<u>CHL 603   Advanced Transport Phenomena</u>

<u>Major: November 22, 2008</u>

40 marks **Open Book, Open Notes** **2 hours**

There are 5 problems in this test, printed on <u>both sides</u> of this sheet. Each question is for 8 marks. Full marks on any problem <u>only</u> if you provide the simplest and the most elegant solution.

1. The barrel of an extruder can be modeled as if a solid rod is moving with velocity, V, through a fluid inside a horizontal cylindrical tube. There is <u>also</u> a pressure gradient imposed on the Newtonian fluid in the annulus between the rod and the tube.

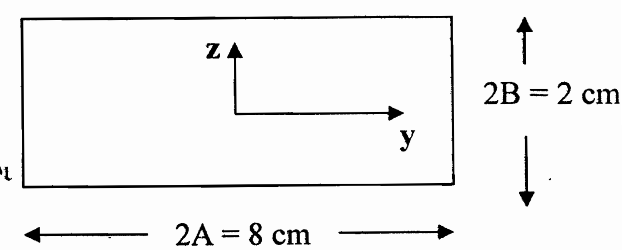
   The outer radius of the rod is R_1 and the inner radius of the extruder barrel is R_2. Viscosity of the fluid is μ and density of the fluid is ρ.

   Derive an expression for the velocity distribution of the fluid in the annulus under steady-state, fully developed laminar flow conditions.

   Interpret physically your final result as the contributions from the pressure gradient and that due to shear induced by the rod. What happens when $\frac{R_1}{R_2} \to 0$?

2. Given the steady state, laminar, fully developed flow in a Newtonian fluid in the rectangular duct shown in the figure. The velocity distribution in the duct is calculated to be:

$$v_z = \frac{9}{5}\bar{v}\left[1-\left(\frac{y}{A}\right)^6\right]\left[1-\left(\frac{z}{B}\right)^2\right]$$

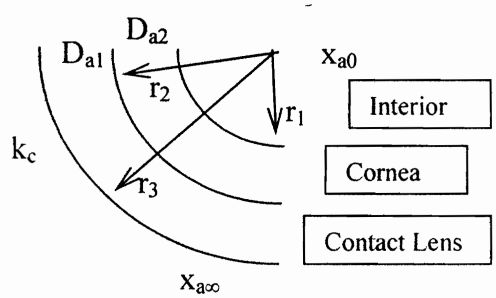


   Where $\bar{v}$ is the average velocity in the duct.

   (a) Derive the expressions for the shear stress at the top and size walls of the duct, i.e., $\tau_{zx}\big|_{z=B}$ and $\tau_{yx}\big|_{y=A}$.
   (b) Obtain expressions for the <u>average</u> values of the shear stresses at the top and side walls.
   (c) If water is flowing through the duct at an average velocity of 4.5 cm/s, calculate the total viscous force per unit length of the duct.
   (d) Use an overall force balance to relate the total viscous force calculated in (c) to the pressure gradient in the duct, $\frac{\Delta P}{L}$, and then calculate $\frac{\Delta P}{L}$ in units of cm-H_2O per metre.

3. Oxygen transfer from the atmosphere to interior regions of the eye depends enormously on whether one wears contact lenses. Treating the eye as a composite material system, determine the mass transfer rate from the atmosphere through the cornea with and without contact lenses. You may assume dilute solutions. The oxygen concentration in the interior of the eye is maintained at a uniform level by circulation of the fluid inside the eye.

Data: $x_{a\infty} = 0.2$; $x_{a0} = 0.05$; $r_1 = 0.0102$ m; $r_2 = 0.0127$ m; $r_3 = 0.0165$ m;
$D_{a1} = 1 \times 10^{-9}$ m^2/s; $D_{a2} = 5 \times 10^{-9}$ m^2/s $k_c = 0.001$ m/s; $c_t = 0.008$ mol/m^3



4. Oxygen diffuses through the wall of drug containers and oxidizes many drugs rendering them inactive. That is why a drug "expires", so to say. It is the oxygen diffusion/reaction scenario that limits the shelf life of many pharmaceutical products. To limit the oxidation of drugs, oxygen scavengers like sodium bisulfite ($NaHCO_3$) are often added. The reaction to remove oxygen is:

$$2\ HSO_3^- + O_2 \rightarrow SO_4^{2-} + H_2O$$

Consider a liquid drug stored in a cylindrical polyethylene container. The container is 15 cm high and has an inner diameter of 6 cm. The initial concentration of $NaHSO_3$ in the drug formulation is 1 g/liter. How thick must the walls of the container be to ensure that 90% of the $NaHSO_3$ remains after 1 year? Neglect the diffusion through the top and bottom of the container and assume that the $NaHSO_3$ reacts instantly with the O_2 so that the O_2 concentration in the drug is always zero. The effective diffusivity of O_2 through polyethylene is 9×10^{-13} m^2/s and you may assume the partition coefficient as 1 (i.e., concentration of O_2 in adjacent layers in air and polyethylene is the same). Assume also P = 1atm, T = 20°C, and the process of diffusion operates at steady state.

5. A sailboard is gliding across a lake at a speed of 20 mph (9 m/s). The sailboard is 3 m long, 0.75 m wide and represents a smooth, flat surface. Use the physical properties of water listed below:

$\rho = 997.5$ kg/m^3 $\mu = 9.8\times10^{-4}$ Ns/m^2

Under the conditions of interest (turbulent), the Fanning's friction factor f is correlated to the Reynolds number Re as:

$$\frac{f\,\mathrm{Re}}{2} = 0.029\,\mathrm{Re}^{4/5}$$

a) What is the average shear stress on the sailboard?
b) How much power must the wind provide to propel the sailboard?