Department of Mathematics

Indian Institute of Technology Delhi

MAL517: Differential Equations

Major examination July-Nov 2008 Max.Marks:50

[7+7+7+7+10+8+9]

1. Consider the equation $y'' + a_1(t)y' + a_2 y = 0$ where $a_1(t), a_2(t)$ are continuous periodic functions of period T. Then show that a non-trivial solution $y(t)$ is periodic of period T if and only if $y(0) = y(T)$ and $y'(0) = y'(T)$.

2. If $P_n(t)$ is a Legendre polynomial, then show that
$$\int_{-1}^{1} P_n^2(t)dt = \frac{2}{2n+1}$$

3. Determine e^{tA} or the fundamental Matrix Φ with $\Phi(0) = I$ of the system $x' = Ax$ where A is given by
$$\begin{pmatrix} 2 & 0 & 0 \\ 3 & 0 & 3 \\ 4 & 3 & 0 \end{pmatrix}$$

4. Find eigenvalues and eigenfunctions of the Sturm Liouville Problem:
$$\begin{aligned} x'' + \lambda x &= 0, \qquad 0 \le t \le \pi \\ x(0) = 0 \quad & x'(\pi) = 0 \end{aligned}$$

5. Using Picard iteration method, show that the following IVP admits one and only one solution
$$x'' + \lambda^2 x = f(t,x),\ x(0) = 0,\ x'(0) = 1$$
where $f(t,x)$ is a continuous function on $S_a = \{(t,x) : |t - t_0| \le a, |x| < a, a > 0\}$ and satisfies Lipschitz condition
$$|f(t,x_1) - f(t,x_2)| \le K|x_1 - x_2|,\ \forall (t,x_1),(t,x_2) \in S_a.$$

6. Suppose $f(x)$ is a continuous function on [0,1]. Show that the following BVP admits one and only one solution:
$$y'' = f(x),\ y(0) + y(1) = 0,\ \ y'(0) + y'(1) = 0$$

7. Test the Stability properties of the following systems:
(i) $x_1' = 2x_1 + 8\sin x_2,\ \ x_2' = 2 - e^{x_1} - 3x_2 - \cos x_2$
(ii) $x_1' = -4x_2 - x_1^3,\ \ x_2' = 3x_1 - x_2^3$
(iii) $x_1' = -6x_2 - \frac{1}{4}x_1 x_2^2,\ \ x_2' = 4x_1 - \frac{1}{6}x_2.$