MAJOR TEST AML 774 (4)

1. Lagrange functional L is
(0.5) (a) sum of strain energy and potential energy of external body & surface loads.
(b) difference of " " "
(c) none of these.

2. For a variational principle where the functional $F(\psi)$ is defined over a domain Ω with a boundary Γ and is a function of ψ. First variation

$$\delta F = \int_{\Omega} [\_\_①\_\_]\, \delta\psi \, d\Omega + \int_{\Gamma} [(\_\_②\_\_)\,\delta\psi + (\_\_③\_\_)\,\delta\psi']\, d\Gamma = 0$$

(.5) The Euler equation(s) for F are :

(.5) The essential B.C.(s) for F are :

(.5) The natural B.C.(s) for F are :

3. When the stiffness properties of an elastic body are increased, then its strain energy
(.5) under given force load.
(a) remains the same (b) decreases (c) increases

(.5) 4. Lagrange functional is essentially a function of u only / σ only / u and σ

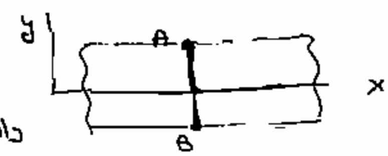
(.5) 5. Reissner functional is essentially a function of u only / σ only / u and σ

6. Euler's equations of Reissner functional are :
(.5) (a) equations of equilibrium only
(b) equations of equilibrium, & stress-strain relations only
(c) equilibrium equations, stress-strain relations and strain-displacement relations.

7. (.5) Natural B.C's of Reissner functional are
(a) essential B.C's of elasticity problem.
(b) natural B.C's of " "
(c) essential and natural B.C's of elasticity problem.

(.5) 8 (a) For Timoshenko beam theory differs from the classical beam theory in the assumption that .....

(1) (b) For Timoshenko beam theory $u(x,y) = u(x,0) - \theta(x)\,y$, $v(x,y) = v(x)$
Draw the deformed position of cross-section AB carefully showing clearly $u(x,0)$, $v(x)$, $\theta(x)$ in the diagram.

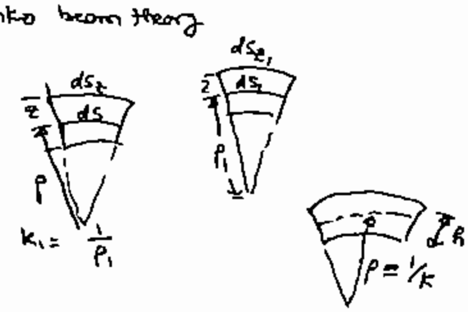


(0.25) (c) Why is shear correction factor required in Timoshenko beam theory

(0.25) (d) Common value of shear correction factor for rectangular section is ....

9. (a) For a plane curvilinear bar with shear ignored
(1.5) ρ and ρ_1 are radii of curvature of bar's axis at $z=0$ and strain at $z=0$ is ε_0. Show that strain ε at z is related to ε_0 by

$$1+\varepsilon = \frac{1+zk_1}{1+zk}(1+\varepsilon_0) \quad \text{where } k = \frac{1}{\rho},\ k_1 = \frac{1}{\rho_1}$$



(0.25) (b) for small-curvature bars neglect value of ___ w.r.t. 1 in various relations.
(0.25) (c) for medium-curvature bars " " " ____ " " ".

10. For shear deformation included in plane curvilinear bars, the solution obtained for
(0.5) deflection is more than / less than the deflection obtained for case of theory neglecting shear deformation.

11. The deflection obtained in classical theory for plane curvilinear bars based on
(0.5) small-curvature approximation is more than / greater than the deflection obtained for case of medium-curvature approximation.

12. For classical plate theory of bending $\varepsilon_{xx} = -z\omega_{,xx}$, $\varepsilon_{yy} = -z\,\omega_{,yy}$, $\varepsilon_{xy} = -z\,\omega_{,xy}$
(1.5) Strain energy $E = \frac{1}{2}\int[\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + 2\sigma_{xy}\varepsilon_{xy}]\,d\vartheta$, $d\vartheta = d\Omega\,dz$

(use: $M_{\alpha\beta} = \int_{-h/2}^{h/2} z\,\sigma_{\alpha\beta}\,dz$ and $M_{\alpha\beta}-\omega$ relations: $M_{xx} = -D(\omega_{,xx} + \nu\,\omega_{,yy})$, $M_{xy} = -D(1-\nu)\,\omega_{,xy}$)

Prove that $E = \frac{1}{2}\int_{\Omega} D\,[\omega_{,xx}^2 + 2\nu\,\omega_{,xx}\,\omega_{,yy} + \omega_{,yy}^2 + 2(1-\nu)\,\omega_{,xy}^2]\,d\Omega$

(1) 13. Describe in few sentences static-geometric analogy in theory of plates.

14.(a) In Reissner's plate theory of bending of plates, the number of equilibrium
(1) equations are ____ and these are in terms of ω and ---.

(b) Displacements u, v in Reissner's plate theory are approximated as:
(1) $u =$ $v =$

15. For thin-walled open-section bars (a) $\sigma_x = \frac{N}{A} + \frac{M_y z}{I_y} + \frac{M_z y}{I_z} - E\theta''\omega$
where $-\theta'\omega$ is axial displacement due to warping of cross-section.
The term $-E\theta''\omega$ contributes a term to the strain energy just as term $\frac{N}{A}$ does and it is expressed in terms of bitorque B defined as
(1) $B =$ - - - .
and $B = -EI_\omega\,\theta''$ where I_ω is defined by $I_\omega =$ ---
The theory is formulated in terms of displacement variables whose number is -----
(b) $\tau_{xs} = \tau + \tau_K$ ← linear distribution across thickness with zero value at midthickness
↓ constant value across thickness due
The axial moment M_x = moment contributed by τ + moment contributed by τ_K,
called moment M_ω of constrained torsion. called moment H of pure torsion
(1) Resultant of τ distribution at principal pole consists of force: - - - - - - - and moment - - - -
Resultant of τ_K distribution at principal pole " " " - - - - - - " " - - -

16. Without allowing for shearing in thin walled open section bars axial displacement u:
(1) $u(x,y,z) = \xi(x) - \zeta'(x)z - \eta'(x)y - \theta'(x)\,\omega$. 'Allowing for shearing' means replacing it by
$u =$
The theory is formulated in terms of displacement variables whose number is ----

17. For closed profile single cell thin walled bars; the key idea of Umanski is to take
(0.5) the warping contribution of axial displacement in the form - - - - - - - - -

18 What are the special features of theory for multiple-contour closed profile thin-walled bars?
(1)

19. What are the special features of theory for compound profile thin walled bars?
(1)

(6)

20. State 3 conditions to be satisfied by Φ_i in the Ritz solution based on $\sum_{i=1}^{n} \alpha_i \Phi_i$

(1) 1.

2.

3.

21 In the application of Ritz method to Lagrangian functional, Φ_i's must satisfy

(.5) all B.C.'s / only essential B.C.s. / only natural B.C.s.

The value of L obtained in solution by Ritz method with approximation $\sum_{i=1}^{n} \alpha_i \Phi_i$

(.5) $\leq$ / $\geq$ / $=$ the value of L obtained in exact solution.

22 For the eigenvalue problem of dynamics:

$\cancel{Kz = \lambda}$ $\quad \underset{\text{stiffness matrix}}{\gamma} z = \lambda \underset{\text{inertia matrix}}{m} z$ $\quad \lambda = \omega^2$ eigenvalue, z = eigen vector

Rayleigh ratio / Rayleigh functional r is defined by

(.5)

$r(z) =$ ______

(.5) $\min[r(z)] =$ ...., $\max[r(z)] =$ .....

23 For eigenvalue problem arising from Lagrangian functional solved by Ritz method;

(.5) eigen-values so obtained are $\leq$ / $\geq$ exact eigen values

24. The minimum of $r(z)$ obtained under 4 independent constraints applied to z

(.5) has value $\min[r(z)] \geq$ (where λ_i are exact eigenvalues)

25. Define stability of an equilibrium configuration

(1)

26. State Lagrange-Dirichlet theorem for stability:-

(1)

27. What do you understand by matrix of geometric stiffness of the system?

(1)

28. Including shear deformation in buckling of bars results in increase / decrease of buckling load.

(.5)

29 For linear stability analysis of elastic systems

(1) $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) << 1$, $\omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}) << 1$ $\quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i} u_{k,j}) << 1$, and .....

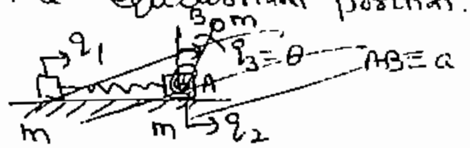
30 $m\ddot{q} + C\dot{q} + Kq = Q$ with $C = \alpha m + \beta K$. $q = U\eta$, given $q(0)$, $\dot{q}(0)$.

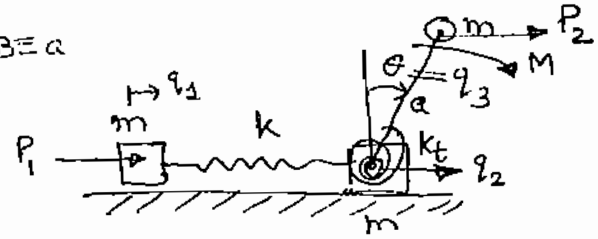
(10) ~~Eval~~ Derive equation for η_i. If ξ_1, ξ_2 are given for first two modes, ~~~~
set up 2 eqn for finding α, β. Also ~~express~~ prove expression of $\eta(0)$ in terms of $q(0)$ which does not require inversion of U.

OPEN NOTES AML 774

7

1. Find the Kinetic energy T, potential energy V with datum at equilibrium position and δW^{nc} of applied loads P_1, P_2, M for small oscillations and hence list M, K matrices and load vector.
(10) The rod is light and its relative rotation is restrained by a torsional/rotational spring of stiffness k_t. $q_1, q_2, q_3 (\equiv \theta)$ are measured from the equilibrium position.





2. $M = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$, $K = \begin{bmatrix} 5 & -5 & 0 \\ -5 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$, find natural frequencies and prove that ~~for~~ the lowest normalised mode is
(10) $\frac{1}{\sqrt5}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and it is a rigid body mode.

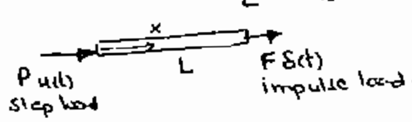
3. In problem 2, the normalised modes and natural frequencies are $\frac{1}{\sqrt5}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\frac{1}{\sqrt5}\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$ and ~~for~~ 0, 3, 5 respectively.

(a) For $q(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\dot q(0) = \begin{bmatrix} \sqrt5 \\ \sqrt5 \\ 1 \end{bmatrix}$ find the free response for $\xi_3 = 0.1$ for ~~all modes,~~ and no damping for the first mode.
(7)

(b) Find the undamped response for $Q_1(t) = 3\sqrt5\, u(t)$
$Q_2(t) = -2\sqrt5\, u(t)$
(9) $Q_3(t) = \delta(t)$
where $u(t)$ is step function and $\delta(t)$ is Dirac-delta function.

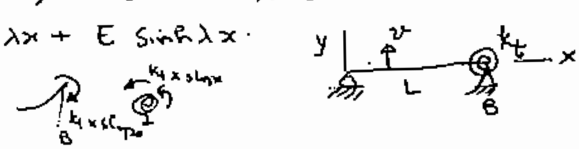
(c) Find forced damped response (steady state) for $\xi_1 = 0$, $\xi_2 = \xi_3 = 0.1$ for $Q_1 = 6\sqrt5\cos(5t) - 12\sqrt5\sin(4t)$
(9)
$Q_2 = -4\sqrt5\cos(5t) + 8\sqrt5\sin(4t)$
$Q_3 = 0$

4. A free-free bar has $\omega_n = \sqrt{\frac{E}{\rho}}\,\lambda_n$, ~~$\lambda_n = \frac{2\pi}{L}$~~
$\lambda_1 = 0$ with normalised mode shape $\frac{1}{\sqrt{\rho A L}}$ and $\lambda_i = (i-1)\frac{\pi}{L}$ for $i \geq 2$, with normalised mode shape $\sqrt{\frac{2}{\rho A L}}\cos\frac{(i-1)\pi x}{L}$
(15) It is subjected to force shown.
Initial vel. and displacement are zero.



Find the undamped transient response.

5. For free oscillations of a beam $EI\lambda^4 = \rho A\omega^2$, $v(x,t) = X(x)\cos\omega t$
$X(x) = A\cos\lambda x + B\sin\lambda x + D\cosh\lambda x + E\sinh\lambda x$.
(10) (Hint one of B.C.'s at B is Bending moment at B $= -k_t \times$ slope at B)



Derive the characteristic equation for λ and corresponding mode shape.