Normalizing Flows (Part 6)

Dr. Alireza Aghamohammadi

Coupling Flows

Coupling flows split the input vector h into two parts, such that $h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$, and define the flow $f\left[h, \phi\right]$ as follows:

$$h'_1 = h_1$$
$$h'_2 = g\left[h_2, \phi\left[h_1\right]\right]$$

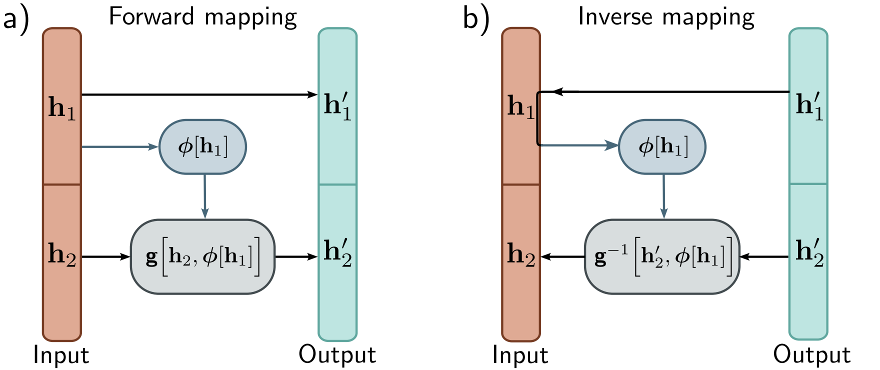

Image credit: Understanding Deep Learning

Considerations for Coupling Flows

- The function $g[\bullet, \phi]$ is an elementwise flow (or another invertible layer) with parameters $\phi[h_1]$, which are a nonlinear function of the input h_1.
- The function $\phi[\bullet]$ is typically a neural network and does not need to be invertible.
- If $g[\bullet, \phi]$ is an elementwise flow, the Jacobian will be lower triangular, allowing efficient computation of both the inverse and the Jacobian.
- This method only transforms the second half of the parameters based on the first half.
- To achieve a more general transformation, the elements of h are randomly shuffled using permutation matrices between layers, ensuring that every variable is ultimately transformed by every other variable.
- A permutation matrix has exactly one non-zero entry in each row and column, all of which are ones.
- As the name suggests, it permutes the entries of a vector.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \Rightarrow \quad P\mathbf{v} = \begin{bmatrix} b \\ c \\ a \end{bmatrix}$$