EEL205 Signals and Systems - MAJOR
Semester II, 2007-2008

(35 marks, 120 minutes)

Name: ______________________ **Entry no.** ______________________ **Group no.** ______________________

1. Write your answers neatly and legibly in the space provided in the question paper itself. Answers that are not legible or accompanied by correct reasoning will not receive any credit.
2. WRITE YOUR NAME AND ENTRY NUMBER ON ALL PAGES OF THE QUESTION PAPER.
3. It is best to write the answers where possible as Transform pairs separated by $< - - - >$. Take care to label all sketches properly.

1. **(4 marks)** Write down the inverse Fourier Transform of $U(j\omega)$ using the Fourier Transform of $u(t)$ and duality (where $U(j\omega) = 1$ for $\omega > 0$ and 0 for $\omega < 0$).

   A signal $x(t)$ is such that $X(j\omega) = 0$ for $\omega > 0$. Using $X(j\omega) = X(j\omega)U(j\omega)$, express the real part $x_R(t)$ of the signal $x(t)$ in terms of its imaginary part $x_I(t)$.

2. **(2 marks)** A signal $x(t) = cos(50t) + 2cos(70t)$ is sampled at a rate $1/T$ with $T = \pi/60$ to obtain a discrete sequence $x[n]$. Sketch in the space provided below the Fourier Transform of $x[n]$.

3. **(4 marks)** Let $H(z) = \frac{(1-\frac{5}{4}z^{-1})}{(1-\frac{1}{4}z^{-1})(1-\frac{3}{4}z^{-1})}$ represent the Z-transform of a causal impulse response $h[n]$. The ROC of $H(z)$ is given by:____________.

The impulse response $h[n] =$ ____________.

The ROC of $log(H(z))$ for it to be the Z-transform of a stable sequence is given by: ____________

4. **(3 marks)** Evaluate the integral $\int_{-\infty}^{\infty} \frac{sin(\omega+\phi)}{\omega} d\omega$ (express your answer in terms of ϕ).

5. **(3 marks)** Consider a discrete-time periodic sequence $x[n]$ with fundamental period N and Fourier Series coefficients a_k. The Fourier Series coefficients of the sequence $x[n] - e^{j2\pi ln/N}x[n]$ are given by:____________

6. (**2 marks**) Denote the input and output of an LTI system with impulse response $h(t)$ by $x(t)$ and $y(t)$. Denote the input, output and impulse response of a second system by $x_1(t)$, $y_1(t)$ and $h_1(t)$. If $x_1(t) = x(at)$, and $z(t) = y_1(t/a)$=y(t), how is $h_1(t)$ related to $h(t)$?

7. (**2 marks**) Given that $\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t)$ has a Laplace transform that is the *entire* finite s-plane, the poles of $X(s)$ are located at __________. The ROC of $X(s)$ for a causal $x(t)$ is: __________.

8. (**4 marks**) Evaluate $\int_{-\infty}^{\infty} \frac{sin(\pi(\frac{t}{2}-20))}{\pi(\frac{t}{2}-20)} cos(10\pi t)dt$

9. (**3 marks**) Sketch in the space provided below, the block diagram implementation of the differential equation $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + 3x(t)$ in the parallel form.

10. **(8 marks)** Let $x(t)$ is a periodic signal with Fourier Series coefficients a_k and transfrom $X(j\omega)$. Define $Z(j\omega) = X(j\omega)Y(j\omega)$ where $Y(j\omega)$ is the transform of a aperiodic signal $y(t)$.

a) **(1 mark)** Express $X(j\omega)$ in terms of a_k.

b) **(2 marks)** Express $z(t)$ in terms of a_k and samples of the transform $Y(j\omega)$.

c) **(3 marks)** Express $z(t)$ as the periodic convolution of $x(t)$ and another signal derived from $y(t)$.

d) **(2 marks)** If $x(t) = \frac{1}{T}\sum_k e^{j2\pi kt/T}$, what is the relationship between $z(t)$ and $y(t)$? When can you uniquely recover $y(t)$ from $z(t)$?