# Finite Element Methods
## AML 705/706
## Department of Applied Mechanics

Major Test – I | Max. Time: 2 hour | Max. Marks: 80

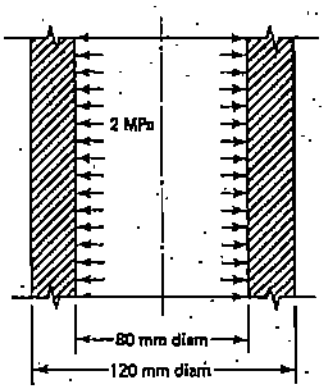
1.

(I) A long cylinder of inside diameter 80 mm and outside diameter 120 mm snugly fits in a hole over its full length. The cylinder is then subjected to an internal pressure of 2MPa. Draw and indicate the geometry and BCs if the problem is analyzed using elements given below. Also write the D matrix for each case.

a.) Axi-symmetric (2-D)

b.) Plane strain. (2-D) (Clearly mention the BCs on outer surface).

c.) 1-D axi-symmetric element (13)




(II) For plane strain axisymmetric problem a 1-D element with two nodes and one degree of freedom can be used. Derive the 'B' matrix for this element. (7)

#2.

The dynamics equation for a system are

$$\begin{bmatrix} 312 & 0 & 54 & -13 \\ 0 & 8 & 13 & -3 \\ 54 & 13 & 312 & 0 \\ -13 & -3 & 0 & 8 \end{bmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_4 \\ \ddot{u}_5 \\ \ddot{u}_6 \end{Bmatrix} + 420 \begin{bmatrix} 12 & 0 & -6 & 3 \\ 0 & 4 & -3 & 1 \\ -6 & -3 & 12 & 0 \\ 3 & 1 & 0 & 4 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

a) With $u_4$ and $u_6$ as slaves find the R matrix

b) Write the K and M used in calculating $R^T K R$ and $R^T M R$. (10)

#3 For a damped system subject to a force $R^{ext}(t)$ derive the fotlowing central difference relation. Also, suggest how the solution is started. The terms have usual meanings. (10)

$$\left[\frac{1}{\Delta t^2}M + \frac{1}{2\Delta t}C\right]\{u\}_{n+1} = \{f^{ext}\}_n - [K]\{u\}_n + \frac{1}{\Delta t^2}[M]\left(2\{u\}_n - \{u\}_{n-1}\right) + \frac{1}{2\Delta t}[C]\{u\}_{n-1}$$

#4

a.) Consider a shaft supported at 'A' (coordinate x=0 ) by a bearing of radial stiffness 20 kN/mm and rotational stiffness 10 N/μm. At 'B' (coordinate x =1 m) the bearing has only rotational stiffness of 15 N/μm. The shaft is subjected to a uniformly distributed transverse load of 10 kN/m (in + y direction) in x-y plane. Write the stiffness matrix, given that shaft has circular cross section of diameter 20 mm and length of 1 m. E=200 x $10^9$ N/$m^2$, υ=0.3. Use only one two noded beam element. Assume bending in x-y plane only.

b.) Determine also the torsional stiffness matrix of the above shaft (The dof corresponding to torsion are 5 and 6)

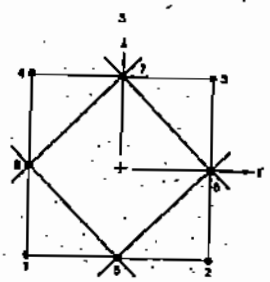
c.) If bearing at 'B' also has a torsional stiffness of 5N/μm (besides rotational stiffness given in (a)) what is the modification in stiffness matrix given in (b)

d) Write the bending stiffness matrix of the shaft in x-z plane. Also write the load matrix if the transverse load in x-z plane is 10 N/m (in –z direction) in x-z plane. The degrees of freedom are 7.8.9.10. (20)

#5.

a.) Derive the terms $M_{11}$ of the mass matrix for a triangular element (4)

b.) Write the shape function $N_1$ and $N_8$ for an 8-noded quadrilateral element shown in figure (4)



c.) For 3-noded 1-D element with central node displaced L/4 from centre, determine the Jaeobin matrix. L is the length of the element. (7)

d.) In the equation [M] {ü} +[K]{u} ={F}
If F is measured in Newtons, u in mm, ü in mm/$s^2$, what are the units of M ?
If volume is in $mm^3$ what are the units of density?
If density of steel is 7800 kg/$m^3$ what is its value in these units? (5)

D matrix for 3-D

$$D = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5-\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5-\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5-\nu \end{bmatrix}$$

stiffness matrix of beam in x-y plane

$$k' = \frac{EI}{\ell_e^3}\begin{bmatrix} 12 & 6\ell_e & -12 & 6\ell_e \\ 6\ell_e & 4\ell_e^2 & -6\ell_e & 2\ell_e^2 \\ -12 & -6\ell_e & 12 & -6\ell_e \\ 6\ell_e & 2\ell_e^2 & -6\ell_e & 4\ell_e^2 \end{bmatrix}$$