Diffusion Models (Conditional Distribution)

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Conditional Distribution

- Our goal is to learn how to reverse the noise process, so we consider the reverse of the conditional distribution $q(z_t \mid z_{t-1})$.
- We look at the conditional version of the reverse distribution, given the data vector x.
- This conditional diffusion distribution $q(z_{t-1} \mid z_t, x)$ is used to train the decoder.
- Given a noisy image, it's hard to guess its less noisy predecessor. Knowing the starting image makes this task easier.
- Using Bayes' theorem, we can express it as:

$$q(z_{t-1} \mid z_t, x) = \frac{q(z_t \mid z_{t-1}, x)q(z_{t-1} \mid x)}{q(z_t \mid x)}$$

It turns out this distribution is a Gaussian distribution.

$$
\begin{aligned}
q(z_{t-1} \mid z_t, x) &= \frac{q(z_t \mid z_{t-1}, x) q(z_{t-1} \mid x)}{q(z_t \mid x)} \\
&\propto q(z_t \mid z_{t-1}) q(z_{t-1} \mid x) \\
&= \mathcal{N}\left(z_t; \sqrt{1-\beta_t} \cdot z_{t-1}, \beta_t \mathbf{I}\right) \mathcal{N}\left(z_{t-1}; \sqrt{\alpha_{t-1}} \cdot x, (1-\alpha_{t-1})\mathbf{I}\right) \\
&\propto \mathcal{N}\left(z_{t-1}; \frac{1}{\sqrt{1-\beta_t}} \cdot z_t, \frac{\beta_t}{1-\beta_t}\mathbf{I}\right) \mathcal{N}\left(z_{t-1}; \sqrt{\alpha_{t-1}} \cdot x, (1-\alpha_{t-1})\mathbf{I}\right) \\
&\propto \mathcal{N}\left(z_{t-1}; m_t(x, z_t), \sigma_t^2 \mathbf{I}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
m_t(x, z_t) &= \frac{(1-\alpha_{t-1})\sqrt{1-\beta_t} z_t + \sqrt{\alpha_{t-1}} \beta_t x}{1-\alpha_t} \\
\sigma_t^2 &= \frac{\beta_t (1-\alpha_{t-1})}{1-\alpha_t}
\end{aligned}
$$