CS105L: Discrete Structures
I semester, 2006-07

Major Exam

1 PM to 3 PM

2nd December 2006

Name	Ent. No.

Section	1	2	3	Total
Marks				

To prove a claim we have to go through a series of inferences. To refute a claim sometimes we have to argue against it and sometimes we have to come up with a case in which the claim doesn't hold. We first refute the claim and then prove that our refutation (or counter claim) is true by producing a *witness* or *counterexample* which bears it out.

Refute the following claims with a one line statement then support your refutation with an example. Note that if the statement of the refutation is wrong or incomplete marks will be deducted even if the example is correct.

All rough work should be done on the additional sheet (which will not be collected). On this sheet just give the refutation, the counterexample and a very brief justification of why it is a counterexample.

1 All questions carry 2 marks

Claim 1.1 *All graphs on 10 vertices have chromatic number 3 or more.*

Refutation 1.1

Counterexample 1.1

Name: Entry No:____________________

Claim 1.2 $((A \Rightarrow B) \Rightarrow (C \Rightarrow B)) \Rightarrow ((A \vee C) \Rightarrow B)$.

Refutation 1.2

Counterexample 1.2

Claim 1.3 *All graphs on less than 15 vertices have girth at most twice their diameter.*

Refutation 1.3

Counterexample 1.3

Claim 1.4 $\chi(G) < \chi'(G)$ *for all graphs* G.

Refutation 1.4

Counterexample 1.4

Claim 1.5 *Every totally ordered set contains a maximal element.*

Refutation 1.5

Counterexample 1.5

Name:______________________ Entry No:______________

2 All questions carry 3 marks

Claim 2.1 *The following is a correct proof by induction of the claim "All horses have the same colour."*

Consider a set of horses S. If $|S| = 1$, the claim holds. Now assume it is true from $|S| = n$. Now consider S, such that $|S| = n+1$. For some element $e \in S$, $|S \setminus \{e\}| = n$ and hence all horses in $S' = S \setminus \{e\}$ have the same colour. Similarly for some $f \neq e$, all the horses in $S'' = S \setminus \{f\}$ have the same colour. Hence all the horses in S must have the same colour, which is the colour of the horses in $S' \cap S''$.

(Note: No counterexample is needed here, just a refutation.)

Refutation 2.1

Claim 2.2 *All non-bipartite graphs on 10 vertices with maximum degree Δ have edge chromatic number $\Delta + 1$.*

Refutation 2.2

Counterexample 2.1

Claim 2.3 *Every vertex of a separator S of a graph G on 10 vertices has an edge to a vertex in each component of $G \setminus S$.*

Refutation 2.3

Counterexample 2.2

Name:______________________________ Entry No:____________________

3 This question carries 6 marks

A *k-uniform hypergraph* is a pair (X, S) where X is a set of vertices and S is a set of hyperedges of size exactly k i.e. each $e \in S$ is a subset of X with the property that $|e| = k$.

A hypergraph is said to be *c-colourable* if its vertices can be coloured with c colours, i.e. each vertex can be assigned one out of c colours, in such a way that no edge is monochromatic i.e. each edge has at least two different colours on its vertices.

Claim 3.1 *A k-uniform hypergraph with less than 2^{k-1} hyperedges is not 2 colourable.*

Refutation 3.1

Counterexample 3.1