Department of Mathematics

Major Test: MAL 522

Max. Marks: 50 Time: Two Hours

(Marks awarded will be restricted to 50)

1. Let $X_{(i)}$ be the i-th order statistics of a random sample of size n from an absolutely continuous distribution function F(x). Show that $\frac{n-i+1}{i}\frac{F(X_{(i)})}{1-F(X_{(i)})}$ has a F-distribution with (i , n-i+1) degrees of freedom. (6)

2. Let m_r' be the r-th sample moment about the origin of a size n random sample from any population. Assuming all population moments exist, show that $\sqrt{m_r'}$ is the CAN estimator for $\sqrt{\mu_r'}$,the corresponding population value. (5)

3. Show that $\frac{n}{n+1}\overline{X}^2$ is UMVUE for $1/\theta^2$ for exponential pdf $f(x;\theta) = \theta e^{-\theta x}$, $x>0$, where $\overline{X}$ is the sample mean based on a random sample of size n. Also compute its efficiency. (7)

4. Prove the invariance property of a MLE. (5)

5. State and prove Neyman-Pearson lemma for testing a simple null against a simple alternative hypothesis. (6)

6. Find a UMP test of size α for the hypothesis $H_0: \theta \geq \theta_0$, against the alternative $H_1: \theta < \theta_0$,using a size n sample from the pdf $f(x;\theta) = \theta/x^2$, $x > \theta$. (6)

7. Define an invariant hypothesis testing problem and a maximal invariant statistics. Prove that a test ϕ for a hypothesis is invariant iff it is a function of a maximal invariant statistics. (6)

8. Find α size likelihood ratio test for the hypothesis $H_0: \mu_1 = \mu_2$, against $H_1: \mu_1 \neq \mu_2$, for the means of two normal populations $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, based on two independent random samples one from each of sizes m and n respectively. (6)

9. Based on a random sample of size n from normal population $N(4, \sigma^2)$, find a uniformly most accurate 95% upper confidence bound for σ^2. (6)