Deptt. of Elec. Engg., I.I.T. Delhi.

EEL768, Detec. & Est. Theory, Major Exam, M.M. 35

MAKE NECESSARY ASSUMPTIONS IF REQUIRED

Prob 1 (a) Consider the following binary hypothesis testing problem: $p_{r_i|H_0}(R_i) = 2e^{-2R_i} u(R_i)$ & $p_{r_i|H_1}(R_i) = 4e^{-4R_i} u(R_i)$

$1 \le i \le N$. Find N such that $P_M \le 0.1$ $P_F \le 0.1$.

(b) Show the following (i) $E\{\Lambda^n | H_1\} = E\{\Lambda^{n+1} | H_0\}$

(ii) $E[\Lambda | H_1] - E[\Lambda | H_0] = Var\{\Lambda | H_0\}$ (7)

Where Λ is Likelihood ratio.

Prob 2 Let $p_{r|A_1, A_2}(R) = (2\pi A_2)^{-1} \exp\left\{-\frac{(R-A_1)^2}{2A_2}\right\}$ (7)

① Find ML estimates of A_1, A_2 by using n independent observations. ② Are they biased; ③ Find error covariance matrix.

Prob. 3 Consider the non-parametric hypothesis testing (7) problem, where we are given that Prob$\{r_i > 0 | H_0\} = \frac{1}{2}$ and Prob$\{r_i < -5 | H_1\} = \frac{3}{4}$. Develop the Wilcoxon test for $N = 5$ observation and $P_F \le 0.125$.

Also compute the P_F achieved.

Prob. 4(a) Find relation between $P[V_{1:n}]$ & $P[V_{1:n+1}]$ and also the corresponding inner product update formula. (7)

(b) Find update formulas for $\Delta_{0,r}$ and forward & backward errors.

Prob. 5 (a) Let the state $x(k)$ is to be estimated from data $y(n)$ upto time k. Find the expression for smoothed estimate of $x(k)$ from the predicted estimate. (7)

(b) Let $y = \sum_{i=1}^{n} x_i$, where x_i's are i.i.d. $N(0, \sigma^2)$ and 'n' is r.v. with Prob$\{n = k\} = \frac{\lambda^k}{k!} e^{-\lambda}$ $k = 0, 1, \ldots$ If $H_1 : n \le 1$