Department of Electrical Engineering
EEL823, Discrete Time Systems,
Major Test, *2006-2007/I.*

Max. time : 2 hours, Max. marks: 80.
Marks: Q1: 15, Q2: 11, Q3: 10, Q4: 12, Q5: 10, Q6:12 , Q7:10

> ***Write clearly each step of your calculation.***

Q1. Consider a system shown in Fig. 1:

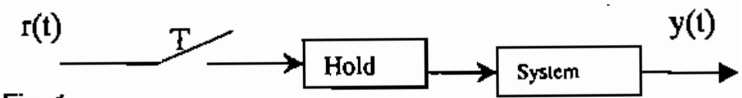

Fig. 1

(a) Suppose G(s) is the system transfer function and hold is a first order hold device. Express the pulse transfer function in terms of G(s) and other information.

(b) Suppose hold device is a zero order hold and G(s) = 1/s. Determine the pulse transfer function?
(c) Find the magnitude and phase difference between the output and input when r[k]=sin[k]

Q2. Consider a nonlinear system described by

$$\dot{x}_1 = -x_2$$

$$\dot{x}_2 = x_1^3 - x_2 + u$$

(a) Find the operating point corresponding to nominal input $u_0 = 0$? Also derive a linearized model near this operating point?
(b) Suppose this linearized model is discretized (smapling interval 1 sec) using trapezoidal rule with a zero order hold. Determine the discrete time state variable model.

Q3. Consider a single input controllable and observable discrete time system which has two state space representation (A,B,C) and $(\underline{A},\underline{B},\underline{C})$. The controllability matrix corresponding these two realization are W_c, $\overline{W}_c$ and the observability matrix are W_o, $\overline{W}_o$ respectively.

(a) Express the matrix $W_c W_o$ in terms of $\overline{W}_c \overline{W}_o$.
(b) Show that the eigenvalues of the $W_c W_o$ and $\overline{W}_c \overline{W}_o$ are same?
(c) Explain briefly the relationship between *lack of controllability/observability* and *pole-zero cancellation*?

Q4. (a) Given a continuous time system (A,B), the ZOH equivalent at sampling interval T is given by (F , G). Show that

$$e^{MT} = \begin{bmatrix} \Phi & \Gamma \\ 0 & 1 \end{bmatrix}, \text{ where } M = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}$$

(b) Suppose a discrete time system $(\Phi \quad \Gamma)$ is given where

$$\Phi = \begin{bmatrix} 1.8 & 1 \\ 0.85 & 0 \end{bmatrix}, \Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Design a deadbeat state feedback controller for this system.

Q5. Consider a discrete time systems

$$X_1(k+1) = 2\,X_1(k) + 0.5\,X_1(k) - 5$$

$$X_2(k+1) = 0.8\,X_2(k) + 2$$

(a) Analyse the asymptotic stability of the equilibrium state using Lyapunov stability theory.
(b) How the contraction mapping is defined?

Q6. (a) Fig.2 shows a control systems. Express K in terms of polynomials to track a unit step input.
(b) Suppose $A(z) = z^2 - 2z + 1$ and $B(z) = 0.0\,2z + 0.02$. What will be the Sylvester matrix to solve Diophantine equation.

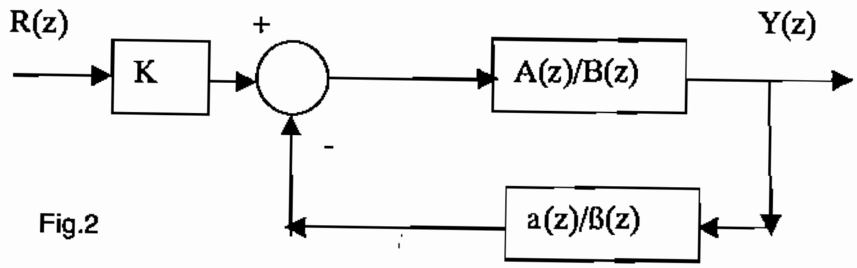

Fig.2

Q7. (a) Figure 3 shows a time delay system. Design a Smith predictor for this system to compensate the effects of time delay. Justify your answers.

(b) Explain briefly the different components of micro-controller. Also state a scheme to generate a PWM output.

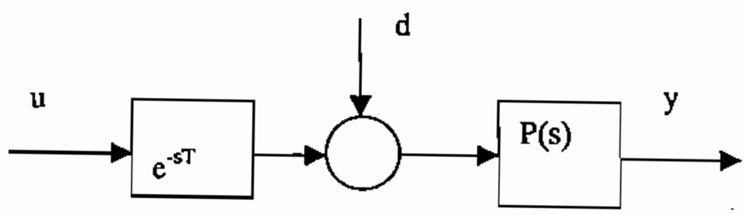

Fig.3

d: disturbance
u: control input