Normalizing Flows (Part 2)

Dr. Alireza Aghamohammadi

General Form for Normalizing Flows

Let's extend this concept to multivariate distributions $P_r(x)$ and $P_r(z)$.

- Consider a function $x = f(z, \phi)$ applied to a random variable $z \in \mathbb{R}^D$ with base density $P_r(z)$.
- Here, $f(z, \phi)$ is a deep network parameterized by ϕ.

The resulting variable $x \in \mathbb{R}^D$ will have a new distribution:

$$P_r(x \mid \phi) = \left|\det(J_f(z))\right|^{-1} \cdot P_r(z)$$

where

- $J_f(z)$ is the Jacobian matrix of f with respect to z, and
- $\left|\det(J_f(z))\right|^{-1}$ is the inverse of its determinant.

Note that in normalizing flows, x and z have the same dimensionality.

Recap: Jacobian Matrix

Consider a function $f(x) : \mathbb{R}^n \to \mathbb{R}^m$, defined as:

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_m(x) \end{bmatrix}$$

The Jacobian matrix $J_f(x)$ of f is:

$$J_f(x) = \begin{bmatrix} \nabla f_1^T \\ \nabla f_2^T \\ \vdots \\ \nabla f_m^T \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

The Jacobian matrix generalizes the gradient for vector-valued functions, where each row corresponds to the gradient of a component function f_i.