<u>**Department of Physics**</u>

PHL551: Classical Mechanics | **I Semester 2008-09** | **Major**

Please answer all questions | **Maximum Marks 50** | **Time: 2 Hours**

1. Find the moment of inertia tensor for a uniform lamina in the form of an equilateral triangle of side a and mass density m per unit area about its centre of mass. Find the axes in which this tensor is diagonal, you need not diagonalise it if you can give reasons as to why the axis you state above must be the principal axis. **8.**

2. A particle moves in a potential of the form $-\dfrac{K\cos\theta}{r^2}$. Is this a central force if the quantities have their usual meaning in spherical polar coordinates? Write down the Hamiltonian for this problem and find the conserved quantities. Write the Hamilton Jacobi equation for this problem and separate it. **8.**

3. A simple pendulum of length l and mass m is suspended from the bob (lower end) of an identical pendulum. Given that both pendulums oscillate in one plane. Find the normal modes of this system for small oscillations. **6.**

4. A solid of revolution rotates with uniform angular velocity Ω about the Z-axis which is its symmetry axis. It is subject to a torque $\vec{N} = (0,\ N\cos\omega t, 0)$. Are there any conserved quantities? If so, find them. Solve the equations of motion, using the change of independent variable $u = f(t)$ if needed. **6**

5. Find the generating function for the canonical transformation, $Q = \dfrac{\alpha p}{x}$, $P = \beta x^2$. Are there any conditions on α and β? **6.**

6. A uniform right circular cone of height h, half angle α and density ρ, rolls on its curved side without slipping in such a manner that it returns to its original position in a time τ. Find expressions for the kinetic energy and the components of the angular momentum of the cone. Hence, write down the Lagrange equations of motion for the system. **8**

7. Are the following statements true or false, give brief reasons for your answer (one or two sentences)

(i) As every particle in a scattering experiment is either scattered or captured, the differential scattering cross-section can never be infinite. **2**

(ii) The Hamilton Jacobi equation is not applicable if the Hamiltonian is time dependant as it makes the transformed Hamiltonian zero. **2.**

(iii) Every coordinate transformation is a canonical transformation of the form $F(q,P) = f(q)P$. **2.**

(iv) A satellite in Earth orbit fires its rocket engine for a short time in a direction perpendicular to the plane of its orbit. The new trajectory will not be a conic section. (Ellipse, parabola, hyperbola etc.) **2.**

Formulae:

$L = T - V$, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$, $p = \frac{\partial L}{\partial \dot{q}_i}$. **Or** (Method of undetermined multipliers)

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$, with $\sum_i \lambda_i f_i = 0$, use $Q_i = \sum_j \left\{ \lambda_j \left[\frac{d}{dt}\left(\frac{\partial f_j}{\partial \dot{q}_i}\right) - \frac{\partial f_j}{\partial q_i} \right] - \frac{\partial \lambda_j}{\partial t}\frac{\partial f_j}{\partial \dot{q}_i} \right\}$.

$\sigma(\theta) = \frac{s}{\cos\theta}\left|\frac{ds}{d\theta}\right|$ $\quad l = s\sqrt{2mE}$, $\quad \theta = \pi - 2\int_{r_m}^{\infty} \frac{dr}{r^2\sqrt{r^2\left(\frac{2mE}{l^2} - \frac{2mV(r)}{l^2} - \frac{1}{r^2}\right)}}$

Orbit equation for Kepler's law $V = -\frac{k}{r}$, $\frac{1}{r} = \frac{\mu k}{l^2}(1 + \varepsilon\cos\theta)$, $\varepsilon = \sqrt{1 + \frac{2El^2}{\mu k^2}}$.

The Euler equations of motion are

$$\begin{aligned} I_x\dot{\omega}_x - \omega_y\omega_z(I_y - I_z) &= N_x \\ I_y\dot{\omega}_y - \omega_x\omega_z(I_z - I_x) &= N_y \\ I_z\dot{\omega}_z - \omega_y\omega_x(I_x - I_y) &= N_z \end{aligned}.$$

$I_{xx} = \int \rho(r^2 - x^2)dxdydz$ $\quad I_{xy} = \int(\rho xy)dxdydz$, $\quad$ Hamiltonian: $H = \sum p_i\dot{q}_i - L$

Canonical Transformations $\quad F_1 = F(q,P,t)$, $p = \frac{\partial F}{\partial q}$, $Q = \frac{\partial F}{\partial P}$,

$F_2 = F(q,Q,t)$, $p = \frac{\partial F}{\partial q}$, $P = -\frac{\partial F}{\partial Q}$, $\quad F_3 = F(p,Q,t)$, $q = -\frac{\partial F}{\partial p}$, $P = -\frac{\partial F}{\partial Q}$,

$F_4 = F(p,P,t)$, $q = -\frac{\partial F}{\partial p}$, $Q = -\frac{\partial F}{\partial P}$,.

Hamilton Jacobi Equation $\frac{\partial S}{\partial t} + H(q, \frac{\partial S}{\partial q}, t) = 0$.