IIT, Delhi

Department of Physics

Major Examination

PHL 552: Electrodynamics

All Questions are compulsory

03-05-2007

Time: 2 hours

Full Marks: 60

Q.1 (a) A solid conducting sphere, with charge Q and radius a, is surrounded by an uncharged concentric hollow conducting spherical shell of radius $b > a$. Let the potential difference between the surface of the solid sphere and the outer surface of the shell be V. If the shell is now given a charge $-5Q$, the new potential difference is: (i) 6V, (ii) V, (iii) 4V, (iv) -4V ? **(3)**

(b) Two identical thin rings of conducting wires, each of radius R, are coaxially placed at a distance R apart. If Q_1 and Q_2 are the charges on the first and the second rings, respectively, uniformly spread over them, then the work done in taking a charge q from the center of one ring to the center of the other ring is: (i) Zero, (ii) $\sqrt{2}q(Q_1 + Q_2)/(4\pi\varepsilon_0 R)$, (iii) $(\sqrt{2} - 1)q(Q_1 - Q_2)/(4\sqrt{2}\pi\varepsilon_0 R)$ (iv) $(\sqrt{2} - 1)q(Q_1 + Q_2)/(4\sqrt{2}\pi\varepsilon_0 R)$? **(3)**

(c) A p[article of mass m and charge q, moving with a constant velocity $\vec{v} = v_0\hat{i}$, enters a region of space with a uniform magnetic field $\vec{B} = -B_0\hat{k}$, extending from $x = a$ to $x = b$ $(a > b)$. The minimum value of the velocity required for the particle to just exit into the region $x > b$ is: (i) qbB_0/m, (ii) qaB_0/m, (iii) $q(b + a)B_0/2m$, (iv) $q(b - a)B_0/m$? **(3)**

(d) Which of the following scalar fields can be used for satisfying the Lorentz condition under a gauge transformation of the electromagnetic potentials in free space (A, B and α are constants): (i) $A\, cos(\omega t)/r$, (ii) $B\, sin(\omega t)/r$, (iii) $sin\alpha\, cos(\omega t - \omega\sqrt{\varepsilon_0\mu_0}\, r)/r$, (iv) $cos\alpha\, cos(\omega t - \omega\varepsilon_0\mu_0\, r)/r$? **(3)**

(e) Consider a relativistic particle of rest mass m_0, moving with a velocity $\vec{u}$. Which of the following is/are conserved (η^μ being the proper 4-velocity): (i) m_0, (ii) $m_0\vec{\eta}/\sqrt{1 - (u^2/c^2)}$, (iii) $m_0/\sqrt{1 - (u^2/c^2)}$, (iv) $m_0 c^2/\sqrt{1 - (u^2/c^2)}$? **(3)**

P.T.O.

Q.2 Write down the real electric and magnetic fields of an electromagnetic wave of amplitude E_0, frequency ω and phase angle zero that is (a) travelling in the negative $y-$direction and polarized in the $x-$direction, and (b) travelling in the direction from the origin to the point $(1,1,1)$ with polarization parallel to the xy plane. Calculate the intensity of the wave in the case (b). **(10)**

Q.3 A parallel plate capacitor, at rest in the frame S_0 and tilted at 45^o angle to the x_0 axis, carries charge densities $\pm\sigma_0$ on its plates (Fig.1). System S, with its axes parallel to the corresponding axes of S_0, is moving with a constant velocity $\vec{v} = v\hat{x}$ relative to S_0. (a) Find the electric field $\vec{E}_0$ in S_0. (b) Find the electric field $\vec{E}$ in S. (c) What angle does $\vec{E}$ make with the x axis? (d) Check by doing explicit calculations whether the electric field is perpendicular to the plates in S or not. **(10)**

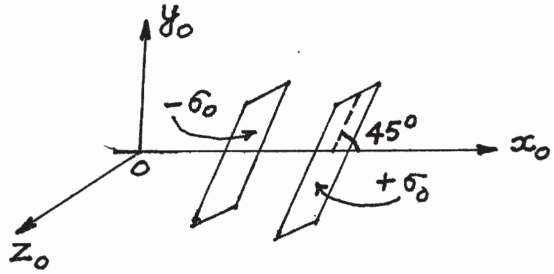

Fig.1

Q.4 Derive Poynting theorem of electrodynamics and state its physical content. Also, derive its differential form. **(10)**

Q.5 Write the expression for the electromagnetic field tensor $F^{\alpha\beta}$ in matrix form. Show that the spatial components of the equation (Einstein's summation convention is implied)

$$\frac{\partial F^{\alpha\beta}}{\partial x^{\beta}} = \mu_0 \, j^{\alpha}, \quad \alpha, \beta = 0, 1, 2, 3$$

reduce to the modified Ampere's law. Using the law of transformation of $F^{\alpha\beta}$ under Lorentz transformations, compute the components of the electric field in a moving frame Σ' (velocity v) in terms of the electromagnetic fields in the stationary frame Σ. **(15)**