Tutorial: Doing Bayesian Data Analysis with R and BUGS

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Before arriving, install free software from this web site:
`http://www.indiana.edu/∼jkkteach/TutorialCogSci2010.html`

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An *introduction* to *doing* Bayesian data analysis

This full-day tutorial shows you how to do Bayesian data analysis, hands on. The software is free. The intended audience is graduate students and other researchers who want a ground-floor introduction to Bayesian data analysis. No mathematical expertise is presumed. If you can handle a few minutes of summation notation like $\sum_i x_i$ and integral notation like $\int x\,dx$, you're good to go. Complete computer programs will be worked through, step by step.

Topics

- Familiarization with software: R, BRugs, BUGS. See installation instructions *before* arriving at the tutorial.
- Uncertainty and Bayes' rule: Application to the rational estimation of parameters and models, given data.
- Markov chain Monte Carlo: Why it's needed, how it works, and doing it in BUGS.
- Hierarchical models: Flexibility for modeling individual differences, group effects, repeated measures, etc.
- Bayesian (multiple) linear regression: Bayesian inference reveals trade-offs in credible regression coefficients.
- Bayesian analysis of variance: Encourages thorough multiple comparisons, with no need for balanced designs.
- Bayesian power analysis and replication probability: Straight forward meaning and computation.

Bayesian data analysis is *not* Bayesian modeling of mind

Data analysis involves "generic" descriptive models, such as linear regression, without any necessary interpretation as cognitive computation. The rational way to estimate parameters in descriptive models is Bayesian, regardless of whether or not Bayesian models of mind are viable.

Why go Bayesian?

Scientists in fields from astronomy to zoology are making Bayesian data analysis their standard operating procedure. Figure 1 (humorously) suggests this trend. Bayesian data analysis delivers many practical benefits:

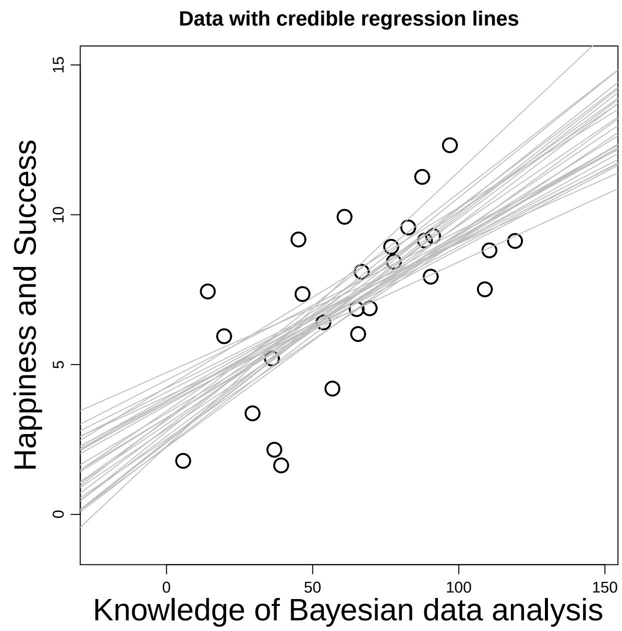

Figure 1: Bayesian linear regression reveals many credible lines, instead of a single "best" line. Also revealed are the inherent trade-offs in estimated slope and intercept: When the slope is steeper, the intercept is lower.

- Bayesian methods permit model flexibility and appropriateness: Hierarchical models can be built easily to suit the design of the experiment and the type of data measured. Figure 2 shows a diagram of the model used for simple linear regression. Such models can be easily extended to capture individual differences, group differences, repeated measures, for various data types.

- Bayesian methods reveal credibilities of all *combinations* of parameter values, unlike traditional analysis which has only point estimates. As a simple example, Figure 1 demonstrates that Bayesian linear regression reveals correlations in estimated values of slope and intercept. Knowledge of these trade offs can be especially useful in applications involving multiple correlated predictors and other realistic situations.

- Bayesian methods encourage thorough data analysis in-

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cluding multiple comparisons, because there are no penalizing "corrections" for multiple comparisons as in *p*-value based decisions. Bayesian methods instead create rational shrinkage informed by the data.

- Model flexibility allows conceptual transition from generic descriptive models to domain-specific models wherein parameters serve psychometric purposes.
- Bayesian methods permit different sample sizes in different groups, and different sample sizes per subject, unlike traditional ANOVA which has troubles with unbalanced designs.
- Bayesian methods allow data collection to stop at any time, unlike *p*-value based decisions that require a pre-set stopping criterion, such as fixed sample size, and no peeking at the data.
- Bayesian hypothesis testing permits a principled way to assess evidence in favor of a null hypothesis, unlike NHST.
- Bayesian methods allow cumulative science and use of prior knowledge for leveraged inference when data are sparse, unlike traditional methods.
- Power and replication probability are straight forward to estimate with Bayesian methods, but difficult to assess in *p*-value based methods.
- 20th century methods, based on *p* values, have numerous deep problems that are avoided with Bayesian methods.

The tutorial does not address all the points listed above, but does illustrate many of them with examples from linear regression and ANOVA. For a brief discussion of several benefits of Bayesian data analysis, along with a worked example, and an emphasis that Bayesian data analysis is not Bayesian modeling of mind, see Kruschke (2010c). For a lengthier exposition that explains one of the primary pitfalls of null hypothesis significance testing and has a discussion of Bayesian null hypothesis testing, along with different examples, see Kruschke (2010a).

Before arriving, install necessary software

We'll be *doing* the analyses, so bring your notebook computer. There will not be internet access from the tutorial room, so you must prepare your computer before arriving at the tutorial. **Please visit the tutorial web site, framed at the top of this page, *before* arriving at the tutorial. Follow the instructions on the web site to install the free software on your computer.**

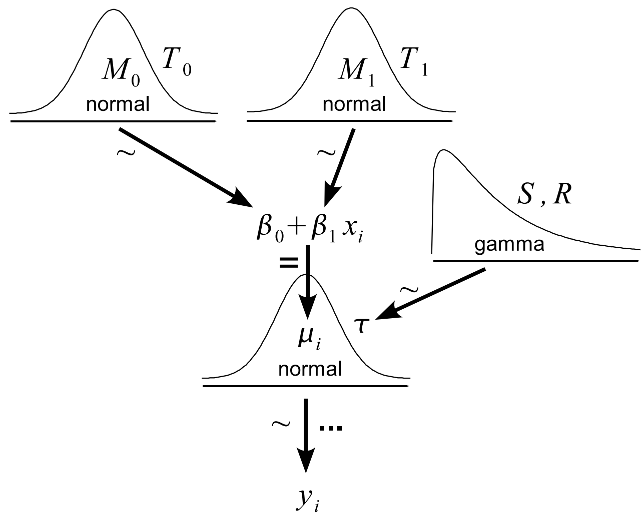

Figure 2: Hierarchical model for Bayesian linear regression. The data are denoted y_i at the bottom of the diagram. The model assumes that the data are generated probabilistically from a normal distribution, governed by parameters β_0 (the intercept), β_1 (the slope), and τ (the noise). Bayesian estimation keeps track of *combinations* of β_0, β_1, and τ that credibly account for the data.

The instructor

John Kruschke has taught introductory Bayesian statistics to graduate students for several years (and traditional statistics and mathematical modeling for over 20 years). He is five-time winner of Teaching Excellence Recognition Awards from Indiana University, where he is Professor of Psychological and Brain Sciences, and Adjunct Professor of Statistics. He has written an introductory textbook on Bayesian data analysis (Kruschke, 2010b); see also the articles linked above. His research interests include models of attention in learning, which he has developed in both connectionist and Bayesian formalisms. He received a Troland Research Award from the National Academy of Sciences. He chaired the Cognitive Science Conference in 1992.

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