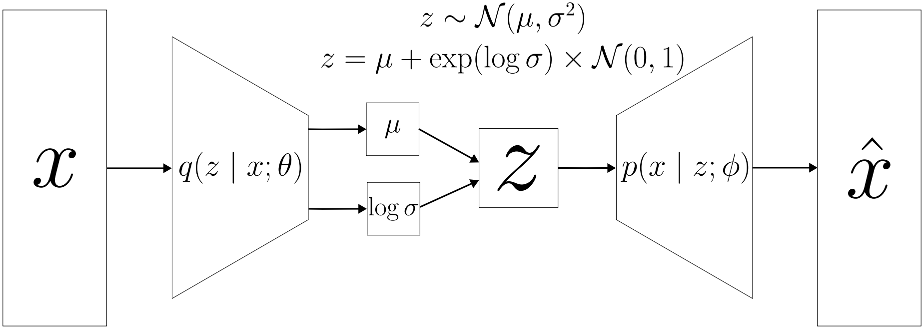
Variational Autoencoders (VAEs)

Dr. Alireza Aghamohammadi

What Is a Variational Autoencoder (VAE)?

- Variational Autoencoders (VAEs) are probabilistic generative models designed to learn a distribution over the data.
- They enable the generation of new samples from this learned distribution.
- Unlike traditional autoencoders that compress input into deterministic bottleneck layers, VAEs transform the input into parameters of a statistical distribution.
- A VAE samples from this distribution and decodes the sample back to its original input.



Maximum Likelihood Estimation (MLE)

- Assume the input data $X = \{x_1, x_2, \ldots, x_N\} \sim p(x \mid \phi)$.
- We aim to find the maximum likelihood $p(x \mid \phi) = \prod_{i=1}^{N} p(x_i \mid \phi)$.
- To maximize the likelihood:

$$\phi^\star = \operatorname*{argmax}_{\phi} \log p(x \mid \phi) = \operatorname*{argmax}_{\phi} \frac{1}{N} \sum_{i=1}^{N} \log p(x_i \mid \phi)$$

Evidence Lower Bound (ELBO)

- Computing the exact posterior $p(x \mid \phi)$ is intractable. Therefore, we need an alternative approach.

$$
\begin{aligned}
\log p(x \mid \phi) &= \log \left(\int p(x, z \mid \phi) \, dz \right) \\
&= \log \left(\int q(z \mid \theta) \cdot \frac{p(x, z \mid \phi)}{q(z \mid \theta)} \, dz \right) \\
&\geq \int q(z \mid \theta) \cdot \log \left(\frac{p(x, z \mid \phi)}{q(z \mid \theta)} \right) \, dz \\
&= \int q(z \mid \theta) \cdot \log \left(\frac{p(x \mid z; \phi) \cdot p(z)}{q(z \mid \theta)} \right) \, dz \\
&= \int q(z \mid \theta) \cdot \log \left(p(x \mid z; \phi) \right) \, dz + \int q(z \mid \theta) \cdot \log \left(\frac{p(z)}{q(z \mid \theta)} \right) \, dz \\
&= \mathbb{E}_{z \sim q(z \mid \theta)} \left[\log p(x \mid z; \phi) \right] - D_{KL} \left[q(z \mid \theta) \| p(z) \right]
\end{aligned}
$$

- The loss function is the negative ELBO.
- The first term is the reconstruction loss.
- The second term is a regularization term that measures how well the inferred latent distribution matches the prior latent distribution.