MAJOR TEST

Maximum marks = 45   Time allowed = 2hrs

All questions compulsory

Q.1.   The deformation field for an isotropic elastic body is specified as follows:

$$x_1 = X_1 - \alpha X_2 X_3$$
$$x_2 = X_2 + \alpha X_3 X_1$$
$$x_3 = X_3$$

where α is a constant.

a)   Determine the ALMANSI strain tensor, e_{ij}.
b)   The isotropic elastic material composing the above body obeys the
      following constitutive equation in the component form:
      $$\sigma_{ij} = \beta_0 \delta_{ij} + \beta_1 e_{ij} + \beta_2 e_{ik} e_{kj}$$

      where β_0, β_1 and β_2 are functions of the invariants of the Almansi strain
      tensor e_{ij}.
      Determine the stress tensor field in the body.

                                                        9+9 = 18 marks

Q.2.   The constitutive equation for an isotropic hyperelastic solid undergoing small
        deformations is stated as

        $$W = W(\epsilon_{ij}) = M [I_1']^2 + N I_2'$$

        where W is the strain energy density and I_1' and I_2' given below are the first
        and second invariants of the small strain tensor ϵ_{ij}.

        $$I_1' = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$$

        $$I_2' = \epsilon_{11} \epsilon_{22} + \epsilon_{22} \epsilon_{33} + \epsilon_{33} \epsilon_{11} - \epsilon_{23} \epsilon_{32} - \epsilon_{31} \epsilon_{13} - \epsilon_{12} \epsilon_{21}$$

a)   Derive the general expression for the stress tensor.
b)   Determine the expressions for the material constants M and N in terms of
      the elastic constants of a conventional isotropic linear elastic solid.
                                                        6+3 = 9 marks

Q.3. Starting from first principles, prove the SUPERPOSITION PRINCIPLE for a KELVIN – VOIGT RHEOLOGICAL MODEL.

9 marks

Q.4. a) Discuss the PRINCIPLE OF MATERIAL FRAME INDIFFERENCE and its importance in constitutive modeling of materials.

b) Discuss the NON-ASSOCIATIVE FLOW RULE in the classical theory of elastoplasticity.

5+4 = 9 marks