EEL 316 Major Test Semester II 2007-2008

Answer all questions (Q.1: 30 marks, Q.2: 30 marks, Q.3: 10 marks) Full Marks: 70

1. Orthogonal signals $\phi_1(t), \phi_2(t), \phi_3(t)$ over a signaling interval $[0, T)$, each having energy E_ϕ, are converted to another set of signals $\{s_1(t), s_2(t)\}$ by the transformation

$$s_i(t) = \phi_i(t) - \frac{\alpha\phi_1(t) + 2\alpha\phi_2(t) + (3 - 3\alpha)\phi_3(t)}{3}, \quad i = 1, 2,$$

where $0 < \alpha < 1$. Binary equiprobable signaling is performed over an AWGN channel with noise p.s.d. $N_0/2$ using the signals $s_1(t)$ and $s_2(t)$.

(a) What is the dimension of the signal space $\{s_1(t), s_2(t)\}$? Starting with $s_1(t)$, obtain an orthonormal basis for the signal space. [6]

(b) Obtain the SEP for coherent reception in terms of E_ϕ and N_0. Calculate this SEP when $E_\phi/N_0 = 20$. [6]

(c) Calculate the value of α for which the signals $s_1(t)$ and $s_2(t)$ are orthogonal. [6]

(d) Let $s_3(t) = \beta s_2(t)$, where $\beta > 0$. For the value of α found in (c), calculate the value of β such that $s_1(t)$ and $s_3(t)$ have the same energy. [4]

(e) For the value of α found in (c) and the value of β found in (d), binary equiprobable signaling is performed over an AWGN channel with noise p.s.d. $N_0/2$ using the signals $s_1(t)$ and $s_3(t)$.

i. Obtain the SEP for coherent reception in terms of E_ϕ and N_0. Calculate this SEP when $E_\phi/N_0 = 20$. [4]

ii. Obtain the SEP for noncoherent reception in terms of E_ϕ and N_0. Calculate this SEP when $E_\phi/N_0 = 20$. [4]

2. The requirement of a communication system using M-ary PSK signaling with $M \geq 2$ (M is a power of 2) and equal apriori probabilities over an AWGN channel is that the SEP $P_e \leq 10^{-8}$ at an SNR $E_s/N_0 = 15$ dB.

(a) Calculate $M_{max,PSK}$, the maximum value of M that can be used. What is the approximate value of P_e for $M = M_{max,PSK}$ when $E_s/N_0 = 20$ dB? [6]

(b) Find the union bound on P_e for $M = M_{max,PSK}$ as a function of E_s/N_0. [4]

(c) If the system switches to noncoherent M-ary orthogonal FSK, then what is the minimum SNR in dB at which the target SEP of 10^{-8} will be reached for $M = M_{max,PSK}$? [4]

(d) If the system switches to M-ary PAM, then what is $M_{max,PAM}$, the maximum value of M at which the SEP is $\leq 10^{-8}$ at an SNR of 20 dB? [6]

(e) If the system switches to M-ary square QAM, where $M = M^2_{max,PAM}$, then calculate the SEP at an SNR of 20 dB. [6]

(f) Find the bandwidth efficiencies in (a), (c), (d), and (e). [4]

3. An analog signal is sampled, quantized, and encoded into a binary PCM wave. The number of representation levels used is 32. A synchronizing pulse is added at the end of each codeword representing a sample of the analog signal. The resulting PCM wave is transmitted over a channel of bandwidth 15 kHz using a quaternary PAM system with raised cosine spectrum. Information is transmitted through the channel at the rate of 48 kbits/sec.

   (a) Find the rolloff factor of the raised cosine pulse. [6]

   (b) Find the rate at which the analog signal is sampled. What is the maximum possible value for the highest frequency component of the analog signal? [4]

Some Formulae

- $\operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \text{if } |t| \le \frac{T}{2}, \\ 0 & \text{if } |t| > \frac{T}{2}, \end{cases}$ $\qquad \operatorname{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$

- Fourier Transform pairs:

$$\operatorname{rect}\left(\frac{t}{T}\right) \leftrightarrow T \operatorname{sinc}(fT), \qquad \exp(j2\pi f_0 t) \leftrightarrow \delta(f - f_0), \qquad G(t) \leftrightarrow g(-f)$$

- MAP receiver: $\hat{i} = \arg\left\{\max_i \; -\|\underline{x} - \underline{s}_i\|^2 + N_0 \ln p_i\right\}$

- If $X \sim \mathcal{N}(0,1)$, then its p.d.f.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty, \qquad \text{and} \quad \Pr[X > x] = \int_x^\infty f_X(y)dy = Q(x) = 1 - Q(-x)$$

- Use the approximation $Q(x) \approx \dfrac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad x \ge 2.5$, wherever applicable.

- $P_e = \dfrac{1}{\pi} \displaystyle\int_0^{\frac{\pi(M-1)}{M}} \exp\left(-\frac{E_s}{N_0} \frac{\sin^2\left(\frac{\pi}{M}\right)}{\sin^2 \phi}\right) d\phi$

- $P_e = \dfrac{(M-1)}{M} \operatorname{erfc}\left(\sqrt{\dfrac{d^2 E}{N_0}}\right), \quad \dfrac{E_{av}}{E} = \dfrac{(M^2-1)}{3} d^2$

- $P_e = \displaystyle\sum_{i=1}^{M-1} \frac{(-1)^{i+1}}{(i+1)} \binom{M-1}{i} e^{-\frac{i}{(i+1)} \frac{E_s}{N_0}}$

- $P_e \approx \operatorname{erfc}\left(\sqrt{\dfrac{E_s}{N_0}} \sin\left(\dfrac{\pi}{M}\right)\right)$ for large SNR, 4 and higher