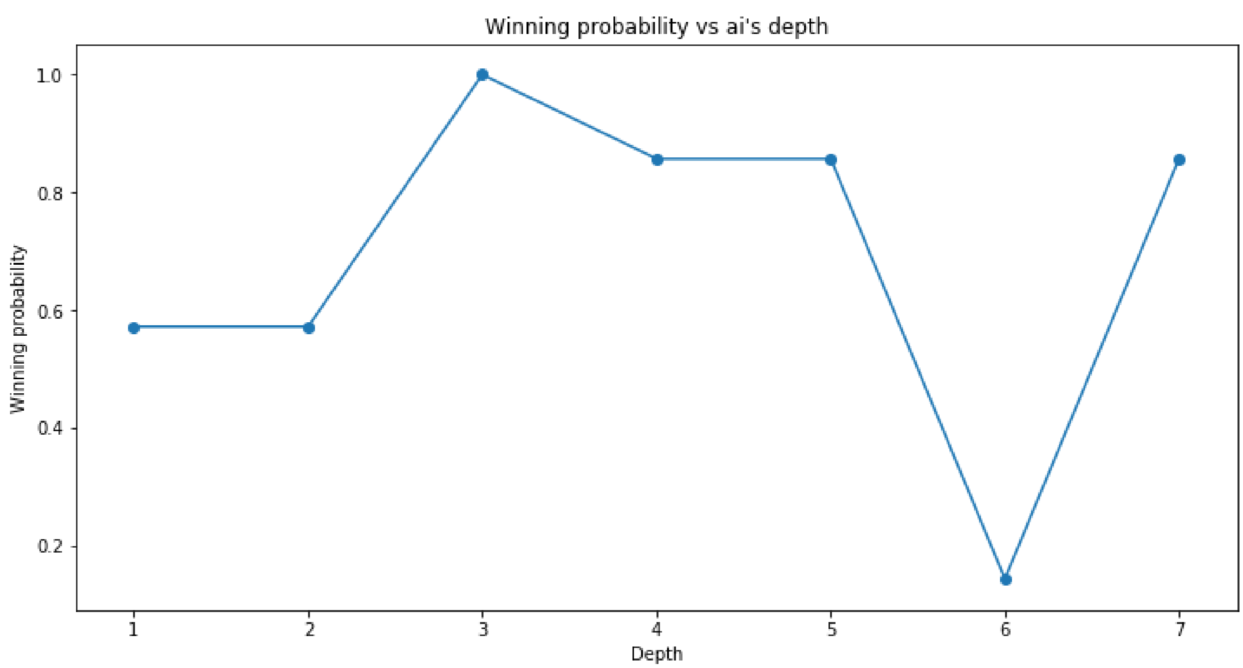


Figure 2: The winning probability with respect to the depth of the algorithm. Generally speaking, increasing the depth does make the ai more powerful: like ai with depth-7 beats almost all the ai

with smaller depth. Theoretically, ai with bigger depth should result in a better outcome, especially against those with smaller depth. In this case, we see the anomalies at 6. One plausible reason is that the evaluation function is imperfect. For example, it might give great value for configuration as below (with three consecutive elements). However, it takes many more steps to reach there and should not be highly evaluated. Further improvement could be made.

-	-	-	-
-	-	X	-
-	X	O	-
X	O	O	-

Table 4: One example that the evaluation function could behave not as accurate, which leads to poor performance against ai of other values of depths.

LOs Application:

- **#aicoding**: I program a two-player game with an ai component. Apply the concepts from this class: minimax, alpha-beta pruning, iterative DFS to reduce the search space and running time. All functions are well commented on and function properly.
- **#search:** Apply the alpha-beta pruning and iterative DFS to reduce the space and running time traversing the game tree, which is exponentially more significant by each depth.

HC Applications:

- **#heuristics:** The depth of the search tree for Connect 4 can be up to 42, which will create an exponentially large number of nodes. Hence, I apply heuristics to develop a temporary evaluation for each state and use if for the depth-5 program to cut the time. I apply #heuristics properly for CS context by selecting the appropriate heuristics to optimize our outcome.
- **#utility:** In the situation of games: #utility means maximizing our benets while mitigating others: which perfectly describes the minimax algorithm. Assuming the opponent will minimize our scores. I apply #utility properly for the CS context to program the minimax algorithms to calculate ten steps ahead on which node will optimize our outcome.

References:

Saveski, M. (n.d.). AI Agent. Retrieved December 17, 2019, from
http://web.media.mit.edu/~msaveski/projects/.

```
# import packages
import sys
import numpy as np
import time
from tabulate import tabulate
import matplotlib.pyplot as plt
import pandas as pd
```

```

```

Description:

Connect 4 is a two-player connection game. One player is X and the other is O. The player first choose a column and then take turns dropping one symbol from the top into a seven-column, six-row vertically suspended grid. The pieces fall straight down, occupying the lowest available space within the column.

For this project, I apply minimax algorithm , alpha-beta-pruning (abPrune), and iterative deepening DFS (iter_DFS). Minimax algorithm with big depth can be very costly, so abPrune helps cut the search space, whereas iter_DFS helps search the game tree faster.

I conduct 2 use cases: one is having AI depth 10 playing with AI depth 5 (depth 10 wins). Another is having a human player playing against an AI, with selection for difficulty (~ tree depth from 1-10).

LOs Application:

- **#aicoding**: I program a two-player game with an ai component. Apply the concepts from this class: minimax, alpha-beta pruning, iterative DFS to reduce the search space and running time. All functions are well commented on and function properly.
- **#search**: Apply the alpha-beta pruning and iterative DFS to reduce the space and running time traversing the game tree, which is exponentially more significant by each depth.

HC Applications:

- **#heuristics**: The depth of the search tree for Connect 4 can be up to 42, which will create an exponentially large number of nodes. Hence, I apply heuristics to develop a temporary evaluation for each state and use if for the depth-5 program to cut the time. I apply #heuristics properly for CS context by selecting the appropriate heuristics to optimize our outcome.

- **#utility**: In the situation of games: #utility means maximizing our benefits while mitigating others: which perfectly describes the minimax algorithm. Assuming the opponent will minimize our scores. I apply #utility properly for the CS context to program the minimax algorithms to calculate ten steps ahead on which node will optimize our outcome.

Basic function: Check the validity, query all the possible combinations, checking winning state

```
# For conenct 4, it's more beneficial to start from the middle: set the order
order=[3,2,4,1,5,0,6]

# search for all available moves
def available_move(state):
  available_moves = []
  # search though each column
  for x in order:
    # scan through the row
    for y in range(size_y-1, -1, -1):
      # if empty tile: accet and move on to mext column
      if state[y][x] == 0:
        available_moves.append([y,x])
        break
  return available_moves

# check the move input by users is valid:
def check_move(state, x, auto = -1):
  # must be a number between 0 and 8: 1-7
  while not (x.isnumeric() == True and int(x) > 0 and int(x) < 8):
    print ("Invalid move!")
    x = input('n: ')
  #check the possible moves
  cols, rows = (np.array(available_move(state)).T).tolist()
  x = int(x)-1

  # if not in the set of available moves
  while x not in rows:
```

```
  while x not in rows:
    print ("Invalid move!")
    x = input('n: ')
  # change the state
  state[cols[rows.index(x)]][x] = auto

# check if the move is valid in the game board
def check_constraint(y, x):
  if (y < 0) or (y > size_y-1) or (x < 0) or (x > size_x - 1):
    return False
  return True
```

```
# Use to draw the table:
def draw(state):

  # the  fisrt column
  for i in range(size_x):
    sys.stdout.write(" %d " % (i+1))

  print ("")
  print ("_" * (size_x * 3))

  # print every value, separate by ||
  # 1-> X, -1 -> O, 0 -> -
  for i in range(size_y):
    for j in range(size_x):
      if state[i][j] == 1:
        sys.stdout.write("|O|")
      elif state[i][j] == -1:
        sys.stdout.write("|X|")
      else:
        sys.stdout.write("|-|")
    print ("")
  print ("_" * (size_x * 3))
  print ("")
```

```
# check if the game is over or not:
def win(state):
  # winning scenario
```

```
  # winning scenario
  ai_win = [1,1,1,1]
  human_win = [-1,-1,-1,-1]

  # search for all lines (horizontal, vertical, diagonal) - explained below
  lines = all_lines(state)

  #search in the lines
  for line in lines:
    # less than 4 line are useless
    if len(line) < 4:
      continue
    # if ai_win move is in 1 line in the state: ai win
    if sub_list(ai_win, line):
      return 1
      break
    # if human_win move is in 1 line: human win
    if sub_list(human_win, line):
      return -1
      break
  # no one win yet
  return 0

# search for all the lines in the matrix
def all_lines(state):
  # initialize
  lines = []

  # horizontal
  for y in range(size_y):
    lines.append(state[y])

  # vertical
  for x in range(size_x):
    lines.append([state[i][x] for i in range(size_y)])

  #all diagonal:
  # https://stackoverflow.com/questions/6313308/get-all-the-diagonals-in-a-matrix-list-of-lists-in-python
  state_array = np.array(state)
```

```
  # get diagonal line from left to right
  diags = [state_array[::-1,:].diagonal(i) for i in range(-state_array.shape[0]+1,state_array.shape[1])]
  # get diagonal line from right to left
  diags.extend(state_array.diagonal(i) for i in range(state_array.shape[1]-1,-state_array.shape[0],-1))
  # add all the line into the whole lines
  lines += [n.tolist() for n in diags]

  return lines

# check if list1 is a sublist of list2: use to check winning stage
def sub_list(list1, list2):
  l1 = len(list1)
  l2 = len(list2)
  #iterate through list2 to find list1
  for i in range(l2 - l1 + 1):
    if list1 == list2[i:i+l1]:
      return True
```

```

```

Dummy version: Assign weights to a small set of possible moves. This version performs poorly. We will use a more extensive weight in the next part. This one is to give a general understanding of the approach: each 4-line configuration has a specific weight. [1,1,1,1] will have the highest weight, whereas [-1,-1,-1,-1] will have the most negative weight. We extend to all 4, 5, 6, 7 line.

```
# search through different axis: x-axis, y-axis, and 2 diagonal axis. Each axis has 2 direction to move
directions = [[[1, 0], [-1, 0]],
              [[0, 1], [0, -1]],
              [[1, 1], [-1, -1]],
              [[1, -1], [-1, 1]]]

# evaluation for a state configuration is the sum of all configuration
def evaluate_location(state, y, x, player, opponent):
  score = 0
  # for each axis
  for axis in directions:
    # checking what if we play at that spot and what if the opponent play at hat spot
    cur_line = [player]
```

```
    opp_line = [opponent]

    # moving direction
    direction1, direction2 = axis
    y_dir = direction1[0]
    x_dir = direction1[1]
    # max line is 4, but we have 1 component already -> check 3
    for i in range(3):
      # moving size
      step = i+1
      # new moves
      y_new = step*y_dir + y
      x_new = step*x_dir + x

      # if does not violate the constraint: add to line
      if check_constraint(y_new, x_new) == False:
        break
      cur_line += [state[y_new][x_new]]
      opp_line += [state[y_new][x_new]]

    # moving in the other direction
    y_dir = direction2[0]
    x_dir = direction2[1]
    # max line is 4, but we have 1 component already -> check 3
    for i in range(3):
      # moving size
      step = i+1
      # new moves
      y_new = step*y_dir + y
      x_new = step*x_dir + x

      # if does not violate the constraint: add to line: but add in the opposite direction
      if check_constraint(y_new, x_new) == False:
        break
      cur_line = [state[y_new][x_new]] + cur_line
      opp_line = [state[y_new][x_new]] + opp_line

    # if length < 4: no need to care
    if len(cur_line) >= 4:
```

```
      for i in range(len(ai_strat)):
        # check in the score table
        if sub_list(ai_strat[i], cur_line):
          score += ai_strat_score[i]

      # this is for blocking startegy: if we don't play at [i, j] and the opponent playes, what will happen
      for i in range(len(block_strat)):
        if sub_list(block_strat[i], opp_line):
          score += block_strat_score[i]

  # result
  return score

# a state evaluation = sum of all the coordinates
def eval(state):
  player = 1
  opponent = -1
  winner = win(state)

  # if win, assign biggest value
  if winner == player:
    return np.inf
  # lose -> assign the worst
  elif winner == opponent:
    return -np.inf

  # start calculating score
  score = 0
  for x in range(size_x):
    for y in range(size_y):
      if state[y][x] == 0:
        score += evaluate_location(state, y, x, player, opponent)

  return score

# snap shot or score table
ai_strat = [[1,1,1,1],
            [0,1,1,1],[1,1,1,0],
            [1,1,0,0],[1,0,0,1],[0,1,1,0],[0,0,1,1],[1,0,1,0],[0,1,0,1],
```

```
            [1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1]]
ai_strat_score = [100,
                  30, 30,
                  10,10,10,10,10,10,
                  2,2,2,2]

block_strat = (np.array(ai_strat)*-1).tolist()
block_strat_score = (np.array(ai_strat_score)*-1).tolist()
```

```
#moves in slot x acording to valid moves function
def set_state(state, x, player):
  val = available_move(state)
  state[val[x][0]][val[x][1]] = player
```

```

```

Minimax + AB Pruning

```
#Alpha Beta Pruning Search Algorithm
def abPrune(state, depth = 4):

  # find the minimum (left side of the tree): the opponent's move to make us worst
  def abMin(state, depth, alpha, beta):
    # query all the available states
    available_moves= available_move(state)

    # of no more moves or ran out of states
    if (depth==0 or not available_moves):
      # return the score
      return eval(state)

    # check our opponents' best move
    cur_val = +1000000
    for y,x in available_moves:
      # assign values
      state[y][x]= -1
      cur_val = min(cur_val, abMax(state, depth-1, alpha, beta))
```

```
    # replacing back in the grid
    state[y][x] = 0

    # always look for the worst option
    if cur_val <= alpha:
      return cur_val
    # returning the worst option for us
    beta = min(beta, cur_val)
  return cur_val

# find the maximum (right side): our move to make us best
def abMax(state, depth, alpha, beta):
  # all the avaliable states
  available_moves = available_move(state)

  # if no more depth or ran out of states
  if (depth==0 or not available_moves):
    # scoring
    return eval(state)

  # check our best move
  cur_val = -1000000
  for y,x in available_moves:
    # our move
    state[y][x] = 1
    # returning our best moves
    cur_val = max(cur_val, abMin(state, depth-1, alpha, beta))
    # assigning back
    state[y][x] = 0
    # always look for the best
    if cur_val >= beta:
      return cur_val
    # the best options
    alpha = max(alpha, cur_val)
  return cur_val

# complete ab pruning process
def ab(state, depth, alpha, beta):
  tracking = []
```

```
    cur_val = -1000000

    for y,x in available_move(state):
      # assigning value and iterate
      state[y][x] = 1

      # best move
      cur_val = max(cur_val, abMin(state, depth-1, alpha, beta))
      tracking.append(cur_val)

      state[y][x] = 0

    # optimize
    largest = max(tracking)
    idx = tracking.index(largest)

    # get the index and result of best outcome
    return [idx, largest]

  return ab(state, depth, -1000000, +1000000)
```

```
import time
#Iterative Deepening DFS #https://www.geeksforgeeks.org/iterative-deepening-searchids-iterative-deepening-depth-first-searchiddfs/
def iter_DFS(state, max_depth = 10):
  global order

  # starting depth
  depth = 1
  # abPrune: start pruning
  result = abPrune(state, depth)

  # keep moving until depth > max_depth (random number)
  time_start = time.time()
  while True:
    # start timer


    # terminal score
```

```
    if abs(result[1]) > 5000:
      return result[0]

    # get results
    temp = result[0]

    #chaning the order in considering moves
    while temp != 0:
      order[temp-1], order[temp] = order[temp], order[temp-1]
      temp -= 1

    # adding depth
    depth += 1
    # prune again
    result = abPrune(state, depth)

    # end timer
    time_end = time.time()

    # if exceed max depth
    if depth >= max_depth:
      return result[0]

    # we don;t want the ai to run so long, so we output after 10s
    if time.time() - time_start > 10:
      print("I stressed my brain out! You're good! Here's my move:")
      return result[0]
```

```

```

Weight matrix

This is the weight matrix for all possible combination of 4, 5, 6, 7 lines. The weight is retrived from (Saveski, 2018)

```
weight_4 = pd.read_csv("https://docs.google.com/spreadsheets/d/e/2PACX-1vS03jXNPKCZqb5aWBnocQSaIR2KO5sh5KCyeDZDHsjl5947g3Mh9DHNqJ3E
weight_5 = pd.read_csv("https://docs.google.com/spreadsheets/d/e/2PACX-1vS03jXNPKCZqb5aWBnocQSaIR2KO5sh5KCyeDZDHsjl5947g3Mh9DHNqJ3E
weight_6 = pd.read_csv("https://docs.google.com/spreadsheets/d/e/2PACX-1vS03jXNPKCZqb5aWBnocQSaIR2KO5sh5KCyeDZDHsjl5947g3Mh9DHNqJ3E
weight_7 = pd.read_csv("https://docs.google.com/spreadsheets/d/e/2PACX-1vS03jXNPKCZqb5aWBnocQSaIR2KO5sh5KCyeDZDHsjl5947g3Mh9DHNqJ3E
```

```
# all the data
data = [weight_4, weight_5, weight_6, weight_7]
length = [4, 5, 6, 7]

# initialize the storage
store4 = {}
store5 = {}
store6 = {}
store7 = {}
storage = [store4, store5, store6, store7]

# convert the data and add all the score to a dictionary
# this part is only preprocessing the data
for i in range(len(data)):
  datum = data[i]
  store = storage[i]
  value = []
  for k in range(len(datum)):
    transform_value = ("%0" + str(length[i]) + "d") % datum["value"][k]
    hash_val = str(transform_value).replace("2", "-1")
    store[hash_val] = datum["weight"][k]
```

```
# real evaluation method: using all the weights
def eval(state):
  global length
  global storage
  score = 0

  # get all the possible lines
  lines = all_lines(state)

  # score = sum of all the lines in the matrix
  for line in lines:
    # length < 4: no need to care cause 4 lines is a must to win
    if len(line) < 4:
      continue

```

```
    # get the appropriate storage
    cur_idx = length.index(len(line))
    store = storage[cur_idx]
    # get the hash_val to query from the dictionary
    hash_val = ''.join(str(e) for e in line)

    # all score
    score += store[hash_val]
  return score
```

```

```

Game

The first cell is an AI vs AI: my algorithm with depth = 10 vs one with depth = 5. The one with depth 10 wins over the one with depth 5.

Next cell is for player to play. I lost to my depth 5 ai player.

Part 1: AI vs AI: depth 10 vs depth 5

```
# total score
size_y = 6
size_x = 7
depth = 5

state = [[0 for _ in range(size_x)] for __ in range(size_y)]


# while still slot to fill
move = 0
while available_move(state):
  # even move
  if move%2 == 0:
    move += 1
    print("Move number {}: AI #1's turn".format(move))
    # set a timer
```

```
    start_time = time.time()
    if move != 1:
      print("AI #1: Nice move! I'll have to think for a bit!")
    # iterate through the game tree
    set_state(state, iter_DFS(state, 10), 1)
    draw(state)
    print("After ", round(time.time() - start_time) + 1, " seconds thinking!")

  # odd move
  else:
    move += 1
    print("Move number {}: AI #2's turn".format(move))
    start_time = time.time()
    print("AI #2: Good job! My turn to think :)")
    # iterate through the game tree
    set_state(state, iter_DFS(state, 5), -1)
    draw(state)
    print("After ", round(time.time() - start_time) + 1, " seconds thinking!")

  # winning result
  if win(state) == -1:
    print("AI #2 is the winnerrrr!!!!")
    print("AI #2: I wins hahaha!")
    draw(state)
    break


  if win(state) == 1:
    print("AI #1 is the winnerrrr!!!!")
    print("AI #1: I am better than you!!! :)")
    break
```

```
Move number 1: AI #1's turn
I stressed my brain out! You're good! Here's my move:
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||O||-||-||-|
_____________________

After  11  seconds thinking!
Move number 2: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||X||O||-||-||-|
_____________________

After  1  seconds thinking!
Move number 3: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
I stressed my brain out! You're good! Here's my move:
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||-||-||-||-|
|-||-||X||O||-||-||-|
_____________________

After  28  seconds thinking!
Move number 4: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
```

```
 _____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||X||-||-||-|
|-||-||X||O||-||-||-|
 _____________________

After  1  seconds thinking!
Move number 5: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
I stressed my brain out! You're good! Here's my move:
 1  2  3  4  5  6  7
 _____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||O||-||-||-|
|-||-||O||X||-||-||-|
|-||-||X||O||-||-||-|
 _____________________

After  40  seconds thinking!
Move number 6: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
 _____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||X||O||-||-||-|
|-||-||O||X||-||-||-|
|-||-||X||O||-||-||-|
 _____________________

After  1  seconds thinking!
Move number 7: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
I stressed my brain out! You're good! Here's my move:
 1  2  3  4  5  6  7
 _____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
```

```
|-||-||-||-||-||-||-|
|-||-||-||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||O||X||-||-||-|
|-||-||X||O||-||-||-|
_____________________

After  13  seconds thinking!
Move number 8: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||O||X||-||-||-|
|-||-||X||O||X||-||-|
_____________________

After  1  seconds thinking!
Move number 9: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
I stressed my brain out! You're good! Here's my move:
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||O||X||-||-||-|
|-||-||X||O||X||-||-|
_____________________

After  22  seconds thinking!
Move number 10: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||O||X||X||-||-|
|-||-||X||O||X||-||-|
```

```
|-||-||X||O||X||-||-|
_____________________

After  1  seconds thinking!
Move number 11: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
I stressed my brain out! You're good! Here's my move:
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||O||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||O||X||X||-||-|
|-||-||X||O||X||-||-|
_____________________

After  20  seconds thinking!
Move number 12: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
_____________________
|-||-||-||X||-||-||-|
|-||-||-||O||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||O||X||X||-||-|
|-||-||X||O||X||-||-|
_____________________

After  1  seconds thinking!
Move number 13: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
I stressed my brain out! You're good! Here's my move:
 1  2  3  4  5  6  7
_____________________
|-||-||-||X||-||-||-|
|-||-||O||O||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||O||X||X||-||-|
|-||-||X||O||X||-||-|
_____________________
```

```
After  16  seconds thinking!
Move number 14: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
_____________________
|-||-||-||X||-||-||-|
|-||-||O||O||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||X||-||-|
|-||-||O||X||X||-||-|
|-||-||X||O||X||-||-|
_____________________

After  1  seconds thinking!
Move number 15: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||X||-||-||-|
|-||-||O||O||-||-||-|
|-||-||O||O||O||-||-|
|-||-||X||O||X||-||-|
|-||-||O||X||X||-||-|
|-||-||X||O||X||-||-|
_____________________

After  5  seconds thinking!
Move number 16: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
_____________________
|-||-||-||X||-||-||-|
|-||-||O||O||-||-||-|
|-||-||O||O||O||-||-|
|-||-||X||O||X||-||-|
|-||-||O||X||X||-||-|
|-||X||X||O||X||-||-|
_____________________

After  1  seconds thinking!
Move number 17: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
```

```
_____________________
|-||-||O||X||-||-||-|
|-||-||O||O||-||-||-|
|-||-||O||O||O||-||-|
|-||-||X||O||X||-||-|
|-||-||O||X||X||-||-|
|-||X||X||O||X||-||-|
_____________________

After  2  seconds thinking!
Move number 18: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
_____________________
|-||-||O||X||-||-||-|
|-||-||O||O||-||-||-|
|-||-||O||O||O||-||-|
|-||-||X||O||X||-||-|
|-||-||O||X||X||-||-|
|-||X||X||O||X||X||-|
_____________________

After  1  seconds thinking!
Move number 19: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||O||X||-||-||-|
|-||-||O||O||-||-||-|
|-||-||O||O||O||-||-|
|-||-||X||O||X||-||-|
|-||-||O||X||X||O||-|
|-||X||X||O||X||X||-|
_____________________

After  1  seconds thinking!
Move number 20: AI #2's turn
AI #2: Good job! My turn to think :)
 1  2  3  4  5  6  7
_____________________
|-||-||O||X||-||-||-|
|-||-||O||O||-||-||-|
|-||-||O||O||O||-||-|
|-||-||X||O||X||X||-|
```

```
|-||-||O||X||X||O||-|
|-||X||X||O||X||X||-|
_____________________

After  1  seconds thinking!
Move number 21: AI #1's turn
AI #1: Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||O||X||-||-||-|
|-||-||O||O||-||-||-|
|-||-||O||O||O||O||-|
|-||-||X||O||X||X||-|
|-||-||O||X||X||O||-|
|-||X||X||O||X||X||-|
_____________________

After  1  seconds thinking!
AI #1 is the winnerrrr!!!!
AI #1: I am better than you!!! :)
```

```

```

Part 2: AI vs AI: Analysis between pairwaise matches from depth 1 to 7

```
# total score
size_y = 6
```

```
size_x = 7
depth = 5



def play(depth1, depth2):
  time1 = []
  time2 = []
  state = [[0 for _ in range(size_x)] for __ in range(size_y)]
  # while still slot to fill
  move = 0
  while available_move(state):
    # even move
    if move%2 == 0:
      move += 1
      # set a timer
      start_time = time.time()
      # iterate through the game tree
      set_state(state, iter_DFS(state, depth1), 1)
      time1.append(time.time() - start_time)

    # odd move
    else:
      move += 1

      start_time = time.time()
      # iterate through the game tree
      set_state(state, iter_DFS(state, depth2), -1)
      time2.append(time.time() - start_time)

    # winning result
    if win(state) == -1:
      result = -1
      return result, np.mean(time1), np.mean(time2)
      break


    if win(state) == 1:
      result = 1
```

```
      return result, np.mean(time1), np.mean(time2)
      break
  return 0, np.mean(time1), np.mean(time2)
```

```
# result_record[i]: result i play against other: 1 if i win, -1 if i lose, 0 if draw
N = 7
result_record = [[] for _ in range(N)]
present_result_record = [[] for _ in range(N)]

# time_home_record[i][j]: time i cost when i vs j
time_home_record = [[] for _ in range(N)]
present_time_home_record = [[] for _ in range(N)]

# time_away_record[i][j]: time j cost when i vs j
time_away_record = [[] for _ in range(N)]
present_time_away_record = [[] for _ in range(N)]

for depth1 in range(N):
  for depth2 in range(N):
    #print(depth1, depth2)
    result, time1, time2 = play(depth1 + 1, depth2 + 1)
    result_record[depth1].append(result)
    time_home_record[depth1].append(time1)
    time_away_record[depth1].append(time2)

for i in range(N):
  present_result_record[i] = ["Depth " + str(i+1)] + result_record[i]
  present_time_home_record[i] = ["Depth " + str(i+1)] + time_home_record[i]
  present_time_away_record[i] = ["Depth " + str(i+1)] + time_away_record[i]
```

```
print(tabulate(present_result_record, headers= ["Depth " + str(i+1) for i in range(N)]))
```

```
         Depth 1    Depth 2    Depth 3    Depth 4    Depth 5    Depth 6    Depth 7
-------  ---------  ---------  ---------  ---------  ---------  ---------  ---------
Depth 1          1          1          1         -1         -1          1          0
Depth 2          1          1          1         -1         -1          1          0
Depth 3          1          1          1          1          1          1          1
Depth 4          1          1          1         -1          1          1          1
Depth 5          1          1         -1          1          1          1          1
Depth 6         -1         -1         -1         -1         -1          1         -1
Depth 7          1          1          1          1          1         -1          1
```

```
print(tabulate(present_time_home_record, headers= ["Depth " + str(i+1) for i in range(N)]))
```

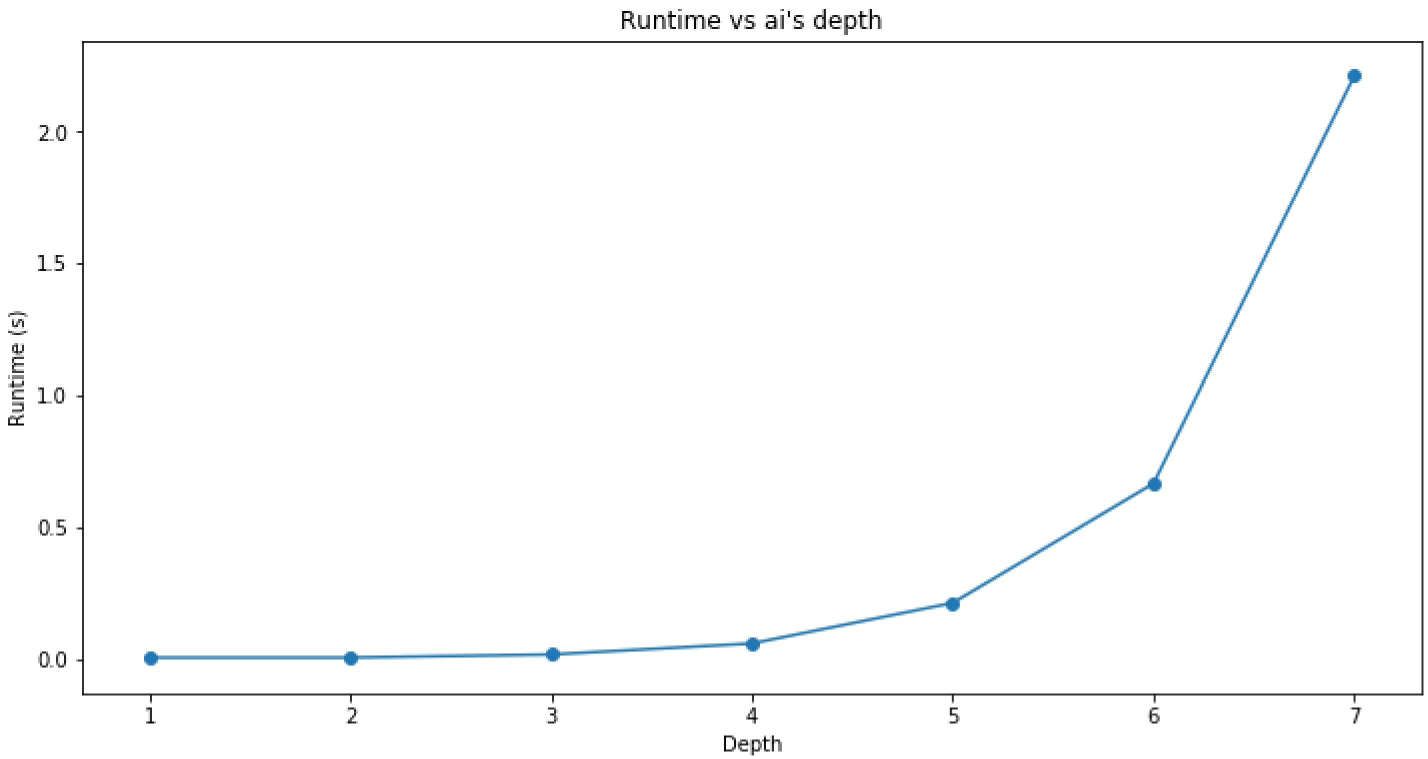
```
           Depth 1     Depth 2     Depth 3     Depth 4     Depth 5     Depth 6     Depth 7
-------  ----------  ----------  ----------  ----------  ----------  ----------  ----------
Depth 1  0.00397251  0.00359475  0.00398839  0.00393493  0.00335439  0.00424588  0.00337117
Depth 2  0.00390619  0.00402462  0.00393882  0.00392917  0.00332775  0.00422464  0.0032877
Depth 3  0.0155327   0.0156759   0.013945    0.0157022   0.0149929   0.0158774   0.0169935
Depth 4  0.059597    0.058298    0.0595243   0.0463924   0.0639935   0.0602978   0.0529915
Depth 5  0.25146     0.253011    0.161102    0.203386    0.16943     0.265315    0.174722
Depth 6  0.586758    0.629097    0.609956    0.632597    0.607308    0.741331    0.845137
Depth 7  2.46071     2.54834     2.75253     2.12555     1.77397     2.14283     1.65966
```

```
print(tabulate(present_time_away_record, headers= ["Depth " + str(i+1) for i in range(N)]))
```

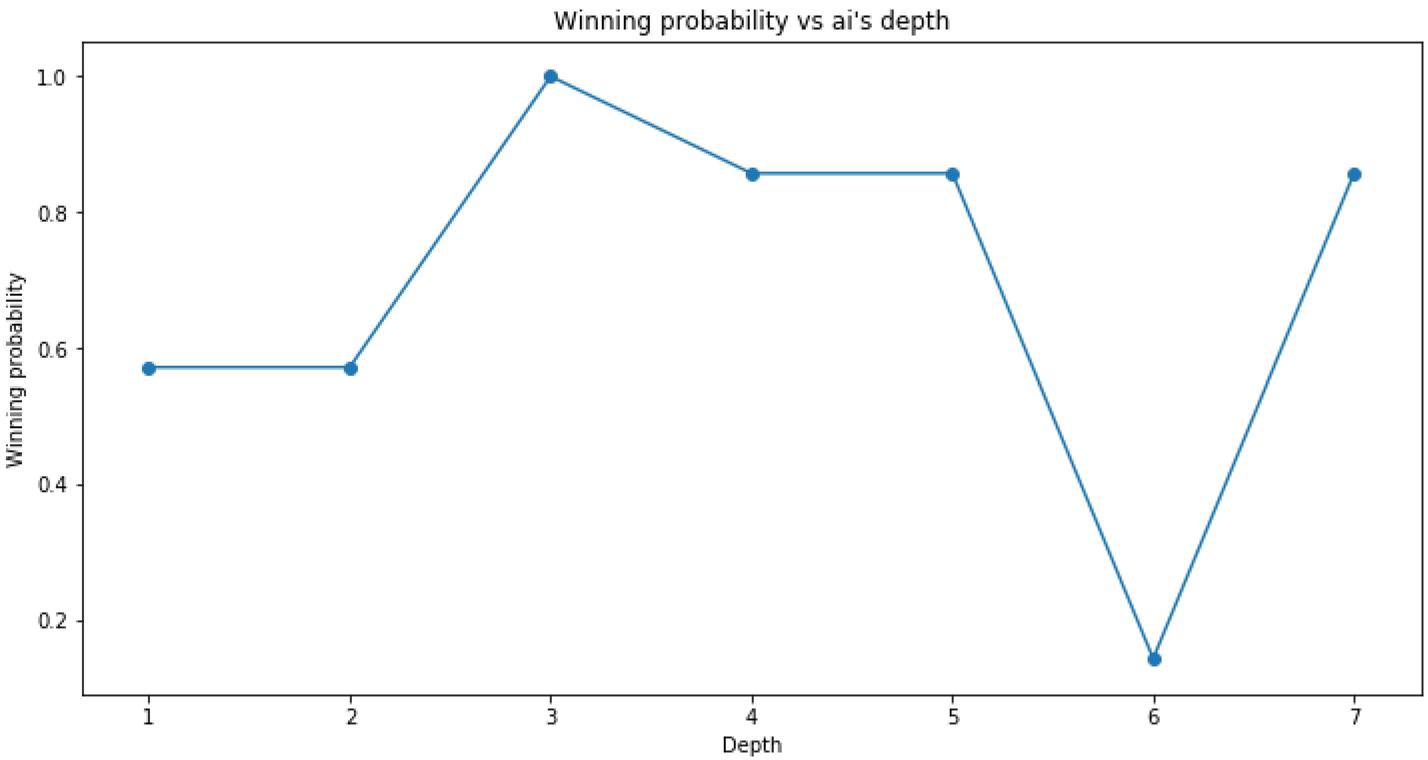
```
           Depth 1     Depth 2     Depth 3    Depth 4    Depth 5    Depth 6    Depth 7
-------  ----------  ----------  ----------  ---------  ---------  ---------  ---------
Depth 1  0.00405841  0.00352865  0.0138756   0.0423746   0.155805   0.841318    2.22451
Depth 2  0.00397658  0.00407711  0.0131456   0.0440853   0.155739   0.838967    2.21726
Depth 3  0.00445428  0.00454807  0.013797    0.0496254   0.160098   0.654848    2.3252
Depth 4  0.00396058  0.00386056  0.0128266   0.043655    0.184953   0.658056    1.20109
Depth 5  0.00390611  0.00390339  0.0107121   0.0607644   0.185251   0.964602    2.40417
Depth 6  0.00287534  0.00287617  0.00960918  0.0409449   0.128311   0.64728     2.45096
Depth 7  0.00377699  0.00381105  0.0169116   0.0653051   0.155627   0.558808    1.96797
```

```
plt.figure(figsize= (12,6))
plt.plot(range(1, N+1), np.mean(np.array(time_home_record), axis = 1))
plt.scatter(range(1, N+1), np.mean(np.array(time_home_record), axis = 1))
plt.xlabel("Depth")
```

```
plt.ylabel("Runtime (s)")
plt.title("Runtime vs ai's depth")
plt.show()
```




```
plt.figure(figsize= (12,6))
plt.plot(range(1, N+1), np.mean(np.array(result_record) == 1, axis = 1))
plt.scatter(range(1, N+1), np.mean(np.array(result_record) == 1, axis = 1))
plt.xlabel("Depth")
plt.ylabel("Winning probability")
plt.title("Winning probability vs ai's depth")
plt.show()
```




```

```

Part 3: Human vs AI

```
# total score
size_y = 6
size_x = 7
depth = 5

depth = input("WELCOME TO CONNECT 4! Please select the difficulty (1-10): ")
while depth.isnumeric() == False or int(depth) < 0 or int(depth) > 10:
  print("INVALID DIFIICULTY: must be integer from 1 to 10.")
  depth = input("WELCOME TO CONNECT 4! Please select the difficulty (1-10): ")

depth = int(depth)
```

```
depth = int(depth)
print("You are X, the other player will be O")
# initialize the state
state = [[0 for _ in range(size_x)] for __ in range(size_y)]
human_first = input("Do you want to play first? (Y/N)?")
#the player plays first

move = 0
if human_first.capitalize() == 'Y':
  # present table;
  draw(state)

  # while still slot to fill
  while available_move(state):
    n = input("Your move: (1-7) ")
    check_move(state, n)
    draw(state)

    # if user win:
    if win(state) == -1:
      print("You win!!! Congrats!")
      draw(state)
      break

    start_time = time.time()
    print("Nice move! I'll have to think for a bit!")
    set_state(state, iter_DFS(state, depth), 1)
    draw(state)
    print("After ", round(time.time() - start_time), " seconds thinking!")
    if win(state) == 1:
      print("AI wins! Better luck next time :)")
      break

#The AI ai_score plays first
else:
  while available_move(state):
    start_time = time.time()
    print("Nice move! I'll have to think for a bit!")
    set_state(state, iter_DFS(state, depth), 1)
    draw(state)
```

```
    draw(state)
    print("After ", round(time.time() - start_time) + 1, " seconds thinking!")
    if win(state)==1:
      print("AI wins! Better luck next time :)")
      break

    n = input("Your move: (1-7) ")
    check_move(state, n)
    draw(state)
    if win(state) == -1:
      print("You win!!! Congrats!")
      break
```

```
WELCOME TO CONNECT 4! Please select the difficulty (1-10): 7
You are X, the other player will be O
Do you want to play first? (Y/N)?y
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
_____________________

Your move: (1-7) 4
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||X||-||-||-|
_____________________

Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||X||-||-||-|
_____________________

After  5  seconds thinking!
Your move: (1-7) 4
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
```

```
|-||-||-||X||-||-||-|
|-||-||O||X||-||-||-|
_____________________

Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||O||-||-||-|
|-||-||-||X||-||-||-|
|-||-||O||X||-||-||-|
_____________________

After  3  seconds thinking!
Your move: (1-7) 3
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||O||-||-||-|
|-||-||X||X||-||-||-|
|-||-||O||X||-||-||-|
_____________________

Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||-||O||-||-||-|
|-||-||-||O||-||-||-|
|-||-||X||X||-||-||-|
|-||-||O||X||-||-||-|
_____________________

After  9  seconds thinking!
Your move: (1-7) 3
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
```

```
|-||-||-||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||X||X||-||-||-|
|-||-||O||X||-||-||-|
_____________________

Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||X||X||-||-||-|
|-||-||O||X||-||-||-|
_____________________

After  4  seconds thinking!
Your move: (1-7) 5
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||X||X||-||-||-|
|-||-||O||X||X||-||-|
_____________________

Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||X||X||O||-||-|
|-||-||O||X||X||-||-|
_____________________

After  5  seconds thinking!
Your move: (1-7) 6
 1  2  3  4  5  6  7
```

```
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||X||X||O||-||-|
|-||-||O||X||X||X||-|
_____________________

Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||X||X||O||-||-|
|-||-||O||X||X||X||O|
_____________________

After  3  seconds thinking!
Your move: (1-7) 2
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||-||-||-|
|-||-||X||X||O||-||-|
|-||X||O||X||X||X||O|
_____________________

Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||O||-||-|
|-||-||X||X||O||-||-|
|-||X||O||X||X||X||O|
_____________________

After  5  seconds thinking!
```

```
After  5  seconds thinking!
Your move: (1-7) 6
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||-||-||-|
|-||-||X||O||O||-||-|
|-||-||X||X||O||X||-|
|-||X||O||X||X||X||O|
_____________________

Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||O||-||-|
|-||-||X||O||O||-||-|
|-||-||X||X||O||X||-|
|-||X||O||X||X||X||O|
_____________________

After  6  seconds thinking!
Your move: (1-7) 2
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||-||-||-|
|-||-||O||O||O||-||-|
|-||-||X||O||O||-||-|
|-||X||X||X||O||X||-|
|-||X||O||X||X||X||O|
_____________________

Nice move! I'll have to think for a bit!
 1  2  3  4  5  6  7
_____________________
|-||-||-||-||-||-||-|
|-||-||-||-||O||-||-|
|-||-||O||O||O||-||-|
|-||-||X||O||O||-||-|
|-||X||X||X||O||X||-|
|-||X||O||X||X||X||O|
```

```
After  0  seconds thinking!
AI wins! Better luck next time :)
```

```
1
```

Reference:

Saveski, M. (n.d.). AI Agent. Retrieved December 17, 2019, from http://web.media.mit.edu/~msaveski/projects/.