**Department of Physics**
**EPL202**
**MAJOR TEST**

**Full Marks: 40** | **Date: 29.04.2008**
**Time: 2 hours**

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*Attempt all questions*

Q1. Write brief and logical answers:

(a) Show that for a degenerate (two-fold) state, if an Hermitian operator commutes with H and $H'$ and the eigenstates are simultaneous eigenfunctions of this operator, the degenerate calculation reduces to ordinary first order perturbation theory.

(b) A spectroscopist found a triplet Zeeman lines under strong magnetic field. The lower state is already identified as $^2S_{1/2}$. What would be the upper state and why?

(c) Prove the statement ' Determinate states are eigenfunction of any operator'.

(d) A beam of electrons is split into two and passed either side of a long solenoid (carrying a steady current I) before being recombined. Even if the beams are kept well away from the solenoid, a clear interference effect is seen in an electron detector-

Justify this observation.

(e) 'The expectation value of the Hamiltonian in an arbitrary state is greater than or equal to the ground state energy'- prove the statement. (5 x 2 = 10)

Q2. (a) A particle of charge q and mass m, which is moving in a one-dimensional harmonic potential of frequency ω, is subject to a weak electric field ε; (i) calculate the energy upto first non-zero correction and (ii) show that it matches well with the expression of energy (from exact calculation) if the Hamiltonian is written in a proper form. (3+2)

(b) In case of a two-dimensional oscillator [ $H_0 = \frac{1}{2m}\left(p_x^{\,2} + p_y^{\,2}\right) + \frac{1}{2}k\left(x^2 + y^2\right)$], the energy of the first excited state has two-fold degeneracy. Considering a perturbation of the form $H^1$ = axy, show the removal of degeneracy. What are the 'good' unperturbed states? (4+1)

Q3. (a) In order to calculate the ground state energy of H atom, a theoretician uses two trial wave functions

$\psi^1 = a(1+\alpha r)e^{-\alpha r}$ and $\psi^2 = be^{-\alpha r^2/2}$. Which one is the better? Justify. (5)

(b) Consider a particle of mass m that is bouncing vertically and elastically on a smooth reflecting floor in the Earth's gravitational field,

V(z) = mgz for z > 0 and

= ∞ for z ≤ 0 Find the energy levels using WKB method. (4)

(c) What is the physical reason behind 'Hyperfine splitting'? (1)

Q4.(a) A particle is initially ($t<0$) in its ground state in a one-dimensional harmonic oscillator potential. At $t = 0$ a perturbation $V(x,t) = V_0 x^3 e^{-t/\tau}$ is turned on. Calculate to first order the probability that, after a sufficiently long time ( i.e. $t \rightarrow \infty$), the system will have made a transition to a given excited state; consider all final state. (4)

(b) Let $|\psi> = c_1 |\psi_1> + c_2 |\psi_2>$ is an eigenket, where $|\psi_1> = \pi^{-1/2} \sin x and |\psi_2> = \pi^{-1/2} \sin 2x$ respectively. Consider an operator $D = -\frac{d^2}{dx^2}$ operated on $|\psi>$ and gives another eigenket $|\phi>$. Represent this operation diagrammatically in a vector space. (2)

(c) In general, in case of degeneraey (say, two-fold), how do you represent the eigenvectors in Hilbert space? (2)

(e) Why partial wave analysis in not applieable in case of scattering of a particle under coulomb potential? (2)

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