PHL556 : MAJOR EXAMINATION (2007)

Attempt all questions. Max. Marks : 45

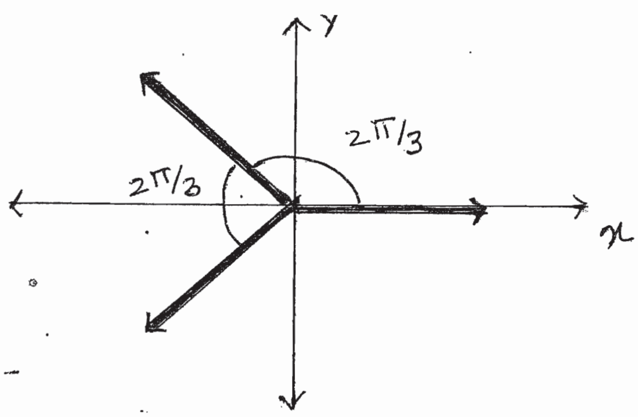
1. On probability distributions -

   (a) Consider a Bernoulli process describing an event which occurs n times in N trials with a probability p. Work out the distribution for the limiting case of small p, with Np remaining finite. (3)

   (b) Atoms are deposited at random onto a surface until the resulting film is 6 atoms thick on average. Thinking in statistical terms, a trial in this system involves depositing an atom of radius b (and area $\sim b^2$) onto the surface of area L^2. Naturally, $b^2 \ll L^2$. What fraction of the substrate is not covered by the metal? What fraction of the metal surface is 6 atoms thick? (3)

   (c) Now answer this question for a bonus of 2 marks - what do you conclude about the nature of the film surface? Any comments or suggestions regarding deposition of "smooth films"?

2. It is possible to describe the behavior of certain materials in nature with magnetic moments m which can point in three possible directions, all in one plane, as depicted in the figure below.



   (a) A paramagnet whose constituents are spins of the type described above is at temperature T. There is no interaction between the spins. Find is partition function when an external magnetic field is applied along the x direction. (3)

   (b) Find the average magnetization per spin from the partition function. (2)

3. On the Ising model -

   (a) Show that the average magnetization m for a q-dimensional Ising ferromagnet is given by m = $tanh\beta(Jqm + B)$, where symbols have the usual meaning. (4)

   (b) Show that the Curie Weiss law viz. $\chi_T \sim \frac{1}{|T-T_c|}$ is obeyed by this system. (3)

   (c) Argue that a 1-dimensional Ising model does not show a phase transition **or** argue that the 2-dimensional Ising model exhibits a non-trivial phase transition. (3)

4. On the Van-der Waals system -

   (a) Depict the Van-der Waals isotherms in the PV - plane schematically clearly pointing out the the critical point. Comment on the unphysical behavior of the isotherms. Qualitatively explain the removal this problem by introducing the concept of phase co-existence and appropriate thermodynamics. (3)

   find (b) ~~Show that~~ the isothermal compressibility which is defined as $\frac{1}{\chi_T} = -\frac{\delta P}{\delta V}|_T$. (3)
   You may need $P_c = \frac{a}{27b^2}$, V_c = 3b and $RT_c = \frac{8a}{27b}$.

   (c) Comment on the universal behavior of the Ising system and the Van-der Waals system for a bonus of 2 marks.

5. On statistics - (a) is compulsory, you may choose between (b) and (c).

   (a) Show that the Fermi energy of an intrinsic semiconductor lies in the middle of the band gap. (Assume that the mass of the electron is equal to the mass of the hole). (4)

(b) Compute the Fermi energy of the extrinsic semiconductor obtained by doping Germanium with Indium (resulting in a p-type semiconductor), at the rate of 3.7 X 10^{22} atoms/m^3 at room temperature. The band gap of Germanium is 0.72 eV, while the acceptor levels of Indium are 0.012 eV above the valence band. At room temperature, kT $\approx$ 0.025 eV, and $2(\frac{2\pi mkT}{h^2})^{3/2} = 2.5$ X 10^{25}. (3)

(c) Obtain the condition under which the two quantum statistics reduce to the classical Maxwell Boltzmann statistics. (3)

(You may need (i) $g(\epsilon)d\epsilon = \frac{8\pi\sqrt{2}m^{3/2}V}{h^3}\sqrt{\epsilon}\, d\epsilon$ and $\int_0^\infty \sqrt{x}e^{-x}dx = \frac{\sqrt{\pi}}{2}$.)

6. On non-equilibrium studies -

(a) Starting with the "master equation", show that the continuum limit of a one-dimensional random walk is nothing but diffusion. (3)

(b) Write down the Langevin equation of a Brownian particle, explaining the various terms and mentioning the significance and properties of the noise term. (3)

(c) Get 2 bonus marks by explaining the statement "Diffusion equation and the Langevin equation are equivalent descriptions of Brownian motion"

7. Assorted questions (answer any 5) - (5)

(a) The schematic sketch of the density ρ as a function of temperature T as a transition from a solid to a liquid state is observed on crossing the coexistence curve of a one-component system looks like ....

(b) The phase diagram depicting the normal phase and the condensed phase in an ideal bose gas as a function of temperature looks like ......

(c) Equivalence of ensembles breaks down when ....

(d) We saw that the Debye model mimics the variation of the specific heat versus temperature of solids very impressively. The physical picture of a solid in my mind is ....

(e) A Fermi system is highly energetic even at T = 0 K because ....

(f) Calculation of the Fermi energy is important because ....

Here is further chance to score **more than 45** : the promised recovery problems. Solve both if you need a grade improvement.

(a) Show that the fluctuations in the magnetization M of a paramagnet at temperature T are related to the magnetic susceptibility $\chi = \frac{1}{N}\frac{d<M>}{dB}$. Here, N and B refer to the number of spins and the magnitude of the applied field respectively. Assume the energy of the microscopic state E_i as $E_i = M_i$B, forgoing the angular dependence. (3).

(b) A gas molecule of mass m is in thermodynamic equilibrium at temperature T. The velocity of the gas molecule is $\mathbf{v} = (v_x, v_y, v_z)$. Using a fully classical description, calculate :

i. the normalized probability P($\mathbf{v}$)dτ in velocity space, (2)

ii. $< v_x^2 >$ and check whether the equipartition theorem holds for this single particle. (2)

You may require $\int_{-\infty}^{\infty} e^{-\alpha x^2}dx = (\frac{\pi}{\alpha})^{\frac{1}{2}}$; $\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}dx = \frac{1}{2}(\frac{\pi}{\alpha^3})^{\frac{1}{2}}$