

EEL-772: Optimal Control Theory

Semester: Jan-July 2010
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Major

1. For the system

$$x(k+1) = 3x(k) + u(k) ; x(0) = 1$$

Find the optimal control $u(k), k = 0, 1$, such that $x(2)$ reaches as close to 5, while also minimizing the transition cost $\sum_0^1 x^2(k)$. 15

2. For a standard nonlinear system $\dot{x} = f(x, u)$, write the HJB equation using standard symbols and explain the symbols. 5

3. A simple linear mass-spring system with $m = 1, k = 2$ has a force input $u(t)$; From any initial condition it needs to be taken to (0,0) such that the integral of the sum of kinetic energy, potential energy, and square of the force input is minimum.

(a) Formulate it as a standard LQR problem.

(b) Form the Riccati equation for the problem.

(c) Write an expression for the optimal feedback law for the force input. Solve the above to obtain the input if you can. 5+3+7

4. A system $\dot{x} = -3x(t) + u(t)$ is to be taken from $x(t_0)$ to 0 in minimum time, with the constraint $|u(t)| \leq 1$. Obtain the optimal control law. 10

5. Give an iterative algorithm to solve the matrix Algebraic Riccati Equation. What is an "ill-conditioned" problem ? 3+2