Major Examination
CRL 715: Radiating Systems for RF Communications

Marks: 35 (4+8+7+3+5+8) Time: 2 Hrs.

Q. 1: Consider a square waveguide opening with a side of dimension 'a' in an infinite perfectly electric conducting (PEC) xy-plane. The field distribution in the aperture is the superposition of two orthogonal TE_{10} modes. The modes are excited so that the radiated field has perfect right-hand circular polarization (RHCP) on the +z-axis.

a. Determine the aperture electric field.
b. Determine the aperture equivalent sources.

Q. 2: Design an E-plane horn such that the maximum phase difference between two points at the aperture (one at the center and the other at the edge) is 120°. Assuming that the maximum length along its wall ρ_e (measured from the aperture to its apex), is 10λ. The waveguide feeding the horn has dimensions of $a=0.5\lambda$ and $b=0.25\lambda$. Find

a. The maximum total flare angle of the horn,
b. The largest dimension of the horn at the aperture,
c. The directivity of the horn (dimensionless and in dB). See Appendix for the Fresnel integrals

Q. 3: Using the equivalence principle and cavity model, explain why some sides of a microstrip patch antenna are called 'radiating edges' and some other sides are called 'non-radiating edges'. Assume that the dominant mode within the cavity is TM^x_{010} for which the fields are given as:

$$E_x = E_0 \cos\left(\frac{\pi}{L} y'\right)$$

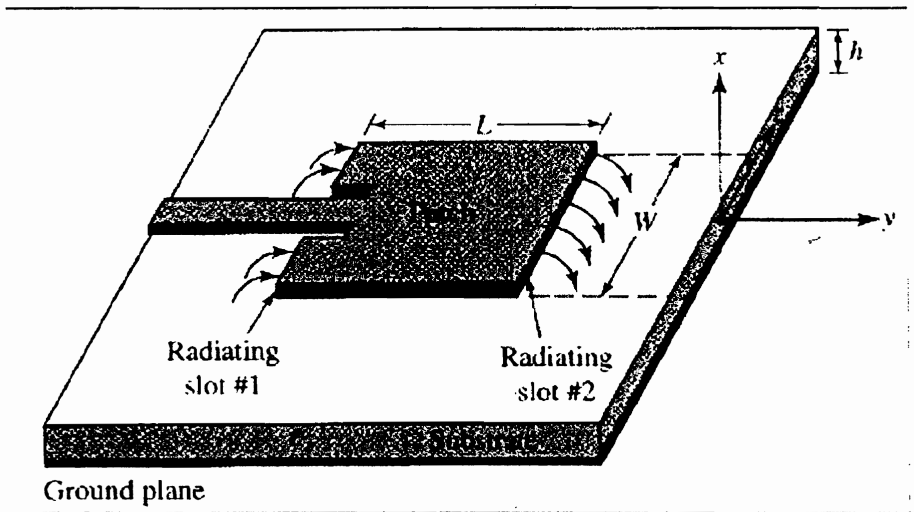
$$H_z = H_0 \sin\left(\frac{\pi}{L} y'\right)$$

$$E_y = E_z = H_x = H_y = 0$$

Q. 4: Explain with an example the co-polarized and cross-polarized radiation patterns?

Q. 5: Design a patch antenna working at the frequency 1.575GHz and etched on a substrate with $\varepsilon_r = 10$ and thickness h = 0.508 mm.

a. Determine W and L
b. Determine the inset length L_1 needed for matching of antenna to the microstrip feed line (50Ω).



Q. 6: A thin slot with the length L=0.5λ and the width w=0.1λ is cut in an infinite ground plane. Assume a coordinate system such that the conducting plane lies on the xy-plane with the larger dimension of the slot parallel to y-axis and assume that the width of the slot is small compared to the wavelength. The field distribution is constant over the slot and is given by:

$$\vec{E}_a = -E_0 \hat{a}_x$$

Draw the geometry indicating the electric field. Find the equivalent source. Sketch the six approximate principal plane patterns in the xz-plane, the yz-plane and the xy-plane. The radiated field are given by:

$$E_\theta = C\cos\phi \frac{\sin X}{X}\frac{\sin Y}{Y}$$

$$E_\phi = C\cos\theta\sin\phi \frac{\sin X}{X}\frac{\sin Y}{Y}$$

$$X = \frac{kw}{2}\sin\theta\cos\phi$$

$$Y = \frac{kL}{2}\sin\theta\sin\phi$$

Useful Formulae:

1. Directivity of an E-plane sectoral horn:

$$D_E = \frac{64 a \rho_1}{\pi \lambda b_1}\left[C^2\left(\frac{b_1}{\sqrt{2\lambda\rho_1}}\right) + S^2\left(\frac{b_1}{\sqrt{2\lambda\rho_1}}\right)\right]$$

2. Expressions for a patch antenna:

$$W = \frac{v_o}{2 f_r}\sqrt{\frac{2}{\varepsilon_r + 1}}$$

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-1/2}, \quad \frac{W}{h} > 1$$

$$\Delta L = 0.412\, h \frac{\left(\varepsilon_{reff} + 0.3\right)\left(\frac{W}{h} + 0.264\right)}{\left(\varepsilon_{reff} - 0.258\right)\left(\frac{W}{h} + 0.8\right)}$$

$$R_{in}\left(L_1\right) \approx \frac{1}{2G_1}\cos^2\left(\frac{\pi}{L} L_1\right)$$

$$G1 = \frac{1}{90}\left(\frac{W}{\lambda_0}\right)^2$$

Appendix A: Fresnel Integrals

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt$$

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)dt$$

x	$C(x)$	$S(x)$
0	0	0
0.0500	0.0500	0.0001
0.1000	0.1000	0.0005
0.1500	0.1500	0.0018
0.2000	0.1999	0.0042
0.2500	0.2498	0.0082
0.3000	0.2994	0.0141
0.3500	0.3487	0.0224
0.4000	0.3975	0.0334
0.4500	0.4455	0.0474
0.5000	0.4923	0.0647
0.5500	0.5377	0.0857
0.6000	0.5811	0.1105
0.6500	0.6219	0.1393
0.7000	0.6597	0.1721
0.7500	0.6935	0.2089
0.8000	0.7228	0.2493
0.8500	0.7469	0.2932
0.9000	0.7648	0.3398
0.9500	0.7760	0.3885
1.0000	0.7799	0.4383

x	$C(x)$	$S(x)$
1.0500	0.7759	0.4880
1.1000	0.7638	0.5365
1.1500	0.7436	0.5821
1.2000	0.7154	0.6234
1.2500	0.6801	0.6587
1.3000	0.6386	0.6863
1.3500	0.5923	0.7050
1.4000	0.5431	0.7135
1.4500	0.4933	0.7111
1.5000	0.4453	0.6975
1.5500	0.4018	0.6731
1.6000	0.3655	0.6389
1.6500	0.3388	0.5968
1.7000	0.3238	0.5492
1.7500	0.3219	0.4994
1.8000	0.3336	0.4509
1.8500	0.3584	0.4077
1.9000	0.3945	0.3733
1.9500	0.4391	0.3511
2.0000	0.4883	0.3434
2.0500	0.5374	0.3513