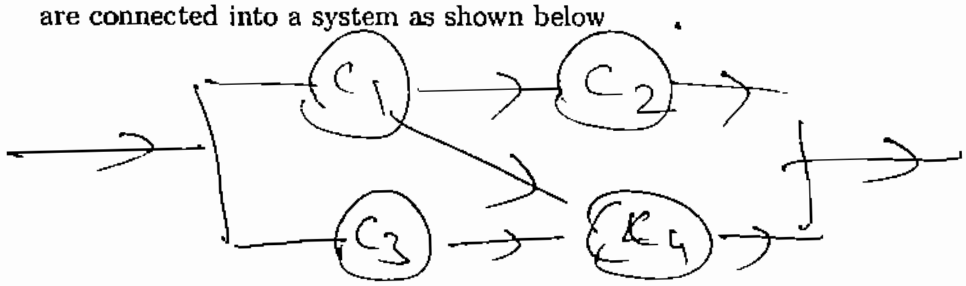
Department of Mathematics
MAL250 (Probability and Stochastic Processes)
Major Test

Time: 2 hours **Marks: 50**

1. Four identical components $C_i, i = 1, 2, \ldots, 4$ functioning independently, are connected into a system as shown below



Assume that the life time of each component has exponential distribution with parameter λ. Find the probability that the system functioning until time t? (5 marks)

2. Let X be uniformly distributed random variable on the interval $(0, 1)$. Define

$$Y = a + (b - a)X, \quad a < b$$

Find the distribution of Y. (5 marks)

3. Let X and Y have joint pdf

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{\pi}, & x^2 + y^2 \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Determine the marginal pdf's of X and Y. Are X and Y ~~are~~ independent?
(2 + 2 + 1 marks)

4. A telephone network switch works until one of its five components fails. The life times (in months), $X_i, i = 1, 2, \ldots, 5$, of these components are independent and each has the Weibull distribution with pdf

$$f(x) = \begin{cases} \frac{2x}{25} \; e^{-(\frac{x}{5})^2}, & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that the switch stops working in the first eight months?
(4 marks)

5. Let X and Y be i.i.d. random variables each having uniform distribution on the interval $(-\pi, \pi)$. Let $Z(t) = \cos(tX + Y), \quad t \geq 0$. Is $\{Z(t), t \geq 0\}$ covariance stationary. Justify your answer. (5 marks)

6. Let $X_n, \quad n = 1, 2, \ldots$ be a sequence of independent identically distributed random variables with

$$P(X_1 = k) = \begin{cases} p, & k = 1 \\ \\ 1-p, & k = -1 \end{cases} \qquad , 0 < p < 1$$

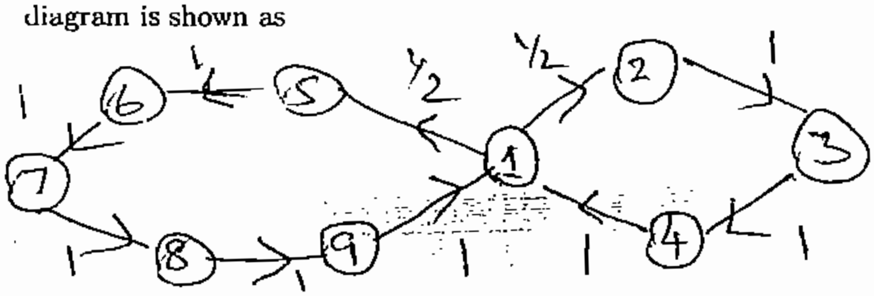
Define

$$S_n = \sum_{i=1}^{n} X_i \quad \text{for} \quad n = 1, 2, \ldots \quad \text{and} \quad S_0 = 0.$$

Show that $\{S_n, \ n = 0, 1, \ldots\}$ is a DTMC. Also, find $P(S_n = j/S_0 = i)$ for $n = 1, 2, \ldots$.

(4 + 3 marks)

7. Consider a DTMC with state space $S = \{1, 2, \ldots, 9\}$. The state transition diagram is shown as



Classify the states of the chain as transient, +ve recurrent or null recurrent. Find the probability that the system visit to state 1 at exactly n step given initial state 1. Also find the expectation of this distribution.

(2 + 2 + 1 marks)

8. Consider a system with two non identical processors working independently. Assume that the Poisson arrival of jobs with rate λ and the service time is exponentially distributed with rates μ_1 and μ_2 ($\mu_1 > \mu_2$) for two processors respectively. Each job requires exactly one processor for its execution and the sheduling policy is FCFS. When both processors are idle, the faster processor is scheduled for service before the slower one. Model this system as a CTMC $\{X(t), t \geq 0\}$ where $X(t) = (i, j)$ where $i \geq 0$ denotes the number of jobs in the queue, including any at the faster processor and $j \in \{0, 1\}$ denotes the number of jobs at the slower processor. Draw the state transition diagram. (4 marks)

9. Consider a parallel redundant system with two identical components. Assume that the time to failure of a component is exponentially distributed with mean $\frac{1}{\lambda}$ and is independent with other components. The system has a single repair facility. Assume that the time to repair of a component is exponentially distributed with parameter μ. The system fails when both the components fail and no recovery is possible. Model this system as a BDP and draw the state transition diagram. (3 marks)

10. Describe $M/M/1/2$ queueing system. Write down the forward Kolmogorov equations for the above system. Derive the steady state probabilities.

(3 + 2 + 2 marks)