Deptt. of Elec. Engg., IIT Delhi

EEL205, Signals & Systems, Major Exam, M.M. 35, Dur: 2 Hr

Prob 1 Find z-Transform of the following signals: (7)

(a) $x[n] = 3^{n-2} u[n] * \cos\left(\frac{\pi}{6}n + \frac{\pi}{3}\right)u[n]$

(b) $x[n] = n\left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{4}\right)^n u[n-2]$

Prob. 2 (a) Suppose the following information about a periodic signal $x(t)$ with period 3 and Fourier coefficients a_k is given: (i) $a_k = a_{k+2}$; (ii) $a_k = a_{-k}$; (iii) $\int_{-0.5}^{0.5} x(t)dt = 1$; (iv) $\int_{0.5}^{1.5} x(t)dt = 2$. Determine $x(t)$.

(b) Let a causal discrete-time LTI system is given as: $y[n] - \frac{1}{4}y[n-1] = x[n]$. Find Fourier series representation of the output $y[n]$ if $x[n] = \cos\left(\frac{\pi}{4}n\right) + 2\cos\left(\frac{\pi}{2}n\right)$ (7)

Prob. 3 (a) Determine $x[n]$ if $X(z) = (1+z^{-1})^4 \quad |z| > 0$

(b) Determine difference equation representation of the system with impulse response

$$h[n] = 2\left(\frac{2}{3}\right)^n u[n-1] + \left(\frac{1}{4}\right)^n \left[\cos\left(\frac{\pi}{6}n\right) - 2\sin\left(\frac{\pi}{6}n\right)\right]u[n]$$ (7)

Prob 4 (a) Let $g(t) = x(t)\cos^2 t * \frac{\sin t}{\pi t}$; $x(t)$ real & $X(\omega) = 0$ for $|\omega| \geq 1$. Show that there exists an LTI system S, such that $(Sx)(t) = g(t)$.

(b) Consider an LTI system whose output is given to be: $y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$, to the input with $X(s) = \frac{s+2}{s-2}$, $x(t) = 0 \quad t > 0$. Determine $H(s)$, its ROC & $h(t)$. (8)

Prob. 5: The following is known about a discrete-time LTI system with input $x[n]$ and output $y[n]$:

(i) If $x[n] = (-2)^n \ \forall n$ then $y[n] = 0 \ \forall n$

(ii) If $x[n] = \left(\frac{1}{2}\right)^n u[n] \ \forall n$ then $y[n] = \delta[n] + a\left(\frac{1}{4}\right)^n u[n]$

(a) Determine the value of a. (b) Determine $y[n]$ if $x[n] = 1 \ \forall n$ (6)