PHL558: Major Test
05.05.07

All Q's are compulsory

Q1. a) A grating with (15,000 lines per inch) is illuminated by sodium light. The grating spectrum is observed on the focal plane of a convex lens of focal length 10 cm. Calculate the separation between the D_1 (5890 Å) and D_2 (5896 Å) lines.

b) Consider a circular aperture of diameter 2 mm illuminated by a plane wave of wavelength λ_0 to observe Fresnel diffraction. The most intense point on the axis appears at a distance of 200 cm from the aperture. Calculate λ_0.

5+5

Q2 a) Calculate the cross correlation between two *rect* functions of identical width '*a*' and make a sketch of the result.

b) Suppose a given aperture produces a Fraunhofer field pattern $g(x,y)$. Show that if the aperture's dimensions are altered such that the aperture function changes from $A(x,y)$ to $A(\alpha x,\beta y)$, the newly diffracted field would be given by $\left[\frac{1}{\alpha\beta} g\left(\frac{x}{\alpha},\frac{y}{\beta}\right)\right]$

c) Consider an object $f(x)$ placed in the front focal plane of a lens, at whose spatial frequency filter plane a spatial filter of form $T(u) = \alpha u$ is placed. Show that such a filtering process would result in edge enhancement in the image of the object.

5+4+4

Q3. a) A plane wave is assumed to fall on a lens; the emergent wave from the lens is allowed to illuminate an object having transmittance $g(x,y)$, which is placed at a distance of $(f - d)$ to the right of the lens; f being the focal length of the lens. Calculate the field distribution on the back focal plane of the lens.

b) What is a phase contrast microscope? Explain its working principle through appropriate algebra.

5+5

Q4. a) Consider formation of a hologram from the interference of two plane waves in the *xz* plane, one of which is assumed to be traveling along the *z* axis as the reference wave. Show that during reconstruction one would get back the primary wave and its conjugate.

b) Draw schematic diagrams to explain i) formation of a Fourier Transform hologram of an object transparency $f(x,y)$ of width b and ii) image reconstruction from it. In the image plane of F.T. holography one gets a sum of four terms given by $f(x.y) \circledast f(x,y) + A^2\delta(x)\delta(y) + Af(-x-a,-y) + Af^*(x-a,y)$; where a represents the offset distance of the reference source used during formation of the hologram and the other symbols having their usual meaning. With respect to your drawn diagram for the image reconstruction step, explain the meaning of each term and hence show that in order to make primary and conjugate images appear separate, one must satisfy the condition $2a \geq 3b$.

3+4+3+2

Q5 a) Define guided mode of an optical waveguide.

b) Write down the wave equation satisfied by the modes of an asymmetric

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slab/planar waveguide (having core, cover, and substrate refractive indices as n_1, n_c and n_s ($> n_c$), respectively and of width d for the core) in the core and in the cover and the substrate.

c) Express modal solutions of the wave equations in part b) above and schematically draw the field distributions corresponding to the three lowest order guided modes with respect to cross-section of the waveguide.

d) Taking the complete expression for the modal field distribution in the core, physically interpret the meaning of a guided mode in terms of superposition of two plane waves and hence interpret physically the meaning of a *mode cut-off*.

e) Consider a symmetric planar waveguide with $n_1 = 1.5$, $n_2 = 1.48$, and $d = 3.912$ μm. Find out which of the TE modes that would be supported by this waveguide at $\lambda_0 =$ 1 μm; justify your answer with appropriate calculations and arguments.

2+2+4+4+3