DEPARTMENT OF MATHEMATICS

MAL390 (STATISTICAL METHODS AND ALGORITHMS)

MAJOR TEST

Maximum Marks 50
Time Two Hours
Datc:06.05.09,
Time Schedule: 10.30 A.M. – 12.30 P.M

Answer all questions. The allocation of marks for each of first two questions is three marks cach, next five questions is four marks each and to each of the last three questions it is eight marks.

1. Prove that every Gramian matrix (of the form $A^\top A$) has a square root decomposition $U^\top U$ where U is an upper triangular matrix.

2. Define the multiple correlation coefficient $R^2_{0(12..k)}$ and give the connection to variable having the form of variance ratio divided by appropriate degrees of freedom and state the probability distribution of that variate.

3. Derive the maximum likelihood estimator of λ in the exponential distribution $f(x, \lambda) = \frac{1}{\lambda} \exp\{-(x/\lambda)\}$, where $\lambda > 0$ based on a random sample of size n from the above exponential distribution.

4. State and prove Neyman-Pearson theorem.

5. With the background of the two-way classification with $c > 1$ observations per cell, show that a particular solution of the normal equations in the least squares estimation of the parameters is $\widehat{\alpha}_i = \bar{x}_{i..} - \bar{x}_{...}$, and $\widehat{\beta}_j = \bar{x}_{.j.} - \bar{x}_{...}$, and $\widehat{\gamma}_{ij} = \bar{x}_{ij.} - \bar{x}_{i..} - \bar{x}_{.j.} + \bar{x}_{...}$. Is the expression $\bar{x}_{i..} - \bar{x}_{...}$ an unbiased estimator of α_i. Why?

6. Describe how the method of finding restricted and unrestricted minimum of $(Y - X\beta)^\top(Y - X\beta)$ can be employed and how it leads to testing of the testable hypothesis $H^\top\beta = \gamma_0$.

7. Establish that the sum of squares due to fitiing $\beta^\top = (\beta_I^\top | \beta_{II}^\top)$ can be done in two stages first fitting β_I ignoring β_{II} and then finding the extra reduction in residual sum of squares by fitting β_{II} after fitting β_I and determine these sums of squares in terms of solutions of the normal equations.

8. In a regression problem, the corrected sum of squares and products matrix based on 10 observations $(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i})$, $i = 1, 2, \ldots, 10$ is

$$S = \begin{bmatrix} 36 & 12 & 30 & 6 & 18 \\ \cdot & 20 & 2 & 10 & 22 \\ \cdot & \cdot & 29 & 1 & 7 \\ \cdot & \cdot & \cdot & 14 & 20 \\ \cdot & \cdot & \cdot & \cdot & 40 \end{bmatrix}$$

Find (a) the multiple correlation coefficient $R^2_{3(12)}$, (b) the partial correlation coefficent $r_{3(2).1}$ and (c) the analysis of variance table when $y = x_5$ is regressed on the variables x_1, x_2, x_3. (2+3+3)

9. Consider the 2^3 factorial design in the factors feed rate (A), depth of cut (B), and tool angle (C), with $r = 2$ replicates. Let $a_i b_j c_k$ denote for $i, j, k = 1, 2$ the treatment combination at the (i, j, k) cell and the results namely the observed surface roughness are

TABLE SHOWING SURFACE ROUGHNESS

$a_1b_1c_1$	$a_2b_1c_1$	$a_1b_2c_1$	$a_2b_2c_1$	$a_1b_1c_2$	$a_2b_1c_2$	$a_1b_2c_2$	$a_2b_2c_2$
$9, 7$	$10, 12$	$9, 11$	$12, 15$	$11, 10.$	$10, 13$	$10, 8$	$16, 14$

Are there significant three level interactions that is ABC? Find 95% confidence intervals in the differences in the main effects of A, B and of C. You are given that the upper .05 probability point(s) of the $F_{1,r}$ distribution for $r = 6, 7, 8, 9$ is 3.78, 3.59, 3.46, 3.36 respectively. (2+2+2+2)

10. The following table gives the scores for Leprosy Bacilli before (X, the Concomitant variable) and after (Y) treatment by drugs named A, D, and F(levels of factor called drug)

TABLE OF SCORES

DRUGS					
A		D		F	
X	Y	X	Y	X	Y
11	06	06	00	16	13
08	00	06	02	13	10
05	02	07	03	11	18
14	08	08	01	09	05
19	11	18	18	21	23
06	04	08	04	16	12
10	13	19	14	12	05
06	01	08	09	12	16
11	08	05	01	07	01
03	00	15	09	12	20

Form the table of Analysis of Vaiance and covariance. Test the hypothesis that there is no useful contribution from the addition of concomitant variable X. The tabulated upper α percent point for $\alpha = .05$ of the F distribution for $(1, 26)$, $(1, 27)$, $(2, 26)$ and $(2, 27)$ degrees of freedom is 2.91, 2.90, 2.52, and 2.51 respectively. (4+4)