APPLIED MECHANICS DEPARTMENT

AML835: MECHANICS OF COMPOSITE MATERIALS

SEMESTER – II, 2007-2008 MAJOR TIME: 2 hrs

MAX. MARKS: 100

Note: Answer all questions.

1. a) Name two types of laminates, with a specific example in each case, for which $B_{ij}=0$, $A_{16}=A_{26}=D_{16}=D_{26}=0$.

   b) Show that for cross-ply anti-symmetric laminate, $B_{22}=-B_{11}$ and other elements in B_{ij} matrix are zero.

   c) Using the stress-strain relation of a lamina and definition of stress resultants N and M, obtain the plate constitutive relations relating N, M with strain terms $\varepsilon^0, \overline{\chi}$ according to classical laminate theory, <u>in presence thermal loading</u>. (3, 5, 5)

2. A two-layer cross-ply [0/90] laminate is cooled down during curing from a temperature of T_2 to T_1.

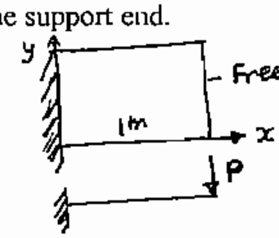
   a) Show that for the given laminate $A_{11}=A_{22}$ and $D_{11}=D_{22}$.

   b) Show that for this case, thermal stress resultants $N_{xT}=N_{yT}$ and $M_{xT}=-M_{yT}$.

   c) Show that if no external mechanical load is applied during the curing process, the curvatures χ_1, χ_2 of the plate after cooling are given by $\chi_1=-\chi_2=\dfrac{(A_{11}+A_{12})M_{xT}-B_{11}N_{xT}}{(A_{11}+A_{12})(D_{11}-D_{12})-B_{11}^2}$. (3, 3, 6)

3. A rectangular four-ply cross-ply $[0/90]_S$ composite plate with each ply of 1 mm thickness is clamped at one edge at $x=0$ and free at other three edges. It is subjected to a line load at the edge at x=1 m (Fig. 1) such that a bending moment of $M_x = 20$ kN-mm/mm is induced at the support end.

   For the laminas, $Q=\begin{bmatrix}140 & 3 & 0\\ 3 & 10 & 0\\ 0 & 0 & 5\end{bmatrix}$ GPa.



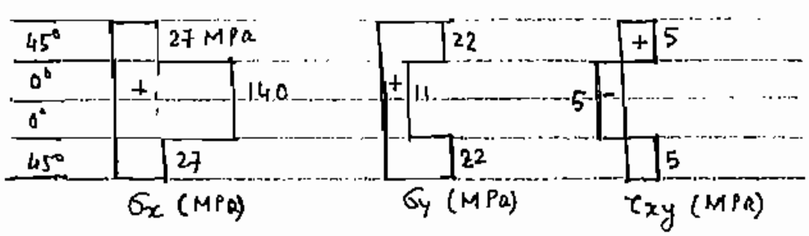
(Fig.1)

   a) Find the inplane strains and curvature of the plate at $x=0$.

   b) Show the distribution of σ_x across the thickness of the laminate with all key values.

   c) Obtain the transverse shear stress τ_{zx} at $x=0$ at the mid-surface using the post-processing approach based on three-dimensional stress equilibrium equations. (8, 5, 5)

4. A four-ply laminate $[45/0]_S$ with each ply of 3 mm thickness is subjected to inplane loads $N_x = 1000$ N and $N_y/N_x = 0.2$ which induce the following stresses in the layers.



   a) Find the layer stresses in the principal material directions.

   b) Compute the axial load N_x (with N_y=0.2 N_x) corresponding to the first-ply failure (FPF). Use Tsai-Hill criteria for failure with the following strengths: F_{1t}=500 MPa, F_{2t}=50 MPa, F_{1c}=350 MPa, F_{2c}=75 MPa, F_{12}=35 MPa. (5, 5)

5. The governing equation of motion in z direction for a rectangular composite plate is given by

$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} + p = I_0\ddot{w}_0 + I_1(\ddot{u}_{0,x} + \ddot{v}_{0,y}) - I_2(\ddot{w}_{0,xx} + \ddot{w}_{0,yy})$$

a) Obtain the differential equation for deflection w_0 for the case of <u>symmetric cross-ply laminate</u>.

b) Solve the above equation for static deflection of a simply-supported rectangular plate under a sinusoidal pressure load $p = p_0 \sin(\pi x/a)\sin(\pi y/b)$.

c) Also obtain the expression for undamped bending natural frequencies of the plate. (6, 3, 4)

6. A rectangular four-ply laminated plate $[0/90]_S$ with each ply of 1mm thickness is subjected to a temperature rise of 100°C and 50°C at top and bottom surfaces, respectively. The plate is supported on all sides such that all displacements and rotations are restrained.

b) Find the stress-temperature coefficients β_i of the laminas, which have the following properties:

$E_1 = 150$ GPa, $E_2 = 10$ GPa, $G_{12} = 6$ GPa, $\nu_{12} = 0.3$, $\alpha_1 \cong 0$, $\alpha_2 = 30\times10^{-6}$/deg C.

c) Find the axial force N_x and bending moment M_x developed in the laminate due to the temperature rise. Assume a linear variation of temperature across the thickness. (6, 7)

7. a) The transverse tensile strength F_{2t} is less than the strength of the tensile strength σ_{mu} of matrix. True/False. Explain.

b) Define the coefficients of mutual influence of first and second kind and show that $\dfrac{\eta_{12,1}}{E_1} = \dfrac{\eta_{1,12}}{G_{12}}$.

c) State the numbers of independent elastic constants for (i) anisotropic, (ii) monoclinic and (iii) orthotropic materials. (2.5, 6, 1.5)

8. a) A unidirectional E-glass-epoxy composite with v_f=0.65 has the following properties of its constituents: E_f=70 GPa, E_m=3.5 GPa, ν_f=0.2, ν_m=0.36. Find its transverse modulus using (a) strength of materials approach and (b) Tsai-Halpin equation with ξ=2.

b) For the above mentioned composite, tensile strengths of fibre and matrix are given as 3500 MPa and 105 MPa, respectively. Find the tensile strength F_{1t} of the composite. Assume elastic behavior to failure for both fibre and matrix. (6,5)

<u>Useful Formulae:</u>

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} = \begin{bmatrix}c^2 & s^2 & 2cs\\ s^2 & c^2 & -2cs\\ -cs & cs & c^2-s^2\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}, \quad \frac{M}{M_m} = \frac{1+\xi\eta v_f}{1-\eta v_f}, \quad \eta = \frac{M_f/M_m - 1}{M_f/M_m + \xi}, \quad E_2 = \frac{E_m}{1-\sqrt{v_f}\,(1-E_m/E_{2f})}$$

$$G_{12} = \frac{G_m}{1-\sqrt{v_f}\left(1-\frac{G_m}{G_{12f}}\right)}, \quad G_{23} = \frac{G_m}{1-v_f\left(1-\frac{G_m}{G_{23f}}\right)}, \quad \frac{\sigma_1^2}{F_1^2} + \frac{\sigma_2^2}{F_2^2} - \frac{\sigma_1\sigma_2}{F_1^2} + \frac{\tau_{12}^2}{F_{12}^2} < 1$$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \quad \hat{\beta}_1 = Q_{11}\alpha_1 + Q_{12}\alpha_2$$

$$\hat{\beta}_2 = Q_{12}\alpha_1 + Q_{22}\alpha_2, \quad N_T = \sum_{k=1}^{L}\int_{z_{k-1}}^{z_k} \bar{\beta}^{(k)}\, T(z)\,dz, \quad M_T = \sum_{k=1}^{L}\int_{z_{k-1}}^{z_k} \bar{\beta}^{(k)}\, z\, T(z)\,dz$$

$$F_{1t} \simeq v_f\sigma_{fT}, \quad \text{i)}\ F_{1c} \approx v_f\sigma_{fc} \quad \text{ii)}\ F_{1c} = 10F_{12} + 2.5\sigma_{mT}, \quad \text{iii)}\ F_{1c} \simeq \frac{G_m}{1-v_f\left(1-\frac{G_m}{G_{f12}}\right)}$$

$$F_{2t} \simeq \left[1-\left\{\sqrt{v_f} - v_f\right\}\left\{1-\frac{E_m}{E_{f2}}\right\}\right]\sigma_{mT}, \quad F_{2c} \simeq \left[1-\left(\sqrt{v_f}-v_f\right)\left(1-\frac{E_m}{E_{f2}}\right)\right]\sigma_{mc}$$