2/5/09 EEL 772 OPTIMAL CONTROL

MAJOR TEST　　Time: 2 hrs. Marks: 50

Q1:– Find the extremal of functional $J=\int_{-2}^{0}[12t\,x(t)+\dot{x}^2(t)]\,dt$ to satisfy boundary condition $x(-2)=3$ and $x(0)=0$. Find optimal $x^*(t)$ and nature of extrema. (8)

Q2:– A mechanical system is described by $\ddot{x}(t)=u(t)$. Find the optimal control and states by minimizing $J=\frac{1}{2}\int_0^5 u^2(t)\,dt$ satisfying boundary condns $x(0)=2,\ x(5)=0$, $\dot{x}(0)=2,\ \dot{x}(5)=0$ (8)

Q.3:– Let $\dot{x}=x-u$ where $x\in\mathbb{R}$. It is desired to drive any initial state $x(0)$ to zero in MINIMUM TIME if $|u(t)|\leq 1$

(i) Write state eqn, costate eqn, boundary conditions and Pontryagin "Stationarity Condition"

(ii) Solve costate eqn in terms of unknown $\lambda(T)$. Sketch $\lambda(t)$

(iii) Express $u^*(t)$ in terms of $\lambda(T)$ for all possible cases to find the possible values for $u^*(t)$.

(iv) Solve state eqn for all possible values of $u^*(t)$ if $x(T)=0$.

(v) Sketch switching curve and sample trajectories in phase plane

(vi) Find optimal cost J^* in terms of $x(0)$ and optimal feedback control.

(vii) In terms of $x(0)$, when does this optimal control problem have a solution. (14)

Q.4:– Consider a scalar bilinear system $x_{k+1}=x_k u_k+u_k^2$ with cost index $J_0=x_N^2+\sum_{k=0}^{N-1}x_k u_k$.

Let $N=2$. The control takes on values of $u_k=-1$ or $+1$ and the state to take on values of $x_k=-1,0,1,2$.

(a) Use Dynamic programming to find optimal state feedback control law.

(b) Let $x_0=2$. Find the optimal cost, control sequence and state trajectory. (10)

Q.5:– Define tracking error $e(t)=y(t)-r(t)$ where $r(t)$ is ref. signal, $y(t)$ is output of scalar system $y^{(n)}+a_1y^{(n-1)}+\cdots+a_ny=u$ with $u(t)$ as control input. Define cost fn $J(t_0)=\frac{1}{2}pe^2(T)+\frac{1}{2}\int_{t_0}^{T}(qe^2(t)+ru^2)\,dt$, and operator $\Delta(s)=s^n+a_1s^{n-1}+\cdots+a_n$ where $s\equiv$ Derivative. Then the plant is $\Delta(s)y=u$. Suppose $r(t)$ satisfies $\Delta(s)r=0$