Consider the system:

$$H = J\sum_{<i,j>}\vec{s}_i\cdot\vec{s}_j + \vec{h}\cdot\sum_i \vec{s}_i$$

Rewrite $\vec{s}_i = \vec{m} + (\vec{s}_i - \vec{m})$, where $\vec{m} = \sum_i \vec{s}_i$

Substituting this into the Hamiltonian gives

$$H \approx NJzm^2 + \sum_i (2Jz\vec{m} + \vec{h})\cdot\vec{s}_i = Jzm^2 + \sum_i h_{eff}\cdot\vec{s}_i$$

where z is the coordination number (i.e. the number of spins that are interacting at any site), and we have thrown away terms of order $O((\vec{s}_i - \vec{m})^2)$.

We will write $\vec{h_{eff}} = (h\cos(\phi), h\sin(\phi))$, and $\vec{s}_i = (\cos(\theta_i), \sin(\theta_i))$.

The partition function is then

$$\begin{aligned}
Z &= \int\prod_i d\theta_i \exp\{-\beta J z^2 m - \beta h_{eff}\sum_i \cos(\theta_i - \phi)\} \\
&= e^{-N\beta Jzm^2}\left[\int d\theta \exp\{-\beta h_{eff}\cos(\theta-\phi)\}\right]^N \\
&= e^{-N\beta Jzm^2}\left[2\pi I_0(\beta h_{eff})\right]^N
\end{aligned}$$

So the Free energy is

$$\begin{aligned}
F &= -\frac{1}{\beta}\ln Z \\
&= \frac{1}{\beta}\left[N\beta J z^2 m - N\ln\left(\pi I_0(2\beta Jzm)\right)\right] \\
&= NJz^2 m - \frac{N}{\beta}\ln(2\pi I_0(2\beta Jzm))
\end{aligned}$$

We can compute $\vec{m} = < \vec{s}_i >$

$$\begin{aligned}
\vec{m} &= \frac{1}{\beta N Z}\frac{\partial Z}{\partial h}|_{h=0} \\
&= \frac{1}{\beta N}\frac{\partial \ln Z}{\partial h_{eff}} \\
&= -\frac{I_1(\beta h_{eff})}{I_0(\beta h_{eff})}\hat{h}_{eff} \\
&= -\frac{I_1(2\beta Jm)}{I_0(2\beta Jm)}\hat{h}_{eff}
\end{aligned}$$

Note: $I_1(x)$ is odd. This has no solution (besides $m = 0$ for $J > 0$. For $J < 0$ this has a solution (besides $m = 0$ when $\beta > \beta_c = |1/(Jz)|$.

And likewise $< s_i s_j >= \chi_M$

$$
\begin{aligned}
< s_i s_j > &= \frac{1}{\beta N Z}\frac{\partial^2 Z}{\partial h^2}|_{h=0} \\
&= \frac{1}{\beta N}\frac{\partial^2 \ln Z}{\partial h_{eff}^2}|_{h=0} \\
&= \frac{\partial}{\partial h_{eff}}\frac{I_1(\beta h_{eff})}{I_0(\beta h_{eff})}|_{h=0} \\
&= \beta\left[\frac{I_0(\beta h_{eff}) + I_2(\beta h_{eff})}{2I_0(\beta h_{eff})} - \left(\frac{I_1(\beta h_{eff})}{I_0(\beta h_{eff})}\right)^2|_{h=0}\right]
\end{aligned}
$$

Meanwhile the energy is found from

$$
\begin{aligned}
< E > &= -\frac{1}{NZ}\frac{\partial Z}{\partial \beta} \\
&= -\frac{\partial \ln Z}{\partial \beta} \\
&= Jzm^2
\end{aligned}
$$

where in the last step we took $\vec{h} = 0$.

and the specific heat is

$$
\begin{aligned}
C_v &= \frac{\beta^2}{NZ}\frac{\partial^2 Z}{\partial \beta^2} \\
&= h_{eff}^2\left[\frac{I_0(\beta h_{eff}) + I_2(\beta h_{eff})}{2I_0(\beta h_{eff})} - \beta^2\left(\frac{I_1(\beta h_{eff})}{I_0(\beta h_{eff})}\right)^2\right] \\
&= (2Jzm)^2\beta\chi_M
\end{aligned}
$$

where in the last step we took $\vec{h} = 0$.