MAJOR TEST

EEL721 Linear System Theory

Nov. 28, 2006
10.30 - 12.30

Q1 Consider the system

$$\underline{\dot{x}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \underline{x}$$

(a) Find the eigenvalues of $\underline{A}$ and from there determine the stability of the system
(b) Find the transfer function model and from there determine the stability of the system.
(c) Are the two results same? If not, why?

----- 10 marks

Q2 Consider the system

$$\underline{\dot{x}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & -1 \end{bmatrix} \underline{x}$$

Design a reduced-order state observer that makes the estimation error to decay at least as fast as e^{-10t}.

---- 10 marks

Q3 Figure Q3 shows the optimal control configuration of a position servo system. Both the state variables (angular position θ, and angular velocity $\dot{\theta}$) are assumed to be measurable.

It is desired to regulate the angular position to a unit-step function θ_r. The step is applied at $t = 0^+$.

(a) Find the optimum values of gains k_1 and k_2 that minimize

$$J = \int_0^\infty [(x_1 - \theta_r)^2 + u^2]\, dt$$

(b) Find the minimum value of J.

(c) Verify that the optimal closed-loop system is stable.

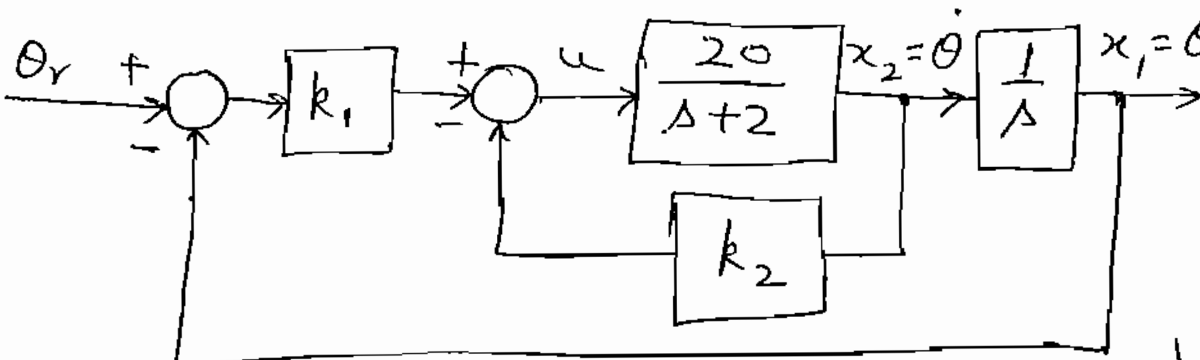

Fig. Q3 ---- 15 marks

Q4 Consider the completely controllable plant

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u}$$

where $\underline{x}$ is $n \times 1$ state vector, $\underline{u}$ is $p \times 1$ input vector; the null state $\underline{x} = \underline{0}$ is the desired steady-state. It is desired to find the control law

$$\underline{u} = -\underline{K}\, \underline{x}(t)$$

that minimizes the following performance index subject to the initial conditions $\underline{x}(0) \triangleq \underline{x}^0$:

$$J = \int_0^\infty (\underline{x}^T Q \underline{x} + \underline{u}^T R \underline{u})\, dt$$

Using Lyapunov function approach, develop the matrix Riccati equation that gives the solution to this optimal state regulator problem.

---- 15 marks