EEL325: Control Engineering-II

Minor 2 | 29th-April 2008 | Time 2 Hrs

1. A linear state space system is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

A feedback gain $\bar{K}$ has to be found such that the feedback input $u = \bar{K}\bar{x}$ takes the system to the zero state from $\bar{x}_0 = [\ 1\ \ -1\ ]^T$ along a trajectory that minimizes

$$J = \int_0^\infty \{x_1^2 + 5x_2^2 + u^2\} dt$$

(a) Formulate the problem as a standard problem in control theory identifying the name of the problem, and the relevant matrices for the above case. 5

(b) Write a two-step method of obtaining the above $\bar{K}$ requiring solving of a standard matrix equation. Give the name of the equation. 5

(c) Write a small MATLAB code implementaing the whole process applied to this problem, in a manner that also outputs the eigenvalues of the resultant closed loop system. 5

(d) Compute $\bar{K}$ 5
Hint: Note & obtain only the quantities you *really* need to compute for getting $\bar{K}$.

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2. A state space system is given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -1 & 3 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

and

$$C = [\ 0\ \ 1\ \ 0\ \ 0\ ]$$

Find a full-state linear feedback law such that the resultant autonomous system has its eigenvalues at $-3, -2, -1 \pm j$ 10

3. For a certain positive eigenvalue λ_i of a matrix A, it is given that

$$\lambda_i I - A = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

If $b = [2\ 0\ 0]^T$, then is the system (A, b) stabilizable ? 10

4. Among all smooth functions $y(x)$ between $x = 0$ and $x = 1$, let $\bar{y}(x)$ be the one which has the minimum/maximum value of

$$J = \int_0^1 \frac{\sqrt{1 + y'^2}}{y} dx$$

Write the differential equatiou in terms of y, y' that must be satisfied by $\bar{y}$. What are such differential equations called in the context of functional minimizations ? 10

5. A dynamic system is given by

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2^2 + 3u_1^2 + 2u_2^2 && (1) \\ \dot{x}_2 &= x_1^2 - x_2 + u_1^2 - u_2 && (2) \end{aligned}$$

(a) How many equilibrium points does the above system have ? Find tbem. 5

(b) Obtain a two-input linear SS system for small values of x_1, x_2, u_1, u_2 about the point where all of their values are zero. 5

(c) Is this system stabilizable through a linear full state feedback ? 5

(d) For the above system, can $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$ serve to say anything conclusive about the stability of the origin ? 5

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