جلسهٔ حل تمرین ODE

مسئله ۱

$$y' = -y^2$$

$$Y(1) = y$$

$$Y'(1) = y' = -y^2 = -\big(Y(1)\big)^2$$

مسئله ۲

$$y\ddot{y} + \dot{y}^2 = \dot{y}^3 \ln(y)$$

$$Y(1) = y$$

$$Y(2) = \dot{y}$$

$$\dot{Y}(1) = \dot{y} = Y(2)$$

$$\dot{Y}(2) = \ddot{y} = ?$$

$$y\ddot{y} + \dot{y}^2 = \dot{y}^3 \ln(y) \Rightarrow \ddot{y} = \frac{\dot{y}^3 \ln(y) - \dot{y}^2}{y}$$

$$\dot{Y}(2) = \ddot{y} = \frac{\dot{y}^3 \ln(y) - \dot{y}^2}{y} = \frac{Y(2)^3 \ln(Y(1)) - Y(2)^2}{Y(1)}$$

مسئله ۳

$$\begin{cases} 2\dot{x} - x + \dot{y} + 4y = 1 \\ \dot{x} - \dot{y} = t - 1 \end{cases}$$

$$Y(1) = y$$

$$Y(2) = x$$

$$\dot{Y}(1) = \dot{y} = ?$$

$$\dot{Y}(2) = \dot{x} = ?$$

1st method:

$$\begin{cases} 2\dot{x} - x + \dot{y} + 4y = 1 \\ \dot{x} - \dot{y} = t - 1 \end{cases} \overset{+}{\Rightarrow} 3\dot{x} - x + 4y = t$$

$$\Rightarrow \dot{x} = \frac{x - 4y + t}{3}$$

$$\dot{x} - \dot{y} = t - 1 \Rightarrow \dot{y} = \dot{x} - t + 1 = \frac{x - 4y + t}{3} - t + 1$$

2nd method:

$$\begin{cases} 2\dot{x} + \dot{y} = 1 + x - 4y \\ \dot{x} - \dot{y} = t - 1 \end{cases} \xrightarrow{Cramer's\ Rule} \begin{cases} \dot{x} = \dfrac{\begin{vmatrix} 1 + x - 4y & 1 \\ t - 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}} \\ \dot{y} = \dfrac{\begin{vmatrix} 2 & 1 + x - 4y \\ 1 & t - 1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}} \end{cases}$$

$$\dot{x} = \frac{-1 - x + 4y - t + 1}{-2 - 1} = \frac{-x + 4y - t}{-3} = \frac{x - 4y + t}{3}$$

$$\dot{y} = \frac{2t - 2 - 1 - x + 4y}{-2 - 1} = \frac{-x + 4y + 2t - 3}{-3} = \frac{x - 4y - 2t + 3}{3}$$

$$\dot{Y}(1) = \dot{y} = \frac{x - 4y - 2t + 3}{3} = \frac{Y(2) - 4Y(1) - 2t + 3}{3}$$

$$\dot{Y}(2) = \dot{x} = \frac{x - 4y + t}{3} = \frac{Y(2) - 4Y(1) + t}{3}$$

مسئله ٤

$$y'' + 3y = 0$$

$$Y(1) = y$$

$$Y(2) = y'$$

$$Y'(1) = y' = Y(2)$$

$$Y'(2) = y'' = -3y = -3Y(1)$$

$$y(0) = 7 \Rightarrow y_{left} = 7 \Rightarrow Y_{left}(1) = 7 \Rightarrow Y_{left}(1) - 7 = 0$$

$$y(2\pi) = 0 \Rightarrow y_{right} = 0 \Rightarrow Y_{right}(1) = 0$$