Department of Electrical Engg., IIT Delhi (

EEL862, Selected Topics in Comm Engg. Major MM5

Prob. 1 Consider the cyclic subgroup G of S_3 generated by $\rho = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ (i) Find the matrices associated with natural representation. (Assume the basis to be $\mathcal{B} = \{e_1, e_2, e_3\}$ the standard basis.

(ii) Consider $T : V_3(\mathbb{C}) \longrightarrow V_3(\mathbb{C})$ such that $[T]_{\mathcal{B}} = \begin{bmatrix} 2 & 0 & 5 \\ 0 & 4 & 1 \\ 3 & 6 & 1 \end{bmatrix}$

Find a $\tilde{T} : V_3(\mathbb{C}) \longrightarrow V_3(\mathbb{C})$ from T which is G-linear.

(iii) Write down as many irreducible. space as you can. (16)

Prob. 2 Let $\mathcal{F}(3) = \{f \mid f : \{1, 2, 3\} \longrightarrow \mathbb{C}\}$ be set of functions on the set $\{1, 2, 3\}$ (i) Find the constraint on the kernel of linear operator $R : \mathcal{F}(3) \longrightarrow \mathcal{F}(3)$, such that R intertwines (considering the left regular representation on G = the cyclic subgroup generated by ρ given in Prob. 1. (12)

(ii) For the same group $G \leq S_3$, find the inner product, in appropriate space, which gives rise to unitary representation. (Note: Part (ii) has nothing to do with

Prob. 3 Let $V = V_1^1 \oplus V_2^1 \oplus \cdots \oplus V_{c_1}^1 \oplus V_1^2 \oplus \cdots \oplus V_{c_2}^2 \oplus \cdots \oplus V_1^p \oplus \cdots \oplus V_{c_p}^p$ with usual notation. Show that $\dim_{\mathbb{R}} \mathrm{Hom}_G(V, V) = \sum_{i=1}^{p} c_i^2$. (8)

Prob. 4 Let $G = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \mid ad \neq 0 \text{ and } a, b, d \in \mathbb{R} \right\}$

(i) Show that G is a group under

multiplication

(2)

(ii) Let $N = \left\{ \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \,\middle|\, b \in \mathbb{R} \right\}$ Show that $N \trianglelefteq G$, i.e. N is normal in G. (10)

(iii) Show that G/N is Abelian.

<u>Prob. 5</u> Consider a λ-Tableau t (here the symmetry group is taken to be S_5) given as:

$$\begin{array}{|c|c|c|}\hline 3 & 5 & 2 \\ \hline 1 & 4 \\ \cline{1-2}\end{array} \qquad \lambda = (3,2,0,0,0). \qquad (8)$$

(i) Write all elements of C_t.

(ii) Write A_t, the Young Operator in Full.

(iii) Let $\tilde{t}$ and s be λ-Tableau & μ-Tableau respectively given as:

$$\tilde{t}\;:\; \begin{array}{|c|c|c|}\hline 3 & 2 & 5 \\ \hline 1 & 4 \\ \cline{1-2}\end{array} \qquad s\;:\; \begin{array}{|c|c|}\hline 3 & 1 \\ \hline 5 & 2 \\ \hline 4 \\ \cline{1-1}\end{array}$$

Find $A_t e_{[\tilde{t}]}$ and $A_t e_{[s]}$.