Normalizing Flows (Part 4)

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Linear Flows

- ❖ We will discuss different invertible network layers or flows, starting with linear flows.
- ❖ Linear Flow Definition:
  - ❑ Form: $f[h] = \beta + \Omega\, h$
  - ❑ This is a bijection if and only if $\Omega \in \mathbb{R}^{D\times D}$ is an invertible square matrix.
- ❖ Base Distribution Transformation:
  - ❑ Base distribution: Gaussian $P_r(z) = \mathcal{N}(\mu, \Sigma)$
  - ❑ After linear transformation: $\mathcal{N}(\beta + \Omega\,\mu, \Omega\,\Sigma\,\Omega^T)$
- ❖ Expressiveness:
  - ❑ Linear flows alone are not sufficiently expressive.
  - ❑ Useful when combined with nonlinear transformations.
- ❖ The determinant of the Jacobian is simply the determinant of Ω.
- ❖ We need to ensure that the Jacobian determinant and the inverse of the flow are fast to compute.
- ❖ Computational Complexity:
  - ❑ In general, computing $\det(\Omega)$ and $\det(\Omega^{-1})$ requires $O(D^3)$ time.
  - ❑ If Ω is diagonal, the cost is $O(D)$, but the elements of h do not interact.
  - ❑ If Ω is triangular, the Jacobian determinant is the product of its diagonal elements, taking $O(D)$ time. Inverting the flow requires solving the triangular system $\Omega\, h = f[h] - \beta$ using back-substitution, which takes $O(D^2)$ time.