

ORII (MEL 868) MAJOR TEST
The method used is very important

April 2008

Q1. Arrivals are coming in at a counter with an Erlang (2) distribution having mean interarrival time as 6 minutes. The service distribution is exponential with rate 12 per hour. The maximum number of customers allowed in the system is 2. Find the percentage of customers lost by two different methods. Compare the results & explain if there is a difference. Also find the probability that the system is empty. (20)

Q2. A certain security price follows a geometric Brownian motion, with drift parameter $\mu = 0.05$ & volatility parameter 0.2. The present price of the security is 96.

- (a) If the interest rate is 6%, find the no-arbitrage cost of a call option that expires in 4 months & has an exercise price of 100.
- (b) What is the probability that the call option in part (a) will be exercised.
- (c) Consider an investment that for an initial cost B returns you 110 in 4 months if the price at that time is less than 80% of what it initially was but returns you 0 otherwise. What is the value of B for there to be no arbitrage. (10)

Q3. A company can be in one of three states Good (G), Medium (M) & Bad (B). It moves from G to G, M & B with probabilities 0.6 (10), 0.3 (5) & 0.1 (1), from M to G, M & B 0.2 (7), 0.7 (3) & 0.1 (0) & from B to G, M & B 0.1 (5), 0.1 (2) & 0.8 (-4). The rewards in Rs. associated with these movements are given in the brackets. The company is considering a proposal to do research when it is in any of the three states. This would change the probabilities & the rewards from G to G, M & B to 0.8 (7), 0.1 (3) & 0.1 (0) & from M to G, M & B to 0.5 (4), 0.4 (2), 0.1 (-2) & from B to G, M & B to 0.3 (2), 0.4 (1) & 0.3 (-10). Use the policy iteration method to find out what the company should do in the long run in each state. (10)

Q4. There are 3 servers & 2 customers move about among them. Upon completion of service at server i , the customer leaves that server & enters service at whichever of the other two is free (therefore there are always two busy servers). If the service times at the servers are exponential with rates, 3, 5 & 4 per hour what proportion of time is server 2 idle? (5)