29-11-06

Applied Mechanics Department, I Semester 2006-07

AML773 Modelling and Analysis (Fluids) 3-0-0, Major Test

Time 2 hours, Marks 80

Marks

1 a) Derive the von Karman momentum integral equation valid for 2-D incompressible steady laminar/ turbulent boundary layer on flat plate. <7½>

b) Obtain Blasius solution, and hence determine the values of u/U, $\tau_w/\rho U^2$, δ/x, δ_1/x, δ_2/x. <7½>

2 a) Obtain the <u>turbulent</u> boundary layer equations for incompressible steady 2-D flow over a flat plate. <7½>

b) Determine the values of $\tau_w/\rho U^2$, δ/x, δ_1/x, δ_2/x for $1/7^{th}$ power law velocity profile in a 2-D steady incompressible turbulent boundary layer flow. <7½>

3 a) The Reynolds number of a flat plate at zero incidence is 5×10^6. Transition on each surface occurs at 0.4c. The velocity distribution in the laminar part of the boundary layer is given by $u/U=\sin[(\pi/2).(y/\delta)]$, whilst in the turbulent part of the layer it fits a power law $u/U=(y/\delta)^n$ of index $n=1/7$. Find the drag coefficient of the plate (taking both surfaces in to account). <7>

b) Consider fully developed turbulent flow through a smooth pipe. Adopt the notations: distance from the wall $y\equiv r_o-r$, diameter $D=2r_o$, friction velocity $u^*=\sqrt{\tau_o/\rho}$, non-dimensional time mean velocity $u^+=u/u^*$ and non-dimensional y distance $y^+=yu^*/\nu$. Flow region consists of viscous sub-layer, buffer-layer and log-law region. The log-law due to von Karman is $u^+=\frac{1}{k}\ln y^+ + B$, where $k=0.41$, $B=5.2$. The friction law for smooth pipe is expressed in terms of *friction factor f* given by $f=\dfrac{\Delta p\, D}{\frac{1}{2}\rho\bar{u}^2 L}$. Assuming that the log law holds on the pipe axis, and relation $\dfrac{u^*}{\bar{u}}=\sqrt{\dfrac{f}{8}}$ holds. Let *Re* is Reynolds number based on space average velocity.

Obtain the friction law $$\frac{1}{\sqrt{f}}=1.99\log_{10}(\mathrm{Re}\sqrt{f})-0.95\,.$$ <13>

4 A circular orifice of diameter 80 *mm* is designed to be fitted at the bottom of a cylindrical tank which is placed with its axis vertical. Water flows in to the tank at a uniform rate and is discharged through the orifice. It is found that it takes 107 *s* for the water height in the tank to rise from 0.60 m to 0.75 *m*, and 120 *s* for it to rise from 1.2 *m* to 1.28 *m*. Assume a coefficient of discharge of 0.62 for the orifice. <10>

Find the rate of inflow and the cross-sectional area of the tank.

5. a) Derive Euler's equations of motion for an ideal flow. <5>

b) Derive Euler's equations of motion for an ideal flow <u>along a streamline</u>. <5>

6 Air ($\rho=1.23\,kg/m^3$ and $\nu=1.5\times10^{-5}\,m^2/s$) is flowing over a flat plate. The free stream speed is 15 *m/s*.

At a distance 1 *m* from the leading edge, calculate the values of $\delta(x)$ and $\tau(x)$ for

a) Completely laminar flow, and <5>

b) Completely turbulent flow for a $1/7^{th}$ power law velocity profile. <5>