Deptt. of Elec. Engg., IIT Delhi

EEL768, Detec. & Est. Theory, Major Exam, M.M: 35, Dur. 2 hrs

Prob. 1 Let $r(t)|_{H_1} = \sum_{i=1}^{m} a_i s_i(t) + n(t)$ $0 \le t \le T$ and $r(t)|_{H_0} = n(t)$; $0 \le t \le T$, here a_i's are i.i.d $N(0, \sigma_i^2)$ variables and $n(t)$ is zero-mean white with power spectral density $= \frac{N_0}{2}$. Find the optimum receiver structure. (State clearly all the assumptions taken). (7)

Prob. 2 Let $\text{Prob}\{R_i < -5 \mid H_1\} = 0.7$ and $\text{Prob}\{R_i < -5 \mid H_0\} = 0.5$ for $1 \le i \le 5$. Develop suitable sign test and Wilcoxon-test with $P_F \le 0.1$. Also given the observation $\{-7, -20, 5, -10, 10\}$ using the above tests decides whether H_0 is true or H_1. Also compute exact P_F for each test. (7)

Prob. 3. Show that (i) $X(t) = \sum_{i=1}^{\infty} a_i \phi_i(t)$, in the mean square sense where ϕ_i's are chosen as K.L.T. basis. (ii) Find conditions of $\phi_i(t)$ so that a_i are uncorrelated. (7)

Prob. 4: Show the following: (i) $E\{\Lambda^n | H_1\} = E\{\Lambda^{n+1} | H_0\}$ (ii) $E\{\Lambda | H_0\} = 1$; (iii) $E\{\Lambda | H_1\} - E\{\Lambda | H_0\} = Var(\Lambda | H_0)$. (Here Λ is LRT). (7)

Prob. 5 Let $\underline{R}_1 = H_1 \underline{\theta} + \underline{n}_1$ & $\underline{R}_2 = H_2 \underline{\theta} + \underline{n}_2$. Show that MMSE estimate of $\underline{\theta}$ given $\{\underline{R}_1, \underline{R}_2\}$ can be written as linear combination of $\hat{\underline{\theta}}_1$ & $\hat{\underline{\theta}}_2$ where $\hat{\underline{\theta}}_1$ & $\hat{\underline{\theta}}_2$ are MMSE estimates of $\underline{\theta}$ given $\underline{R}_1$ & $\underline{R}_2$ respectively. Assume $\underline{n}_1$ & $\underline{n}_2$ to be i.i.d. $N(0, \sigma^2)$ each. (7)