Invariance & Equivariance

Dr. Alireza Aghamohammadi

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Introduction to Invariance and Equivariance in Neural Networks

- So far, we have only discussed neural layers known as Fully Connected Layers (FCL).
- However, FCLs are not the only type of layers that can be used in neural networks.
- Before we discuss alternative types of neural layers, let's introduce two important concepts:
  - Invariance
  - Equivariance

Understanding Invariance in Neural Networks

- A function $\mathrm{f}(x)$ of an image x is *invariant* to a transformation $\mathrm{t}(x)$ if:

$$\mathrm{f}\left(\mathrm{t}(x)\right) = \mathrm{f}(x)$$

- This means that the output of the function $\mathrm{f}(x)$ remains the same regardless of the transformation $\mathrm{t}(x)$ applied to the image.
- In the context of neural networks, this implies that the network $\mathrm{f}(x)$ should identify an image as containing the same object, even if the image has been translated, rotated, flipped, or warped.

Invariance

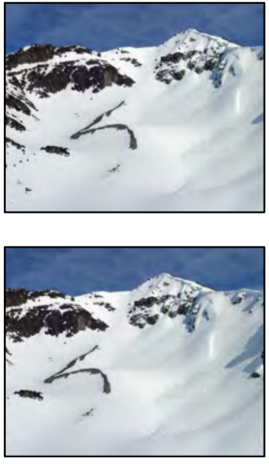

Figure: Illustration of Invariance.[1]

[1] Adopted from the book, Understanding Deep Learning

Equivariance or Covariance in Neural Networks

- A function $\mathrm{f}(x)$ of an image x is said to be *equivariant* or *covariant* to a transformation $\mathrm{t}(x)$ if:

$$\mathrm{f}\left(\mathrm{t}(x)\right) = \mathrm{t}\left(\mathrm{f}(x)\right)$$

- This means that if the image is translated, rotated, or flipped, the network $\mathrm{f}(x)$ should return a segmentation that has been transformed in the same way.

Equivariance

Figure: Illustration of equivariance.[2]

[2] Adopted from the book, Understanding Deep Learning