Normalizing Flows (Part 5)

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Elementwise Flows

- Linear flows lack sufficient expressiveness, so we turn to nonlinear flows.
- Elementwise flows apply a pointwise nonlinear function $f[\bullet, \phi]$ with parameters ϕ to each element of the input:

$$f[h] = [f[h_1, \phi], f[h_2, \phi], \ldots, f[h_D, \phi]]^T$$

- The Jacobian matrix is diagonal because the d^{th} input to $f[h]$ only affects the d^{th} output.
- The determinant of the Jacobian is the product of its diagonal entries.
- While elementwise flows are nonlinear, they do not mix input dimensions and thus cannot create correlations between variables.
- By alternating them with linear flows, we can model more complex transformations.

Elementwise Flows: An Example

A simple example is a piecewise linear function with k regions that maps $[0, 1]$ to $[0, 1]$ as follows:

$$f\left[h, \phi\right] = \left(\sum_{k=1}^{b-1} \phi_k\right) + (hk - b)\phi_b$$

where the parameters $\phi_1, \phi_2, \ldots, \phi_k$ are positive and sum to 1, and $b = \lfloor kh \rfloor + 1$ is the index of the bin that contains h.

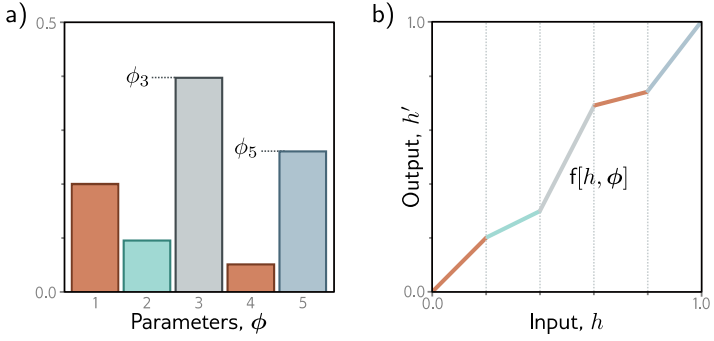

Image credit: Understanding Deep Learning