1530 – 1730
MZ 163

MAL 601 Topology Major Test
07 May 2007

Max Marks
50

All questions carry equal marks

1. On $X = [-1, 1]$, declare a set open iff it either does not contain $\{0\}$ or does contain $(-1, 1)$. Show that one obtains a topology consisting of these open sets which is T_0 but not T_1 and is compact. Further, show that it is first countable but not separable.

2. On the real line $\mathbb{R}$, construct the RHO topology, whose basis is given by intervals $[a, b)$. Prove that each $[a, b)$ is not only open but closed also and that RHO is strictly larger than the usual topology on $\mathbb{R}$. Show that this is regular.

3. Let $X \xrightarrow{f} Y$ be an onto function where X is a given topological space. Construct the quotient topology on Y and show that this is the largest topology on Y with respect to which f is continuous. Further show that any function $Y \xrightarrow{g} Z$ [where Z is some topological space] is continuous iff $g \circ f : X \longrightarrow Z$ is continuous.

4. Define weak topology on X induced by $\{X \xrightarrow{f_\alpha} Y_\alpha\}$ where $\{Y_\alpha\}$ is an indexed collection of topological spaces. Construct the product topology on $Y := \prod_\alpha Y_\alpha$ as an instance of weak topology and show that a nonempty open set in Y projects onto almost all the factors.

5. Show that in a metric space

$$\text{Lindelof} \Leftrightarrow \text{separable} \Leftrightarrow \text{second countable}$$

and determine whether the spaces in questions 1 and 2 are metrizable.