INDIAN INSTITUTE OF TECHNOLOGY DELHI

MAL 120: Mathematics II

Major Test (2006-07)

Maximum Time: 2 Hours Maximum Marks: 50

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Attempt <u>all</u> questions. All carry equal marks.

No electronic gadgets (calculators, mobiles etc.) are allowed.

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(1) (a) **Find the volume of the body bounded by the surfaces** $z = 1 - x^2 - y^2$, $y = x$, $y = x\sqrt{3}$, $z = 0$ **and lying on the first octant.**

(b) **Using Gauss divergence theorem, change the following volume integral into a surface integral:**

$$\iiint_V \left[ f\nabla^2 g - g\nabla^2 f \right] dV,$$

**where** $f$, $g$ **are two scalar functions.**

(2) (a) **If** $f(z) = u(x,y) + iv(x,y)$ **is analytic at** $z_0 = x_0 + iy_0$, **then prove that** $f(z)$ **satisfies Cauchy-Riemann equations at** $z = z_0$.

(b) **Prove Cauchy's integral formula.**

(3) (a) **Find the Laurent series of the function**

$$f(z) = \frac{1}{z^3(1-z)}$$

**in the region** $0 < |z-1| < 1$.

(b) **Evaluate**

$$\oint_C \frac{dz}{z^2 - 1}$$

**where** $C$ **is the ellipse** $\frac{x^2}{4} + \frac{y^2}{9} = 1$ **followed in the counterclockwise direction.**

(4) (a) **Evaluate the following integral:**

$$\int_0^{2\pi} \frac{\cos\theta}{13 - 12\cos 2\theta}\, d\theta.$$

(b) **Find the Cauchy principal value of the integral**

$$\int_{-\infty}^{\infty} \frac{dx}{x(x^2+4)}.$$

(5) (a) **Evaluate**

$$\oint_C \frac{z^2}{(z-2)^3}\, dz,$$

**where** $C$ **is the circle of radius** $4$ **centred at** $0$ **followed in the counter-clockwise direction.**

(b) **Obtain the Fourier series of the following function:**

$$f(x) = \begin{cases} 1 + x^2, & \text{if } -\pi < x < 0, \\ 1 - x^2, & \text{if } 0 < x < \pi, \end{cases}$$

**and** $f(x) = f(x + 2\pi)$ **for all** $x$.