MEL 707 Mathematics for Mechanical Engineers
Major Exam- Part B
(Open Notes)

Max. Marks:60 November 21, 2008

1. A student is asked to obtain the temperature profile in a long rectangular bar having cross-section shown in figure 1, with the faces maintained at the conditions given in the figure. Length of the bar perpendicular to the cross-section shown can be taken to be very large, resulting in steady state 2D heat conduction. Formulate the problem mathematically for determining the steady state temperature profiles and verify whether it is possible to solve it by the method of separation of variables. If it is possible, determine the eigen values of the problem. If it is not possible, show why it is not. (15)

2. **(a)** Some experimental work is being carried out on the ground. Initially the temperature on the surface as well as under the ground can be taken to be T_i. Suddenly the surface of the ground is subjected to a constant temperature T_h. This results in slow one-dimensional conduction of heat to the ground. Assuming the ground to be a semi-infinite medium, answer the following.
(i)Which equation will govern the variation in ground temperature with time?
(ii)What will be the suitable initial/boundary conditions for the same?
(iii)Show that this problem can be solved by similarity method which will result in the following ordinary differential equation :

$$\frac{d^2\theta}{d\eta^2} + \frac{\eta}{2}\frac{d\theta}{d\eta} = 0$$

where $\eta = x/\sqrt{\alpha t}$, α being the thermal diffusivity of the ground material; and
$\theta = (T - T_i)/(T_h - T_i)$
(iv)What will be the initial/boundary conditions for this ODE?
(v) State whether the equation is linear or not and the equation and the initial/boundary conditions are homogeneous or not. (20)

(b) The above equation has to be solved numerically. Clearly enumerate the steps involved in solving the above. Assume that the condition of $\eta \to \infty$ can be replaced by $\eta = 10$. (10)

3. In the analysis of a flat plate solar collector, the following two equations are obtained in terms of the temperature of the first glass cover, T_{c1} and the second glass cover, T_{c2}. Using Newton's method for solving a system of non-linear equations simultaneously, determine the estimates of the two temperatures at the end of first iteration if the initial guess is : $T_{c1} = 350K$ and $T_{c2} = 300K$.

$$5.67 \times 10^{-8}(400^4 - T_{c1}^4) + 5(400 - T_{c1}) = 400$$
$$5.67 \times 10^{-8}(T_{c1}^4 - T_{c2}^4) + 5(T_{c1} - T_{c2}) = 400$$

(15)