DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
MAJOR TEST 2008-2009 FIRST SEMESTER
MAL 230 (NUMERICAL METHODS AND COMPUTATION)

Time: 2 hours **Max. Marks: 50**

1a. Find an upper bound for $\|A^{-1}\|_\infty$ where $A = \begin{bmatrix} 7 & 2 & 1 & 1 \\ 2 & 7 & 2 & 1 \\ 1 & 2 & 7 & 2 \\ 1 & 1 & 2 & 7 \end{bmatrix}$. (4)

1b. Suppose λ is an eigenvalue of A in the above question (1a) and $\alpha \le |\lambda| \le \beta$. Use the upper bound for $\|A^{-1}\|_\infty$ in finding the values of α and β. (2)

2. Consider the linear system $Ax = b$ with $| A | \neq 0$. Let δA be perturbation of A and assume $\|\delta A\| < \frac{1}{\|A^{-1}\|}$. Then examine whether $A + \delta A$ is nonsingular or not. And if we define δx implicitly by $(A + \delta A)(x + \delta x) = b$ then prove or disprove

$$\frac{\|\delta x\|}{\|x\|} \le Cond.(A) \frac{\|\delta A\|}{\|A\|}(1 + O(\|\delta A\|)).$$

(4)

3. Given $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$, for which values of α, the vector sequence $\{x^{(k)}\}$ defined by

$$x^{(k+1)} = (I + \alpha A + \alpha^2 A^2)x^{(k)}, \qquad k = 0, 1, 2, \ldots\ldots$$

where $x^{(0)}$ is arbitrary,. converge to $\mathbf{0}$ as $k \to \infty$. (3)

4. Using the power method, determine the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, correct to two decimal places. (5)

5. Find the number of Given's rotations required in the reduction of an $n \times n$ symmetric matrix to tridiagonal matrix. Using the Given's method and Sturm sequence, find the interval of unit length which contains the largest eigenvalue of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}.$$

(8)

6a. Derive two point Gauss-Chebyshev integration rule and hence evaluate $\int_{\frac{1}{2}}^{1} \frac{dx}{1+x}$. (6)

6b. Find the error associated with the two-point Gauss-Legendre quadrature formula. (2)

P.T.O.

7. Assume that $f \in C^6[a,b]$ and that $x-2h, x-h, x, x+h, x+2h \in [a,b]$. Find the error $E(f,h)$ in the formula

$$f''(x_0) = \frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} - f_{-2}}{12h^2} + E(f,h)$$

where$f_k = f(x_0 + kh), \quad k = -2,-1,0,1,2$. Also find the optimum step size h using the criteria $\mid R.E. \mid + \mid T.E. \mid$ is minimum. (4)

8. True or false justify the following ststement.
The system

$$x = 0.2\cos(x+y)$$

$$y = 0.3\sin(x-y)$$

has unique solution. (4)

9. Determine a polynomial $p(x)$ of degree as low as possible such that

$$\max_{-1\le x\le 1} |\frac{1}{3+x} - p(x)| \le 0.01$$

(Use Lanczos Economization). (4)

10. Obtain polynomial approximation to $f(x) = (1+x)^{\frac{1}{2}}$ over $[0,\frac{1}{2}]$ by means of Taylor's expansion about the point $x=0$. Find the number of terms required in this expansion so that the magnitude of the error is less than 5×10^{-3}. (4)