Deptt. of Elec. Engg. I.I.T. Delhi

EEL768, Detec. & Est. Theory, Major Exam. MM.35

MAKE ASSUMPTIONS, IF REQUIRED.

Prob. 1: Let $y = \sum_{i=1}^{N} x_i$, where x_i's are i.i.d. $N(0,\sigma^2)$ (i) Find ML estimate of N. (treating it as a continuous variable); (ii) Is it unbiased; (iii) What is the error variance?; (iv) Is it efficient? (7)

Prob. 2: Consider the Cauchy distribution $p_{r/A}(R) = \frac{1}{\pi(1+(R-A)^2)}$ (

Assuming n independent observations

(i) Find ML estimate of A. (ii) Find lower bound of any unbiased estimator in this case; (iii) Is the sample estimate of (i) consistent?

Prob. 3: (a) Let $y[n] = \sum a_k y[n-k] + w[n]$; $n = 0, 1, 2 \cdots K$. Find Linear minimum variance estimate of parameters. (Assume $w[n]$ zero mean i.i.d. Is it unbiased. (7)

(b) Given the record $0 \le n \le K$, also give the expression for the least-squares estimation of the parameters.

Prob 4 (a) Let stochastic process $X(t), 0 \le t \le T$, be expressed as linear combination of CON set $\{\phi_k(t), k = 1, 2, 3 \cdots\}$. Find conditions of the CON set so that coefficients along the basis are uncorrelated. (7)

(b) Taking the basis found in (a), show that

$$X(t) = \sum_{k=1}^{\infty} a_k \phi_k(t),$$ in the mean square sense

Prob. 5 Let $r(t)|_{H_1} = \sqrt{E_1}\, s_1(t) + n(t)$ $\quad 0 \le t \le T \quad \int_0^T s_i^2(t)\,dt = 1$ for $i = 0,$

& $r(t)|_{H_0} = \sqrt{E_0}\, s_0(t) + n(t)$

$n(t)$ is zero mean, white Gaussian process with $R_n(\tau) = \sigma^2 \delta(\tau)$

Further let $\rho = \int_0^T s_1(t)\, s_0(t)\, dt$, i.e correlation between s_0 & s_1

Find optimum receiver structure which takes into (7) account all variations of $r(t)$ (rather than few samples). For this receiver write the Probability of error, in ~~modulated~~ PSK