CSL 665: Introduction to (Logic and Functional) Programming
I semester 2008-09

Major Mon 24 Nov 2008 WS 101 10:30-12:30 Max Marks 50

---

1. Use the rules of inference for the polymorphic lambda-calculus to obtain the *most general polymorphic type assignment* for the function `times` defined below. (*Hint: Simply ignore exception conditions and the* `if-the-else` *construct, and concentrate on* `f o (times (f, n-1))`). The type of "o" is given to be

$$\forall\alpha\forall\beta\forall\gamma[(\alpha \to \beta) * (\gamma \to \alpha) \to \gamma \to \beta]$$

```
fun id (x) = x;

exception negative_counter;

fun times (f, 0) = id
|   times (f, n) = if n < 0 then raise negative_counter
                   else f o (times (f, n-1))
```

5 marks

2. The following algorithm (written in ML) was used by Russian peasants in the eighteenth century to multiply two positive integers.

```
exception Negative
fun russian (x, n) =
    if n < 0 then raise Negative
    else if n = 0 then 0
    else
let fun even m = (m mod 2 = 0);
          fun double y = y + y;
in russian (double(x), n div 2) +
   (if even (n) then 0 else x)
end
```

In terms of complexity it requires about $O(log_2 n)$ additions to compute $x * n$.

(a) Write a polymorphic version of this algorithm to repeatedly apply an associative binary operation in a monoid to an element of the monoid.

(b) Use your polymorphic version to define simple functions to compute

i. x^n for any real number x and non-negative integer n,

ii. the n-th power (for non-negative integers n) of a square matrix of real numbers, where the matrix is represented as a list of lists of numbers.

$4 + (3 + 3) = 10$ marks

3. Let `derivative(Y, X, Z)` denote that the derivative of `Y` with respect to `X` is `Z`. Given the following facts:

```
derivative(N, X, 0).
derivative(X, X, 1).
derivative(sin(X), X, cos(X)).
derivative(cos(X), X, -sin(X)).
derivative(exp(X), X, exp(X)).
derivative(log(X), X, 1/X).
```

write logic programming rules to compute derivatives of

(a) sum of two expressions

(b) difference of two expressions

(c) product of two expressions

(d) quotient of two expressions

In particular make sure your definitions can calculate the derivatives of expressions such as $3 * exp(5 * X) * sin(2 * X) + cos(X)/log(X)$ $4 \times 4 = 16$ marks

4. Translate the following argument into First order logic and prove it using resolution.

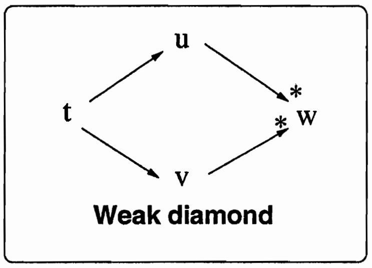
   Whoever visited the building was observed. Anyone who had observed Ajay, would have remembered him. Nobody remembered Ajay. Therefore Ajay did not visit the building.

9 marks

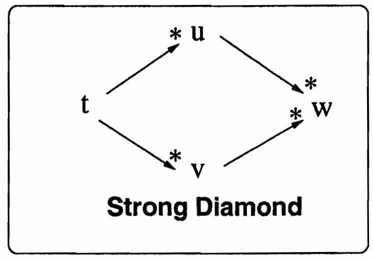
5. Let Σ be a 1-sorted signature (with at least one constant symbol) and let T_Σ be the set of all Σ-terms. Let $\longrightarrow \subseteq T_\Sigma \times T_\Sigma$ be a rewriting relation with N_Σ the set of normal forms. Assume that T_Σ satisfies the following properties.

   (a) Every term in T_Σ may be reduced to a normal form

   (b) $\longrightarrow$ satisfies the following *weak diamond property*: For any three terms $t, u, v \in T_\Sigma$, $t \longrightarrow u$ and $t \longrightarrow v$ implies there exists a term $w \in T_\Sigma$ such that $u \longrightarrow^* w$ and $v \longrightarrow^* w$ (see figure).



Weak diamond



Strong Diamond

Then prove that it satisfies the *strong diamond property*: For any three terms $t, u, v \in T_\Sigma$, $t \longrightarrow^* u$ and $t \longrightarrow^* v$ implies there exists a term $w \in T_\Sigma$ such that $u \longrightarrow^* w$ and $v \longrightarrow^* w$ (see figure).

10 marks