AML300 Constitutive Modeling and Application of New Materials
Major 2007, 2hours.

#1 Consider the structure shown in the figure. A rigid bar pinned at A is supported by composite rods. The rods are made of same material but the fibers in rod 1 are at 45° and in rod 2 at 90° to the principal material direction. The cross-section of both rods is 600 mm^2 and length is 3 m. E_1 = 140 Gpa, E_2 = 10 Gpa, G_{12} = 7 Gpa, ν_{12} = 0.3, $\alpha_1 = -2 \times 10^{-6}$ /K and $\alpha_2 = 28 \times 10^{-6}$ /K.

a) Calculate ν_{21}
b) What is E_x, α_x for rod 2.
c) Calculate E_x, α_x for rod 1.
d) If both rods undergo a temperature rise of 50 C determine the load vector due to temperature change.
e) Write the multi-point constraint between nodes 2 and 4.
f) Write the assembled stiffness and force matrices. Modify the stiffness and load matrix for the multipoint constraints.
g) The displacements at nodes 1, 3 are zero. By elimination method impose these boundary conditions to get 2 x 2 stiffness and 2x1 force matrix.
h) Find the displacement of node 2 and node 4. Determine the stresses in the members (taking temperature into account).

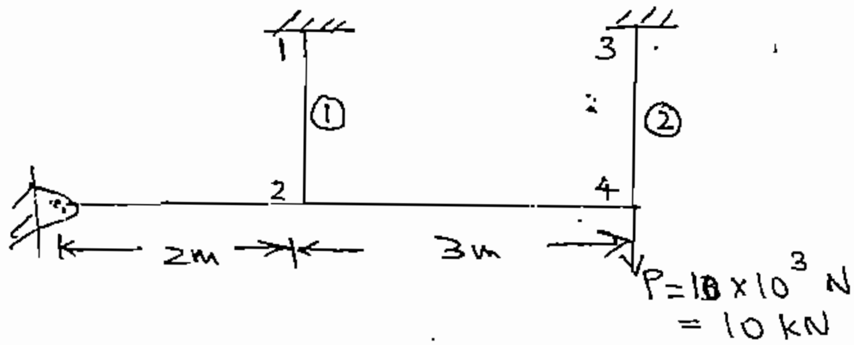

Fig 1.

#2. a) Show that for 3-parameter solid show in figure 2, the differential equation is

$$D\sigma + (\eta_2/E_2)\sigma = (E_1 + E_2)\,D\varepsilon + (E_1 E_2/\eta_2)\,\varepsilon \qquad \text{where } D = d/dt$$

b) Determine the creep compliance J?
c) For the strain history shown in figure 2, determine the response?

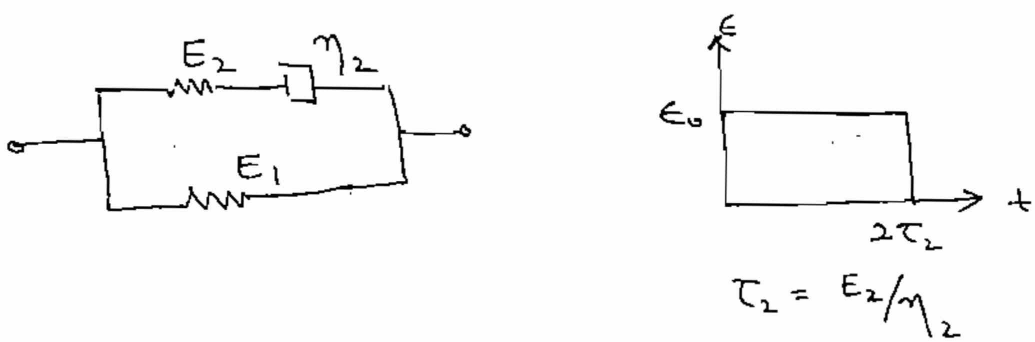

Fig (2)

#3. A block of Kelvin material (Kelvin in distortion, elastic in dilatation) is held in a container with rigid walls so that $\varepsilon_{22} = \varepsilon_{33} = 0$ when stress $\sigma_{11} = -\sigma_0\,[u(t)]$. Determine ε_{11} and relating stress component σ_{22} and σ_{33}. The walls are frictionless.

(a) Write the deviatoric stress s_{11} in terms of σ_{11} and $\sigma_{22}(=\sigma_{33})$.
(b) For linear Kelvin material

$$\tau = (2G + \zeta\, d/dt)\,\varepsilon$$

where τ is the shear and ε is the shear strain. What are the values of operators P^G and Q^G.
(c) Write the relation between s_{11} and e_{11} using the operators P^G and Q^G.
(d) Since the material is elastic in dilatation, use the relation $\sigma_{ii} = 3K\,\varepsilon_{ii}$ to write another equation between stresses and strain.
(e) Using equations found in steps (2) and (4) eliminate σ_{22}. Find ε_{11} as a function of time.

#4. Show that the apparent direct modulus of an orthotropic material as a function of 'θ' can be written as:
$E_1/E_x = (1+a-4b)\cos^4\theta + (4b-2a)\cos^2\theta + a$
Where $a = E_1/E_2$, $b = 1/4(E_1/G_{12} - 2\upsilon_{12})$
Hence show E_x is greater than E_1 and E_2 for the same values of θ if $G_{12} > E_1/(2(1+\upsilon_{12}))$

#5) a) For a Maxwell fluid $P(t) = 1+\eta/E$, $Q(t) = \eta\, d/dt$. For a cantilever beam subjected to a step load $F(t)$ at end $x=l$ and fixed at $x=0$ determine $w(t)$. For an elastic beam $w = Fl^3/3EI$.

b) If a traction force T Newtons/ meter acts on a 3-noded element of length L what is the ratio in which the load is divided between the nodes.

c) In a symmetric laminate, electrical resistance strain gauge rosettes with gauges oriented at 0, 45 and 90 relative to x direction are placed, on both top and bottom of the test specimen. The strains in the top rosette and bottom rosette are

0	8×10^{-4}	-2×10^{-4}
45	10.5×10^{-4}	1.5×10^{-4}
90	8×10^{-4}	0

Determine ε_0 and κ.

Some useful relations

$$\frac{1}{E_x} = \frac{m^2}{E_1}(m^2 - n^2\nu_{12}) + \frac{n^2}{E_2}(n^2 - m^2\nu_{21}) + \frac{m^2}{G_{12}}n^2$$

$$\varepsilon_{45} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\cos\theta\sin\theta$$

$$\alpha_x = m^2\alpha_1 + n^2\alpha_2$$

$$\frac{1}{s}\frac{b_0 + b_1 s}{a_0 + a_1 s} = \frac{b_0}{a_0}\frac{1}{s} + \left(\frac{b_1}{a_1} - \frac{b_0}{a_0}\right)\frac{1}{s + \frac{a_0}{a_1}}$$