EEL 731 MAJOR / SCDR / 02-12-2006 / FULL MARKS=100 / TIME=2 HOURS

Answer <u>any six</u> questions and no more. Should you answer more, answer to the last question will be cancelled. All questions carry equal marks, of course !

Q. 1 In the second lecture on FIR lattice, I committed a mistake by saying that $k_a^{(N)}k_b^{(N)} = 1$ leads to $a_N^{(N)}a_0^{(N)} = b_N^{(N)}b_o^{(N)}$. The actual relationship is $a_N^{(N)} / a_0^{(N)} = b_N^{(N)} / b_o^{(N)}$. Hence the remedy proposed in the class, viz. multiplying one of the transfer functions by 2^q, q = positive or negative integer, will not work. Suggest a simple solution to the problem and apply it to derive a lattice structure for the two transfer functions : $H_2(z) = 1 + 0.1z^{-1} + z^{-2}$ and $G_2(z) = 1 + 0.3z^{-1} + z^{-2}$.

Q. 2 Derive a lattice structure for realizing the transfer function

$$H(z) = (1 - 0.5z^{-2}) / (1 + 0.2z^{-1} + 0.4z^{-2}).$$

Q. 3 The input output relation of a digital system is given by

$$y(n) = \begin{cases} (1/n)\sum_{l=1}^{n} x(l), & n > 0 \\ 0, \quad n \le 0. \end{cases}$$

Determine whether the system is linear or nonlinear, time-invariant or time varying, causal or non-causal and stable or unstable. Determine the impulse and step responses of the system. Give a recursive scheme for computing *y(n)*.

Q. 4 (a) The signal $x(n) = u(n) - u(n-N)$ is applied to an FIR filter with the transfer function $H(z) = 1 + 2z^{-1} + 3z^{-2} + \ldots + Nz^{-(N-1)}$. Find the value of n at which the output $y(n)$ will have the largest value. Find also this largest value.

(b) Find the sequence x(n) whose Fourier transform is $(1 - e^{jN\omega}) / (1 + e^{j\omega})$, where N is even.

Q. 5 (a) Derive the Fourier transform of $x(n) = 1$ for all n.

(b) Find the sequence $x(n)$ whose z-transform is $(1 - z^{-2})^{-1}$, $|z| < 1$.

(c) The first three impulse response samples of an FIR linear phase filter are $h(0) = a$, $h(1) = b$, and $h(2) = c$. . Determine the transfer function of the lowest order if the filter is of (I) type 1, (ii) type 2, (iii) type 3 and (iv) type 4.

Q. 6 (a) Given the transfer function

$$H_1(z) = (1+3z^{-1})(1-2z^{-1}) / [(1-0.25z^{-1})(1+0.5z^{-1})],$$

determine another three different transfer functions which have the same magnitude as that of $H_1(z)$ on the unit circle in the z-plane.

(b) The standard second order band-pass filter

$$H_1(z) = \frac{(1-\alpha)}{2} \frac{1-z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}$$

is cascaded to the standard second order band-stop filter

$$H_2(z) = \frac{(1+\alpha)}{2} \frac{1-2\beta z^{-1}+\alpha z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}.$$

Sketch the resulting magnitude characteristic. Show that the frequencies of maximum response can be found by solving a quadratic equation in $\cos\omega$.

Q. 7 (a) A second order digital bandpass filter with centre frequency 0.4π and 3 dB bandwidth 0.2π is available as the prototype, from which another digital bandpass filter with centre frequency 0.5π and the same 3 dB bandwidth is to be obtained by transformation. Find the transformation function.

(b) The frequency response of an ideal notch filter is required to be of the form

$$H_N(e^{j\omega}) = \begin{cases} 1, & 0 < \omega < \omega_0 \\ -1, & \omega_0 < \omega \leq \pi \end{cases}$$

Express this in terms of the frequency response of an ideal zero-phase low-pass filter with cutoff at ω_0. Hence find the impulse response of the notch filter.