1 The constitutive law for a Neo-Hookean material is given by

$$\underset{\sim}{\sigma} = \lambda_0 \ln J \, \underset{\sim}{1} + \mu_0 (\underset{\sim}{B} - \underset{\sim}{1})$$

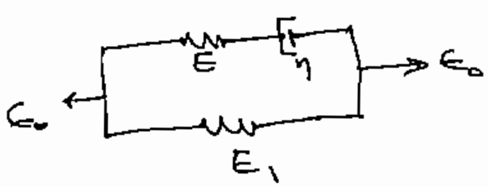
where the ~~the~~ terms have usual meaning.

a) For pure dilatation show that stress state is hydrostatic

b) For simple shear ~~with~~ in 1-2 plane show σ_{11} and σ_{12} are not zero

c) In a simple hydrostatic test $\sigma_{11} = \sigma_{22} = \sigma_{33} = 10$ MPa and $\lambda_1 = \lambda_2 = \lambda_3 = 1.05$.
In simple shear test $\gamma = 10^{-3}$ and $\sigma_{12} = 5 \times 10^4$ Pa
Determine λ_0, μ_0 and σ_{11} (for shear test).

#2 A material can be represent by a spring and Maxwell model in parallel.



a) Determine the relaxation modulus for the material

b) By simpson's rule

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

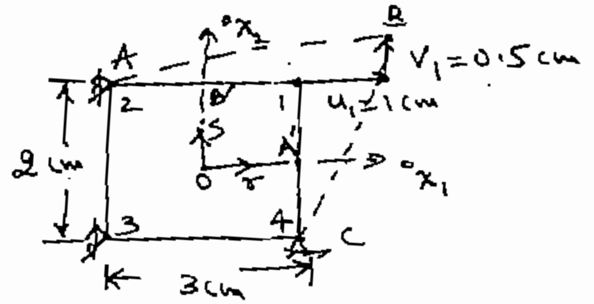
For the above material

time t' (s)	$d\epsilon/dt'$ ($\frac{1}{s}$)
0	10^{-3}
1	2×10^{-3}
2	1.5×10^{-3}

Also $E = E_1 = 3$ GPa $\eta = 3 \times 10^{12}$ Pa-s. Using Simpson's rule determine stress at $t = 2$ seconds.

(Hint: First write the integral relation between σ, relaxation modulus and $\frac{d\epsilon}{dt'}$)

#3 Consider the element shown in figure.



a) Evaluate the deformation gradient F corresponding to deformation at time 't' (shown dotted) at point Q.

b) Determine the new angle between ~~AB~~ OA' and OB' (using the formula & not by measurement)

Given $h_1 = \frac{1}{4}(1+r)(1+s)$, $h_2 = \frac{1}{4}(1-r)(1+s)$

$h_3 = \frac{1}{4}(1-r)(1-s)$, $h_4 = \frac{1}{4}(1+r)(1-s)$

$$J = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix}$$

#4 Given $L = \dot{F} F^{-1}$

a) show $\frac{1}{2}\int \underset{\sim}{\sigma} : \underset{\sim}{d} \, d^tv = \frac{1}{2}\int \underset{\sim}{\sigma} : \underset{\sim}{L} \, d^tv$

b) show $\frac{1}{2}\int \underset{\sim}{\sigma} : \underset{\sim}{L} \, d^tv = \frac{1}{2}\int \underset{\sim}{P} : \underset{\sim}{\dot{F}} \, d^0V$

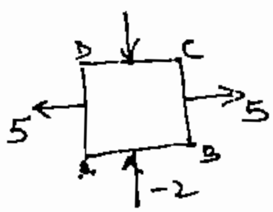
where $\underset{\sim}{P}$ is the first Piola-Kirchhoff stress tensor.

#5a) The B_{NL} for a 2-D plane stress problem in total Lagrangian approach is given as

$$\begin{bmatrix} h_{1,1} & 0 & h_{2,1} & 0 & h_{3,1} & 0 & h_{4,1} & 0 \\ h_{1,2} & 0 & h_{2,2} & 0 & h_{3,2} & 0 & h_{4,2} & 0 \\ 0 & h_{1,1} & 0 & h_{2,1} & 0 & h_{3,1} & 0 & h_{4,1} \\ 0 & h_{2,1} & 0 & h_{2,2} & 0 & h_{3,2} & 0 & h_{4,2} \end{bmatrix}$$

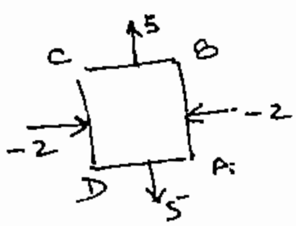
Given $= \; {}^{1}_{0}S = \begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix}$

Write the integral ~~(with proper limits)~~ using one point formula for determining the K_{NL} matrix for problem 3. Leave the answer in matrix form without multiplying matrices.

b) 

The $\underset{\sim}{S} = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$

Write the components of $\underset{\sim}{S}$ and $\underset{\sim}{\sigma}$ ~~when~~ in the new configuration obtained by rotating the old configuration by 90°.



c) In a 1-D steel rod with $E = 200 \times 10^9$, $\alpha = 10^{-5}/^\circ C$, $\Delta T = 700^\circ C$, creep strain $= 5 \times 10^{-3}$, total strain $\epsilon = 0.03$; Determine the stress in the rod.

d) If $f = \sigma_e - \sigma_0 = (\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)^{1/2} - \sigma_0 = 0$

$E = 200 \times 10^3 \ N/mm^2$ $\quad \nu = 0$, $\sigma_0 = 200 \ N/mm^2$

Starting from a point $(120, -80) = (\sigma_1, \sigma_2)$ apply a strain increment $\Delta\epsilon^T = (0.0009, 0.0009)$ and compute the point where the yield surface is intersected.