Major Examination EPL 204 Statistical and Thermal Physics Date: 9th May,2007
Max. Marks50 Time: 2 hrs

Instructions: 1. Closed Book Examination;

2. Cell phones are not allowed ; calculators allowed.

3. Answer should be coherent and to the point.

4. Short answer type questions (No.1) have to be answered with brief reasoning. Otherwise no marks will be given.

5. The derivations leading to a result may be shortened, but without loosing their reasoning and coherence. Otherwise no credit will be given

6.Should try to put the answers of various parts of a given question in the same place.

6. $k_B = 1.38 \times 10^{-16}$ erg deg^{-1} $h = 6.6256 \times 10^{-27}$ erg sec^{-1}

1. Short answer type: 10 × 1 =10
   (a) Why does bulk material have constant resistance or thermal conductance even though they are generated from random motion of electrons and ions?
   (b) Will the nanoscopic system consists of few electrons or atoms will also have constant values for the above quantities?
   (c) Why the chemical potential of an ideal Bose gas cannot be positive?
   (d) Why we cannot factorize the partition function of an ideal gas of fermions or bosons in the same way we do for its classical counterpart?
   (e) Why the Ising model in one dimension does not show a phase transition at $T \neq 0$? Give entropy based argument.
   (f) What is Chandrasekhar limit?
   (g) What is ergodic hypothesis?
   (h) How Bose Einstein condensation is related to the superfluidity of liquid Helium?
   (i) Under what situation fermions can form a Bose Einstein condensate?
   (j) What are the major features of second order phase transition?

2.(a) A neutral atom consists of electrons, protons and neutrons. Explain, why, it is the neutron number that alone determines whether the atom is a boson or fermion. 1

(b) Explain why there is no need to correct the expression for the thermodynamic potential Ω (for grand canonical ensemble) of a degenerate fermion gas by adding a contribution from the ground state, as required for a degenerate boson gas. 1

(c)The recent experiments on BEC of trapped cold bosonic atoms generally take place in magnetic traps which can be modeled as
$V = \frac{1}{2} m\omega^2 (x^2 + y^2 + z^2)$. Assuming such cold atoms are otherwise non-interacting bosons in the above mentioned external potential, show that the critical temperature T_c at which BEC starts taking place is given by up to the leading order as $\varsigma_3(1)\left(\frac{k_B T_c}{\hbar\omega}\right)^3 = N$, where N is the total no. of particles and $\varsigma_3(z) = \sum_{k=0}^{\infty} \frac{z^{k+1}}{(k+1)^3}$. 4

(d) Using the expression of density of states free quantum particles in two dimensions with suitable boundary conditions, show that if such particles obey Bose-Einstein statistics, the number of particles that occupy the excited energy levels does not get saturated at low temperature. Discuss the impact of this result of Bose Einstein condensation in two dimensions. 3+1

3. (a) What is meant by degenerate gas of fermions and what is the general feature of the occupation number of such fermions. 1

(b) For extremely relativistic particles the energy momentum relation is given by $\varepsilon(p) = cp$. Calculate the energy density of an extremely relativistic gas of degenerate fermions with spin $\frac{1}{2}$. Compare to the non-relativistic case. Calculate the pressure of this gas and find the relation between pressure and energy density. 3+1+2 =6

(c) The atomic weight of Sodium (Na) is 23 and and the density of the metal is *0.95 gm/cm³*. There is only one conduction electron per atom. Calculate the numerical value of the Fermi temperature for these conduction electrons. 1

(d) It is desired to cool a sample consisting of 100 cm^3 of *Na* metal from 1K to 0.3K. At this low temperature the lattice heat capacity is negligible compared to that due to the conduction electrons. The metal can be cooled by bringing it into thermal contact with liquid He^3 at 0.3K. If 0.8 Joules are required to evaporate 1 cm^3 of He^3, estimate how much He^3 needs to be evaporated to coal the Na sample. 2

4. (a) The grand thermodynamic potential is given by $\Omega = F - \mu N = E - TS - \mu N$. Here *F* is the Helmholtz free energy and *E* is the internal energy. From this expression prove the following Maxwell's relation 1. $\left(\frac{\partial P}{\partial \mu}\right)_{T,V} = \left(\frac{\partial N}{\partial V}\right)_{T,\mu}$ 2. $\left(\frac{\partial S}{\partial \mu}\right)_{T,V} = \left(\frac{\partial N}{\partial T}\right)_{V,\mu}$ 3. $\left(\frac{\partial S}{\partial V}\right)_{T,\mu} = \left(\frac{\partial P}{\partial T}\right)_{V,\mu}$ 3 ×1.5=4.5

(b) The PV diagram of a Joule ideal gas cycle is as follows: (1) P_1, V_1 to P_2, V_2 through adiabatic compression. (2) P_2, V_2 to P_2, V_3 (3) P_2, V_3 to P_1, V_4 through adiabatic expansion (4) P_1, V_4 to P_1, V_1. Draw the diagram in the PV plane and if all process are quasi-static and C_p is constant, calculate the efficiency of the process in terms of P_1, P_2 , and γ. 2

(c) A molecule in gas moves equal distance ℓ between two collisions with equal probability in any direction. After a total N such displacements find out the mean square displacement $\overline{R^2}$ of the molecule from its starting position. 3.5

5. (a) Show that for an ideal gas of N atoms the canonical partition function $Z = z^N$ where *z* is the partition function of a single atom. Calculate *z* and from that show that the entropy is given by $S = Nk_B[\ln V + \frac{3}{2}\ln(2\pi m k_B T) + \frac{3}{2}]$. Discuss briefly why this expression leads to Gibb's paradox. 1+3+1=5

(b) State the third law of thermadynamics.

Calculate the entropy of a system of N classical three-dimensional harmonic oscillators (classical Einstein solid) and show that it diverges at $T \to 0$. Now quantize the energy levels of this harmonic oscillotor and show that this divergence vanishes. 1+2+2=5