EEL 316 Major Test Semester II 2006-2007

Answer all questions (Q.1: 12, Q.2: 28, Q.3: 30) Full Marks: 70

1. (a) An analog signal is sampled, quantized, and encoded into a binary PCM wave. The number of representation levels used is 16. A synchronizing pulse is added at the end of each codeword representing a sample of the analog signal. The resulting PCM wave is transmitted over a channel of bandwidth 15 kHz using a quaternary PAM system with raised cosine spectrum. The rolloff factor is 0.5.

   i. Find the rate (in bits/sec) at which information is transmitted through the channel. [4]

   ii. Find the rate at which the analog signal is sampled. What is the maximum possible value for the highest frequency component of the analog signal? [4]

   (b) A binary PAM wave is to be transmitted over a baseband channel with an absolute maximum bandwidth of 75 kHz. The bit duration is 10 μs. Find the rolloff factor of the raised cosine spectrum which satsifies these requirements. [4]

2. Consider the case of BFSK in which signals $s_1(t)$ and $s_2(t)$ (corresponding to symbols '0' and '1', respectively), given by

$$s_1(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t\right) , \quad s_2(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi [f_c + f_d] t\right) ,$$

are *transmitted with equal apriori probabilities* over an AWGN channel with noise p.s.d. of $N_0/2$ over a symbol interval of $0 \le t < T_s$, where $f_c \gg \frac{1}{T_s}$ and $f_d < \frac{1}{T_s}$. Let an orthonormal basis for the signal space be $\{\phi_1(t), \phi_2(t)\}$, where

$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) , \quad 0 \le t < T_s .$$

   (a) Find $\phi_2(t)$ using Gram-Schmidt orthogonalization. [6]

   (b) Find the signal vectors $\underline{s}_1$ and $\underline{s}_2$ and draw the signal constellation, labelling the relevant portions. [6]

   (c) If coherent detection with a MAP receiver is performed, then find the SEP $P_{e,coh}$ in terms of E_s, T_s, N_0, f_d. Find the value of f_d for which $P_{e,coh}$ is a minimum. Calculate $P_{e,coh,min}$ (the minimum value of $P_{e,coh}$) when $\frac{N_0}{2} = \frac{E_s}{30}$. [10]

   (d) What is the minimum value of f_d for the signaling to be orthogonal? If noncoherent detection is performed with orthogonal signaling, then calculate the SNR E_s/N_0 in dB so that the symbol error probability $P_{e,noncoh}$ equals $P_{e,coh,min}$ found in (c). [6]

3. The requirement of a communication system using M-ary PSK signaling with $M > 2$ (**M is a power of** 2) and equal apriori probabilities over an AWGN channel is that the SEP $P_e \leq 10^{-7}$ at an SNR $E_s/N_0 = 20$ dB.

   (a) Calculate $M_{max,PSK}$, the maximum value of M that can be used. What is the approximate value of P_e for $M = M_{max,PSK}$ when $E_s/N_0 = 15$ dB? [6]

   (b) Find the union bound on P_e for $M = \frac{M_{max,PSK}}{2}$ as a function of E_s/N_0. [4]

   (c) If the system switches to noncoherent M-ary orthogonal FSK, then what is the minimum SNR in dB at which the target SEP of 10^{-7} will be reached for $M = M_{max,PSK}$? [4]

   (d) If the system switches to M-ary PAM, then what is $M_{max,PAM}$, the maximum value of M at which the SEP is $\leq 10^{-7}$ at an SNR of 20 dB? [6]

   (e) If the system switches to M-ary square QAM, where $M = M^2_{max,PAM}$, then calculate the SEP at an SNR of 20 dB. [6]

   (f) Find the bandwidth efficiencies in (a), (c), (d), and (e). [4]

Some formulae

- If $X \sim \mathcal{N}(0,1)$, then its p.d.f.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty, \qquad \text{and} \quad \Pr[X > x] = \int_x^\infty f_X(y)dy = Q(x) = 1 - Q(-x)$$

- $\operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \text{if } |t| \leq \frac{T}{2}, \\ 0 & \text{if } |t| > \frac{T}{2}, \end{cases} \qquad \operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$

- Fourier Transform pairs:

$$\operatorname{rect}\left(\frac{t}{T}\right) \leftrightarrow T \operatorname{sinc}(fT), \qquad \exp(j2\pi f_0 t) \leftrightarrow \delta(f - f_0), \qquad G(t) \leftrightarrow g(-f)$$

- Use the approximation $Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad x \geq 2.5,$ wherever applicable.

- $P_e = \frac{1}{\pi} \int_0^{\frac{\pi(M-1)}{M}} \exp\left(-\frac{E_s}{N_0} \frac{\sin^2\left(\frac{\pi}{M}\right)}{\sin^2\phi}\right) d\phi$

- $P_e = \frac{(M-1)}{M} \operatorname{erfc}\left(\sqrt{\frac{d^2 E}{N_0}}\right), \quad \frac{E_{av}}{E} = \frac{(M^2-1)}{3} d^2$

- $P_e = \sum_{i=1}^{M-1} \frac{(-1)^{i+1}}{(i+1)} \binom{M-1}{i} e^{-\frac{i}{(i+1)} \frac{E_s}{N_0}}$

- $P_e \approx \operatorname{erfc}\left(\sqrt{\frac{E_s}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)$ for large SNR, 4 and higher

- Values of sinc function:
  sinc(1.37) $= -0.2132$, sinc(1.40) $= -0.2162$, sinc(1.43) $= -0.2172$, sinc(1.46) $= -0.2163$, sinc(1.49) $= -0.2135$