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AML 812 Turbulence & Turbulent Shear Flows

Major: Take home exam. Due date 12th May 2007, 12.00 noon

Maximum marks: 50

Q1) Derive the transport equation governing the evolution of the tripe correlation of the fluctuating velocity $<u_i\ u_j\ u_k>$. A review of the procedure used to derive the equation for $<u_i\ u_j>$ may be useful. (10)

Q2) The longitudinal velocity structure function of order p is defined as $<(u(x + r) - u(x))^p>$. If 'r' lies in the inertial sub range then from Kolmogorov's hypotheses, it can be shown that the structure function depends only on the dissipation rate 'ε' and the scale 'r'.

i) Show that we must have, $<(u(x + r) - u(x))^2> = C_1\, \varepsilon^{2/3} r^{2/3}$ for $\eta << r << L$.

ii) Determine a similar expression for $<(u(x + r) - u(x))^3>$

iii) Hence evaluate the skewness, $<(u(x + r) - u(x))^3>/(<(u(x + r) - u(x))^2>)^{3/2}$ and show that it is independent of 'r' in the inertial sub-range. (5)

Q3) Smoke is being released from a chimney 40m tall and 0.5m in diameter at the exit. The velocity of the smoke at the exit is negligible. A fairly steady cross wind is blowing and at the height of the chimney exit the mean wind speed is 10m/s. The equivalent free-stream velocity at a height of 1km is approximately 50m/s. You may assume that the smoke trail is essentially horizontal with an initial diameter of 0.5m.

i) How far away from the chimney would the smoke be detected on the roof of a building 30m tall?

ii) If the inlet smoke concentration is C_o then what concentration would you expect at the top of the roof?

You may assume that the smoke is well mixed by the turbulence in the atmospheric boundary layer and that at the flow Reynolds number the friction velocity is roughly 1/30th of the free-stream velocity.

iii) If we now assume that the initial smoke temperature is such that its buoyancy is equivalent to that of air at 10°C above that of the ambient fluid then how would the answers to parts i) and ii) above change?

(A review of problem 2.3 in the text may be useful). (15)

Q4) Consider fully developed turbulent diffuser with plane walls. Estimate the opening angle of the diffuser for which the downstream pressure gradient is equal to zero. (10)

Q5) An aircraft of mass 10^4 Kg is in level flight at a constant speed of 100 m/s. The average width of the wings is 3m and the total span (both wings together) is 30m. The curvature of the wings is small enough for the wings to be treated as essentially flat plates. At the altitude of the plane the density of air, ρ, is 1Kg/m^3 and the kinematic viscosity, ν, is 10^{-5} m^2/s.

i) If the critical Reynolds number for transition on a flat plate is $Re_x = 5 \times 10^5$, at what distance from the leading edge of the wing does the flow become turbulent?

ii) If $u^*/U_o = 1/30$ in the turbulent region, what is the total skin friction force acting on the wings?

iii) If the engines are developing 2000kW in order to maintain this level flight, what is the drag force on the rest of the aircraft (other than the wings).

iv) During flight the wings get coated with fine dust. How large can the dust particles be for the wing to considered smooth?

v) Estimate the thickness of the boundary layer at the downstream edge of the wing? (10)

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