Department of Applied Mechanics
Indian Institute of Technology Delhi
Major: Second Semester 2009-2010

Course Title: Finite Element Method | **Course No.:** AML 705
Date: 30 April 2010 | **Duration:** 2 Hrs (3.30 PM-5.30 PM)
Maximum Marks: 120

Note: Answer all the questions. Marks are indicated against each question.

Q. 1(a): Derive the expression for $\{\mathbf{d}\}_{-\Delta t}$ in terms of displacement, velocity and acceleration vectors at $t = 0$ required for starting central difference based direct time integration. [4]

Q. 1(b): Equation of motion for a system is given below:

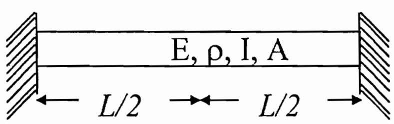
$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} 20 & -10 \\ -10 & 10 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} t \\ 0.5t \end{Bmatrix}$$

Find the natural frequencies of the system. [4]

Q. 1(c): Estimate Δt for numerically stable central difference scheme. [2]

Q. 1(d): For equation of motion given in part (b) above, obtain $\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}_{\Delta t}$ using Newmark's direct time integration. Assume zero initial conditions and suitable Δt. [5]

Q.2 (a): For a clamped-clamped beam, determine the critical temperature rise (ΔT) at which the beam will buckle. Prebuckling stress $\sigma_{xx}^{0} = -E\alpha\,\Delta T$ which is uniform throughout the beam. The beam can be modeled using Euler-Bernoulli Beam Element. [10]



Q.2 (b): Determine the first natural frequency of the beam if it is subjected to half of critical buckling temperature. [5]

Q. 3(a): Write the expression for kinetic energy for two-noded Timoshenko Beam element. Obtain the final mass matrix without neglecting any terms in kinetic energy. [10]

Q. 3(b): If kinetic energy $T = \frac{1}{2}\{\dot{d}\}^T[M]\{\dot{d}\}$ and total potential energy $I = \frac{1}{2}\{d\}^T[K]\{d\} - \{d\}^T\{F\}$, derive the equation of motion using Hamilton's principle. [5]