Department of Mathematics
Indian Institute of Technology Delhi
MAL630: Partial Differential Equations

Major examination July-Nov 2008 Max.Marks:50

[7+7+7+7+8+7+7]

1. Solve the Cauchy problem: $u_y = xuu_x,\ u(x,0) = x$.

2. Describe weak solution of the Cauchy problem:
$uu_x + u_y = 0,$
$$u(x,0) = \begin{cases} 0 & x < 0, \\ x-1 & x > 0. \end{cases}$$

3. Show that the following Problem admits atmost one solution:
$$\sum_{k=1}^{n} a_k(x)u_{x_kx_k} + u|\nabla u| + c(x)u = 0, \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$
where $a_k(x), c(x)$ are continuous functions in $\overline{\Omega}$ such that $a_k(x) > 0$ and $c(x) < 0$ in $\overline{\Omega}$

4. Find the Greens function for $\{(x,y) \in I\!R^2,\ x^2 + y^2 < 1, y > 0\}$

5. Show that the following problem has unique solution
$$u_t - \sum_{k=1}^{n} a_k(x)u_{x_kx_k} - \sum_{k=1}^{n} b_k(x)u_{x_k} - c(x)u = f(x,t) \text{ in } \Omega \times \{t > 0\}.$$
$$u(x,0) = g(x),\ x \in \Omega,\ \ u(\partial\Omega, t) = h(x,t)$$
where $a_k(x) > 0$ and $c(x) < 0$ in $\overline{\Omega}$.

6. Show that any function $u(x)$ satisfying the inequality
$$-\Delta u + |\nabla u|^2 \geq u^2 \text{ in } \Omega$$
does not have maximum at an interior point of Ω.

7. Suppose if $u(x) \in C(\overline{\Omega})$ satisfies mean value property in Ω, then show that $u(x)$ is harmonic in Ω.