ASL850 **Major** **MMarks 40** **TIME 2 hours**

Q1. Non-dimensionalize the shallow water equation by choosing various proper characteristic scales and explain the parameters thus obtained and obtain the O(1) and O(ε) equations.

Q2. Show that the frequency increases and decreases monotonically with κd if λ / d >1/2 and λ / d < 1/2, respectively (ref. shallow water model). What value of κd it reaches to maximum? When is the group velocity zero for all κ? Here κ is the wave number, d is the grid-size and $\lambda (=\sqrt{gH} / f)$ is the radius of deformation. Here, f is the Coriolis parameter.

Q3. If $U_1^{(n)} = U_1^{(0)} e^{in\theta_1}$ is the physical mode in the complex plane, represent graphically the physical and the computational modes, for the leap-frog scheme, with the physical mode phase change (θ_1) is equal to $\pi / 8$ and considering the imaginary parts zero at the initial moment, for a number of values of n.

Q4. Find a way so that the average wave number does not change with time with 2-dimensional nondivergent flow.

Q5. Show that the following one-dimensional equations

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = 0,$$

$$\frac{\partial \eta}{\partial t} + c\frac{\partial \eta}{\partial x} + H\frac{\partial u}{\partial x} = 0.$$

are equivalent to a system of two advection equations, and find the Courant-Friedrichs-Lewy criterion for these cases.

Q6. Show that the maximum wavelength (numerically) of the Rossby wave is just 2πR, where R is the Rossby deformation radius.