MAL 110 MATHEMATICS-I: MAJOR TEST

Max Marks: 50 Max Time: 2 Hours

This question paper consists of 10 questions on two pages.

All questions are compulsory and carry equal marks.

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1. Find the general solution of the differential equation

$$(1-x^2)y'' - 2xy' + 2y = 0\,, \quad |x| < 1.$$

2. Find the general solution of the differential equation

$$x^3y''' + 3x'' + xy' + y = x\ln x\,, \quad x > 0.$$

3. Let V be the set of all 2×2 matrices with complex entries and W be the set of all skew-Hermitian matrices in V.

   (a) Assuming V is a vector space over R, determine the dimension of V over R.

   (b) Is W a vector subspace of V? If yes, determine the dimension of W over R.

   Justify your answers.

4. Determine the total surface area of the solid generated by revolving about $x-axis$, the total region bounded by the curve given by

$$(x^2+y^2)^2 = x^2 - y^2.$$

5. Determine the Laplace transform of the function

$$f(t) = t\int_0^t x^{10}e^{-5x}dx.$$

6. Let $V = R^3_R$, and let $B = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \right\}$.

(a) Prove that B is a basis for V over R.

(b) Determine the linear transformation T on V whose matrix with respect to the ordered basis B is

$$\begin{pmatrix} -3 & -7 & -8 \\ 0 & 2 & 0 \\ 3 & 4 & 7 \end{pmatrix}.$$

7. Is the matrix

$$A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{pmatrix}$$

diagnolizable? Justify your answer.

8. Evaluate $\int_1^2 x^4\, dx$, using the definition of the integral as the limit of a sum, i.e. by the Riemann integration.

9. A shopping plaza is triangular in shape with vertices $(-1, 0)$, $(0, 1)$ and $(1, 0)$. An ATM is to be installed at a point $P(x, y)$ in the interior of the plaza so that the sum of the squares of its distances from the three corners is least. However, due to certain physical restrictions, the location of the ATM has to be on the street represented by the straight line $x - 3y + 1 = 0$. Determine the location of the ATM using the method of the Lagrange multipliers.

10. Let a surface S be given by $F(x, y, z) = 0$, where the function F possesses nonzero first order partial derivatives with respect to each of the variables x, y, z . Determine the value of

$$\left(\frac{\partial y}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial x}{\partial z}\right)$$

at an arbitrary point $P_0(x_0, y_0, z_0)$ on S.