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EQUIVALENT CIRCUIT MODELING OF PATCH-BASED PIEZOELECTRIC ENERGY HARVESTING ON PLATE-LIKE STRUCTURES WITH AC-DC CONVERSION

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ABSTRACT

The equivalent circuit modeling of the vibration-based energy harvesters for accurate estimation of electrical response has drawn much attention over the recent years. Different methods have been proposed to obtain the equivalent circuit parameters using analytical and finite element models of the piezoelectric energy harvesters. In such methods, the structure is a typical cantilever beam with piezoelectric layers under base excitation. As an alternative to beams, piezoelectric patch-based harvesters attached to thin plates can be considered due to the wide use of plate-like structures in automotive, marine and aerospace applications. Considering these needs, a multi-mode equivalent circuit model of a piezoelectric energy harvester integrated to a thin plate is developed in this study. Equivalent circuit parameters are obtained from analytical distributed-parameter model of the harvester which covers the electromechanical coupling behavior of the piezoelectric patch and vibration of the host plate. The multi-mode circuit representation of the harvester is built via electronic circuit simulation software SPICE. Using the SPICE software, electrical outputs of the piezoelectric energy harvester connected to linear and nonlinear circuit elements are computed. Simulation results are then validated for the standard AC-AC and AC-DC configurations. For the AC configuration, voltage Frequency Response Functions (FRFs) are calculated for various resistive loads and they exhibit excellent agreement with the published analytical closed-form solution. For the full-wave rectifier configuration, simulation results of the DC voltage and power outputs are calculated for a wide range of load resistance values and compared with the analytical single-mode expression of the harvester in the literature.

INTRODUCTION

There has been great advancement in self-powered wireless sensors and health monitoring systems in which the external power requirement has efficiently decreased to micro-watt range in the past few decades [1, 2]. The development of vibration-based energy harvesters has raised opportunities for low power electronics to have constant power supply from ambient vibration source as an alternative to conventional batteries [3]. There exist several transduction mechanisms in which vibrational energy can be converted to electrical energy through electrostatic [4, 5], electromagnetic [6, 7], magnetostrictive [8, 9], electroactive and electrostrictive, polymers [10, 11] and piezoelectric [12, 13] transducers. Among these conversion methods, piezoelectric energy harvesters have drawn more attention, since they have high power density, and they are commonly used in microsystems and macroscaled devices [14].

In the literature, most of the piezoelectric energy harvesting studies include cantilever piezoelectric beams subjected to base excitations [13]. Electromechanical coupling behavior of these beam harvesters have been modeled using analytical and numerical methods by several researchers [3, 15-20]. Single-degree-of-freedom model [3], Rayleigh-Ritz solution [15, 16], assumed-modes modeling [17], distributed-parameter modeling [18, 19] and coupled finite element method [20] were presented to predict electrical outputs of the beam harvesters.

Piezoeceramic patches can be also integrated to thin plates to extract vibrational energy of their host structure and convert it to electrical energy. This implementation can be even more convenient and practical for use on the surfaces of aerospace, automotive, and marine applications enabling more compact and broadband energy harvesting. However, piezoelectric energy harvesting on two-dimensional lightweight and flexible structures has been rarely reported in the existing literature. Some of the following studies are worthwhile to mention here regarding the harvesters attached to plate-like structures. De Marqui et al. [21], presented an electromechanically coupled finite element model of a plate with embedded piezoceramic layers. Rupp et al. [22], developed a finite element model of a harvesting structure to conduct topology and shape optimizations for distribution of piezoelectric materials with objective of maximizing the harvested power output. Harne et al. [23], built an analytical model for representing electroelastic dynamics of a vibrating panel with corrugated piezoelectric spring. More recently, Aridogan et al. [24] have presented analytical distributed-parameter electroelastic model of patch-based piezoelectric energy harvesters structurally integrated to a fully clamped plate with experimental verification.

In modeling of piezoelectric energy harvesters using cantilever beams and/or plate-like structures, researchers mainly focused on predicting the electrical outputs for the simplified circuit consisting of only resistive loads [19, 24]. In reality, energy harvesting circuits are more sophisticated. For instance, researchers employed AC-DC converters [25-27] to rectify the AC voltage and later, proposed synchronized switch harvesting on inductor (SSHI) technique [26, 28, 29] to boost up power output of the energy harvesters. All the aforementioned circuits include nonlinear electrical parts. However, analytical analysis of such harvesters becomes very difficult when multi-vibration modes of the structure are considered. Instead, researchers utilized Lumped-parameter modeling of energy harvesters with nonlinear electrical components to predict the electrical output strictly near a single vibration mode. Even though this approach gives good prediction around the resonance modes, it fails when the vibration modes are closely distributed and also neglects effect of the other modes on the electrical response. Using the equivalent circuit representations of electromechanical transducers [30, 31], Elvin and Elvin [32] built an equivalent circuit model (ECM) for a piezoelectric energy harvester which could capture the dynamics of a harvester for multiple vibration modes. Parameters of the equivalent circuit were obtained from the Rayleigh-Ritz method. Thus, the lumped parameters of the mechanical domain were modeled as a RLC (resistance, inductor and capacitor) circuit and the electromechanical coupling term was represented as an ideal transformer. They first validated the results of the equivalent circuit model for the AC configuration with comparisons against the existing analytical solutions of a cantilever piezoelectric beam. Later, they used the lumped parameters as well as full–wave rectifier in SPICE software for simulation of the capacitor charging scenario. As an alternative, Elvin and Elvin [20] and Yang and Tang [33] suggested obtaining the system parameters from the finite element model of the system for accurate modeling of complex structures and then using them in SPICE software for simulating the nonlinear circuits. Yang and Tang [33] also presented the equivalent circuit modeling of piezoelectric beam harvesters using the analytical distributed-parameter model introduced by Erturk and Inman [18]. They obtained the equivalent circuit parameters through analogizing of second-order circuit equation with electromechanical coupled equations of the beam harvester.

In the existing literature of energy harvesting, modeling of piezoelectric patch harvesters integrated to plate-like structures including nonlinear circuits has not been studied. Considering these needs in the harvesting methodologies, this study takes the expertise in the field of piezoelectric harvesters integrated to thin plates to the next level and develop more generalized models for such applications. First, system parameters are identified from the analytical distributed-parameter model developed by Aridogan et al. [24] and results from SPICE simulation are validated for the AC configuration. Then, rectified voltage and power outputs for each single vibration mode obtained through the SPICE software is computed and verified against the Lumped-parameter model presented by Shu and Lien [27].

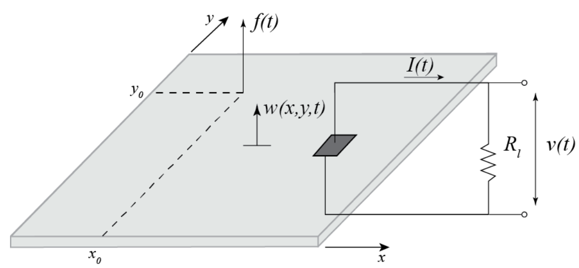

Figure 1. PATCH-BASED PIEZOELECTRIC ENERGY HARVESTER CONNECTED TO A RESISTIVE LOAD

DISTRIBUTED-PARAMETER MODEL

Figure 1 presents the piezoeceramic patch structurally integrated to a thin plate, which is excited by a transverse point force $f(t)$ at (x_0, y_0) position. A perfectly bonded isotropic piezoceramic patch with a length of l_p, width of w_p and thickness of h_p covers a rectangular region. The length, width and thickness of the host plate are a, b and h_s respectively. Kirchhoff's plate theory is used for the distributed-parameter model since thickness of the plate is comparably smaller than other dimensions. The partial differential equation of governing the forced vibration of a piezoceramic patch attached to a thin plate is (details are explained in [24])

$$D\left(\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\frac{\partial^4 w(x,y,t)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right)$$
$$+c\frac{\partial w(x,y,t)}{\partial t}+\rho_s h_s\frac{\partial^2 w(x,y,t)}{\partial t^2}$$
$$-\theta v(t)\left\{\left[\frac{\mathrm{d}\delta(x-x_1)}{\mathrm{d}x}-\frac{\mathrm{d}\delta(x-x_2)}{\mathrm{d}x}\right]\right.$$
$$\times\left[H(y-y_1)-H(y-y_2)\right]$$
$$+\left[\frac{\mathrm{d}\delta(y-y_1)}{\mathrm{d}y}-\frac{\mathrm{d}\delta(y-y_2)}{\mathrm{d}y}\right]$$
$$\left.\times\left[H(x-x_1)-H(x-x_2)\right]\right\}$$
$$=f(t)\delta(x-x_0)\delta(y-y_0) \tag{1}$$

where $v(t)$ is the voltage across the external resistive load and the electromechanical term θ is defined as $\theta=\bar{e}_{31}h_{pc}$, which $\bar{e}_{31}$ is the effective piezoelectric constant and $h_{pc}=(h_s+h_p)/2$ is reference distance. The density of the plate is ρ_s, the flexural rigidity of the plate is $D=Y_s h_s^3/(12-12\nu_s^2)$, where Young's modulus is Y_s and Poisson's ratio is ν_s. The viscous air damping coefficient is c_a. The Dirac delta functions are *δ(x)* and *δ(y)*, and the Heaviside functions are $H(x)$ and $H(y)$ along *x* and *y* axes, respectively. It should be noted that the volume of the piezoceramic patch is assumed to be significantly smaller than the host plate, and the piezoceramic patch's inertial and stiffness effects are neglected. The governing differential equation of the electrical circuit can be derived by substituting the electrical displacement from piezoelectric constitutive equation into the Gauss law [34] as:

$$C_p\frac{\mathrm{d}v(t)}{\mathrm{d}t}+\frac{v(t)}{R_l}+\theta\left\{\int_{y=y_1}^{y_2}\int_{x=x_1}^{x_2}\left[\frac{\partial^3 w(x,y,t)}{\partial x^2\,\partial t}\right.\right.$$
$$\left.\left.+\frac{\partial^3 w(x,y,t)}{\partial y^2\,\partial t}\right]\mathrm{d}x\,\mathrm{d}y\right\}=0 \tag{2}$$

where the capacitance of the piezoceramic is defined as $C_p=\left(\bar{\varepsilon}_{33}^S w_p l_p\right)/h_p$.

By following the modal analysis procedure for the equations (1) and (2), electromechanically coupled ordinary differential equations of the plate in modal coordinates can be written as:

$$\frac{\mathrm{d}^2\eta_{mn}(t)}{\mathrm{d}t^2}+2\zeta_{mn}\omega_{mn}\frac{\mathrm{d}\eta_{mn}(t)}{\mathrm{d}t}$$
$$+\omega_{mn}^2\eta_{mn}(t)-\tilde{\theta}_{mn}v(t)=f_{mn}(t) \tag{3}$$

$$C_p\frac{\mathrm{d}v(t)}{\mathrm{d}t}+\frac{v(t)}{R_l}+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\tilde{\theta}_{mn}\frac{\mathrm{d}\eta_{mn}(t)}{\mathrm{d}t}=0 \tag{4}$$

where the plate modes are designated by *m* and *n*. The modal forcing f_{mn}, and the electromechanical coupling term $\tilde{\theta}_{mn}$ are expressed in the following form:

$$f_{mn}=\int_0^b\int_0^a f(t)\delta(x-x_0)\delta(y-y_0)\phi_{mn}(x,y)\,\mathrm{d}x\,\mathrm{d}y$$
$$=f(t)\phi_{mn}(x_0,y_0) \tag{5}$$

$$\tilde{\theta}_{mn}=\theta\left[\int_{y_1}^{y_2}\left.\frac{\partial\phi_{mn}(x,y)}{\partial x}\right|_{x_1}^{x_2}\mathrm{d}y+\int_{x_1}^{x_2}\left.\frac{\partial\phi_{mn}(x,y)}{\partial y}\right|_{y_1}^{y_2}\mathrm{d}x\right]. \tag{6}$$

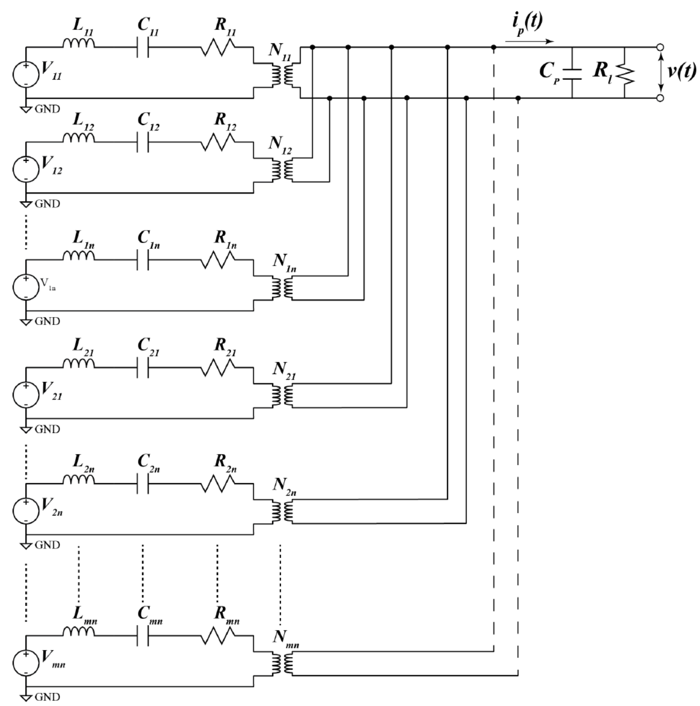

Figure 2. MULTI-VIBRATION MODE CIRCUIT REPRESENTATION OF THE HARVESTER CONNECTED TO A RESISTIVE LOAD

EQUIVALENT CIRCUIT MODELING

The above analytical distributed-parameter model of the plate can capture any vibration modes and has been verified experimentally [24]. Although this model is capable of predicting the power output of the system accurately, its validity is limited with characteristics of the electrical circuit elements connected to the harvester. If only a resistive load is considered, then the distributed parameter model is very efficient and accurate. However, if there is an additional non-linear circuit element, then another approach is required to predict the system behavior accurately.

Considering these needs, this paper focuses on equivalent circuit modeling and analysis of a patch-based harvester integrated to a thin plate while connected to a full-wave rectifier. These models will be based on the equivalent circuit modeling technique which is obtained by superposition of infinite number of lumped-parameter models of each vibration mode. Figure 2 shows the equivalent circuit model of the piezoelectric energy harvester integrated to a thin plate. Each vibrational mode of the harvester is represented as a second-order circuit which is connected to a piezoelectric patch (C_P) and a resistive load (R_l) through an ideal transformer. By applying Kirchoff's voltage law to the multi-vibration mode circuit and analogizing with equation (3), system parameters of the equivalent circuit model can be determined. Table 1 gives a summary of analogy between the analytical distributed-parameter expression and the equivalent circuit model.

Table 1. ANALOGY BETWEEN ELECTRICAL AND MECHANICAL DOMAINS OF PIEZOELECTRIC PATCH-BASED HARVESTER INTEGRATED TO A THIN PLATE

Equivalent circuit parameters at mn-th mode	Mechanical counterparts
Voltage source: $V_{mn}(t)$	Modal point force: $f_{mn}(t)$
Inductance: $L_{mn}(t)$	1
Resistance: $R_{mn}(t)$	$2\zeta_{mn}\omega_{mn}$
Capacitance: $C_{mn}(t)$	$1/\omega_{mn}$
Ideal Transformer ratio: N_{mn}	$-\widetilde{\theta}_{mn}$
Electrical charge: $q_{mn}(t)$	Modal time response: $\eta_{mn}(t)$
Electrical current: $i_{mn}(t)$	$d\eta_{mn}(t)/dt$

Figure 3 demonstrates simple electrical circuit of the harvester patch as a dependent current source i_p in parallel with its internal capacitance C_p. Applying Kirchoff's current law and analogizing with equation (4), the current flowing to piezoceramic capacitance and the current source can be written as:

$$i_c = C_p \frac{\mathrm{d}v(t)}{\mathrm{d}t}\ ,\ i_p = -\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\widetilde{\theta}_{mn}\frac{\mathrm{d}\eta_{mn}(t)}{\mathrm{d}t} \tag{7}$$

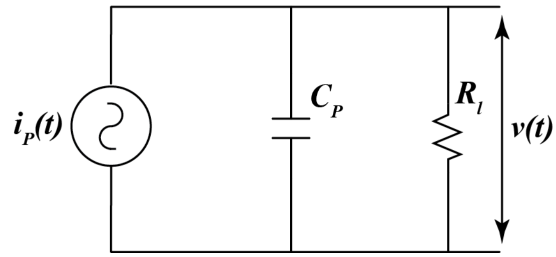

Figure 3. ELECTRIC CIRCUIT FOR CONNECTION OF PIEZOELECTRIC PATCH TO RESISTIVE LOAD

MODEL VALIDATION

In this section, equivalent circuit model of a fully clamped plate with a perfectly bonded piezoceramic patch is built in SPICE software and simulation results are validated for the standard AC and DC problems. AC and DC configurations are validated using the distributed parameter model developed by Aridogan et al. [24] and Lumped Parameter model introduced by Shu and Lien [27], respectively. The material, geometric, dielectric and electroelastic properties of the aluminum plate and piezoceramic patch are given in Table 2.

Table 2. GEOMETRIC, MATERIAL AND ELECTROELASTIC PROPERTIES

Property	Aluminum	Piezoceramic
Length (mm)	580	72.4
Width (mm)	540	72.4
Thickness (mm)	1.96	0.267
Young's modulus (GPa)	70	66
Mass density (kg m^{-3})	2700	7800
Piezoelectric constant d_{31} (pm/V)	-	-190
Permittivity constant $\bar{\varepsilon}_{33}^{S}$ (nF/m)	-	10.38

AC-AC PROBLEM

This section focuses on validation of the equivalent circuit model by comparing the voltage frequency response functions by comparing them against the results of the analytical distributed-parameter model. Voltage response of the harvester under the effect of a harmonic force is shown in Figure 4 for various resistive load cases (ranging from 10 Ω to $1\,\mathrm{M}\Omega$). Results of distributed analytical model and SPICE simulation for equal number of vibrational modes are compared and it can be observed that the equivalent circuit model accurately predicts the voltage response for varying resistive load cases.

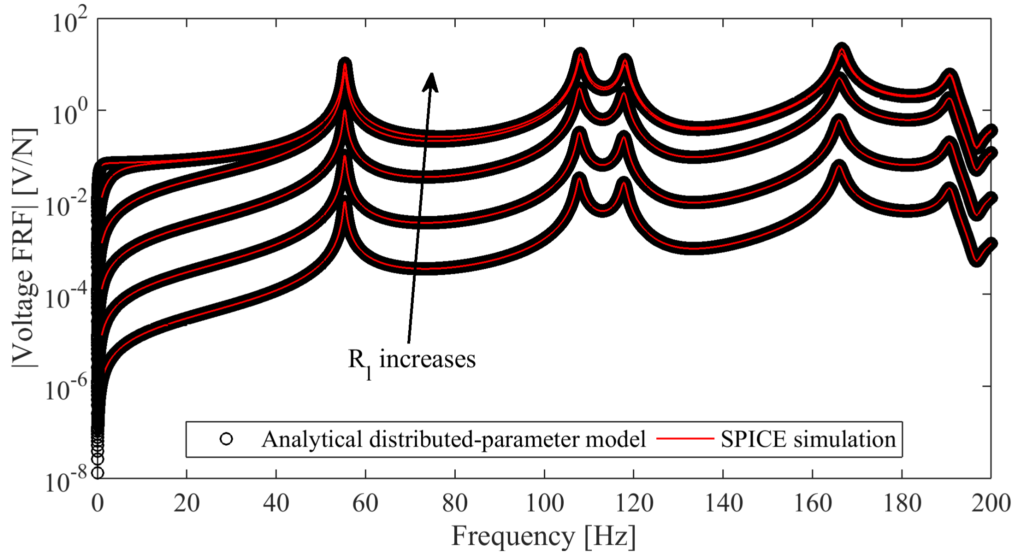

Figure 4. COMPARISON OF SPICE SIMULATION AND ANALYTICAL RESULTS FOR VOLTAGE FRFS WITH 6 DIFFERENT RESISTANCE VALUES

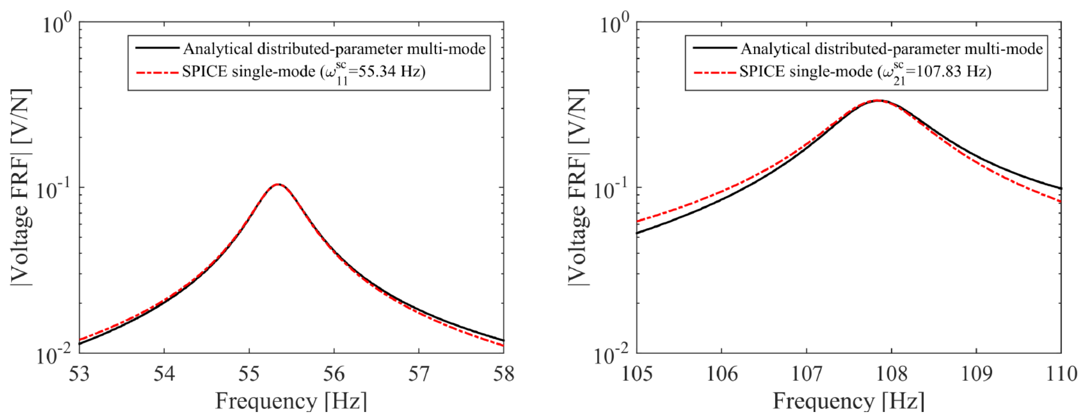

Figure 5. COMPARISON OF ANALYTICAL MULTI-MODE AND SPICE SINGLE-MODE VOLTAGE FRFS FOR THE FIRST TWO VIBRATION MODES (R_l = 100 Ω)

AC-DC PROBLEM

The Lumped Parameter model introduced by Shu and Lien considers only single-mode expression of the harvester. For that reason, single-mode results of the equivalent circuit model are compared with the analytical multi-mode solution in the vicinity of the first two modes as shown in Figure 5. It can be observed that single-mode results of the equivalent circuit model are in good agreement with the analytical multi-mode results.

Once the equivalent circuit model is validated for single (around the resonance neighborhood) and multi-mode cases in the AC configuration, the voltage and power output of the equivalent circuit model of the harvester must be also validated for the DC configuration. For that purpose, using the lumped-parameters for each vibrational mode, AC-DC harvesting circuit is simulated in SPICE software and validated with the analytical model proposed by Shu and Lien [27]. In their study, the authors developed an analytical formulation of the rectified DC voltage output for the piezoelectric energy harvesters using a lumped-parameter approach. The steady-state voltage (V_c) for the piezoelectric patch-based harvester can be written as [27]:

$$V_c = \frac{L_{mn}\omega R_l}{R_l C_p \omega + \pi/2} u_0 \tag{8}$$

where the displacement amplitude u_0 is derived in terms of the point forcing amplitude and the system parameters as:

$$u_0 = \frac{F_0 \phi_{mn}(x_0, y_0)}{\left\{ \left(R_{mn}\omega + \frac{2\omega N_{mn}^2 R_l}{(C_p \omega R_l + \pi/2)^2} \right)^2 + \left(1/C_{mn} - \omega^2 L_{mn} + \frac{\omega N_{mn}^2 R_l}{C_p \omega R_l + \pi/2} \right)^2 \right\}^{\frac{1}{2}}} \tag{9}$$

For detailed derivation information, reader is referred to [27].

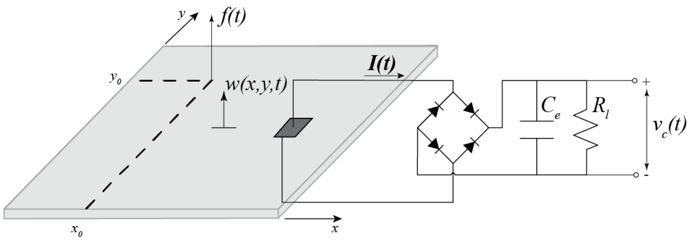

FIGURE 6. PATCH-BASED PIEZOELECTRIC ENERGY HARVESTER CONNECTED TO A FULL-WAVE RECTIFIER, A RESERVOIR CAPACITANCE AND A RESISTIVE LOAD

A patch-based piezoelectric energy harvesting circuit is configured for the validation purpose. Figure 6 illustrates the AC-DC patch-based piezoelectric energy harvesting circuit consisting of a full-bridge diode, a smoothing capacitor and a resistive load. To generate a stable DC voltage v_c, it can be assumed that the capacitor C_e is large enough to achieve constant DC voltage output [25]. It should be noted that, ideal transformer and diodes are used in the simulations such that the results can be compared to Shu and Lien's analytical model. Analysis of the harvesting circuit is performed for each single vibration mode in SPICE software. By conducting transient analysis for the harvester circuit, steady-state voltage values are extracted for various values of resistive loads. Figure 7 and 8 present the SPICE simulation results for the peak DC voltage and power outputs and compare the results with Shu and Lien's analytical model for a wide range of resistive loads. Again, the equivalent circuit model accurately predicts the DC voltage and power outputs. It can be also seen from the graph that there exists an optimum resistance in which the DC power output becomes maximum.

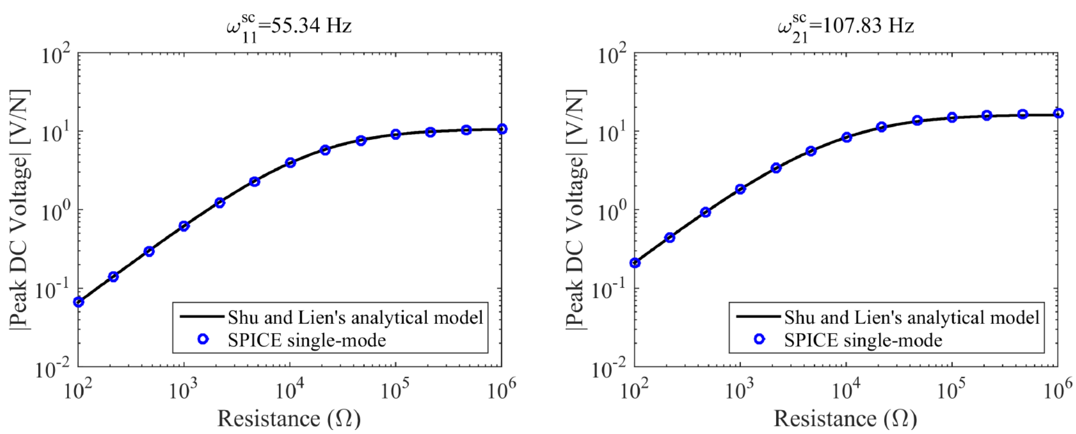

Figure 7. PEAK DC VOLTAGE OUTPUT VS RESISTANCE FOR MODE 1 (LEFT) AND MODE 2 (RIGHT)

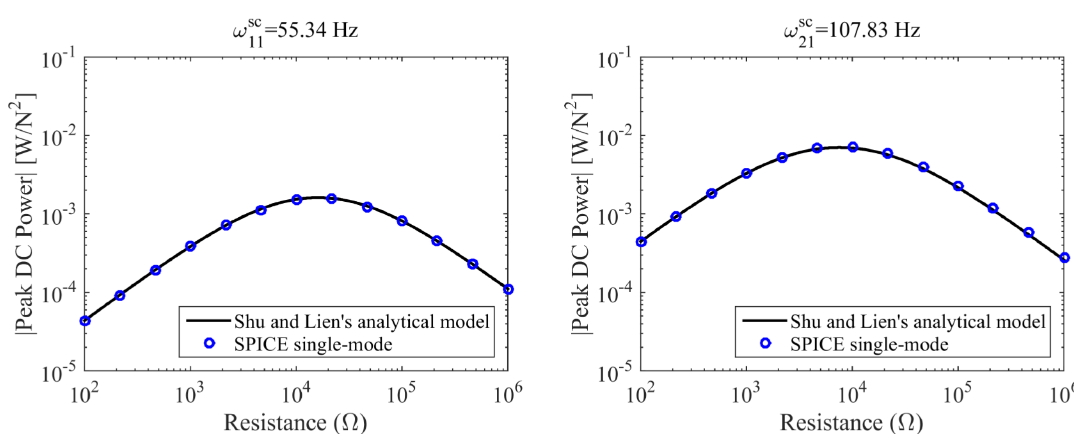

Figure 8. PEAK DC POWER OUTPUT VS RESISTANCE FOR MODE 1 (LEFT) AND MODE 2 (RIGHT)

CONCLUSION

In this paper, an equivalent circuit model of a piezoelectric patch-base harvester attached on a thin plate was developed and validated for the AC and DC configurations. The equivalent circuit parameters considering the multi-vibration modes of the harvester were identified from the analogy between analytical distributed-parameter model and the second-order circuitry equations. Then, using these system parameters, simulations were conducted in SPICE software.

The simulation results of voltage FRFs were presented for a wide range of resistive load cases and verified against the analytical model in the AC configuration. It was shown that equivalent circuit model accurately predicts the coupled electromechanical behavior. Equivalent circuit single-mode representation was also verified with the analytical multi-mode solution in the vicinity of vibration modes. Then, single-mode equivalent circuit model was connected to an AC-DC rectifier, a smoothing capacitor and a resistive load. Steady-state electrical responses were investigated for the first two modes of the harvester. Finally, peak DC voltage and power outputs across different resistive loads were computed and validated with the existing analytical single-degree-of-freedom model proposed by Shu and Lien [27]. The multi-mode equivalent circuit model developed in this study enables exploring the harvesting performance of the piezoelectric patch-based harvesters attached to thin plates with any linear/ nonlinear electrical components. Once these capabilities are included in the harvester models, many design studies can be performed easily to maximize the power output of such systems.

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