# Department of Physics, IIT Delhi

**II Semester 2006-07 Time 2 hours Maximum marks = 50+5,**
**Major: EPL202 Quantum Mechanics and its applications**

1. If $[a, a^\dagger] = 1$, $[b, b^\dagger] = 1$ $[a, b] = 0$ and $[a, b^\dagger] = 0$. Find all the possible commutation relations between $ab^\dagger$, $a^\dagger b$ and $a^\dagger a - b^\dagger b$. 8.

2. A system with unperturbed Hamiltonian $H_0 = J^2$ and unperturbed eigenstates $|j, m>$ is subject to a perturbation $H_1 = cJ_x{}^2$, where $c$ is a constant. Find the change in the eigenvalues and eigenstates to the second order. 12.

3. Given that the ground state configurations of Hydrogen ($Z = 1$) is ${}^2S_{1/2}$ and Helium ($Z = 2$) is ${}^1S_0$, find the possible configurations for $Z = 4$ and $Z = 5$. 10.

4. Solve the Schroedinger equation in the Born approximation for the potential

$$\begin{aligned} V &= k|\vec{r}|, \ |\vec{r}| < a \\ &= 0, \ |\vec{r}| > a. \end{aligned}$$

8.

5. A system is described by the 1-d potential

$$\begin{aligned} V &= kx^2, \ |x| < a \\ &= kx^2 + lx^3, \ |x| > a. \end{aligned}$$

Here $k, l$ and $a$ are constant. When can the eigenstates of this system have definite parity and why? 2

6. The Bloch equations for a two level atom are

$$\frac{\partial u}{\partial t} = -\omega v, \ \frac{\partial v}{\partial t} = \omega u + \Omega w, \ \frac{\partial w}{\partial t} = -\Omega v.$$

Find the steady state solutions of this system if $\Omega$ and $\omega$ are constant. When will the population inversion of this system be a maximum, and what is the polarisation of the medium then if $n_0$ is the number density of these atoms? 10.

**Answer the two questions on the topic on which you submitted your assignment. Are the statements given below true or false? Give reasons, no reasons = no marks** 5.

1. **Cooper Pairs**(a)As the electrons in a Cooper pair always have opposite $\vec{p}$, the pair as a whole can never move. (b) Cooper pairing is destroyed in a strong magnetic field

2. **EPR** In an EPR experiment with photons, one photon is passed through a vertical polariser while its twin passes through a horizontal polariser when they start. After some distance one of them is passed through a right circular polariser to convert it into circularly polarised light. (a) The two photons are no longer completely entangled. (b) The other photon immediately becomes left circularly polarised.

3. **Quantum Hall effect** The quantum hall effect is (a) a property of metals alone and not semiconductors. (b) As it is a property of electrons, even a low energy electron beam propagating in a vacuum will exhibit it

4. **Quantum Computation** (a) The accuracy of an ordinary computer can be improved by increasing the number of bits used to represent the data, this is not possible for a quantum computer dne to the uncertainty principle. (b)A quantum computer is fast as it evaluates all the possibilities in a conditional statement together, but it may not give the right answer because of the probabilistic nature of the result

5. **Quantum Cryptography**(a) While "eve" cannot remove photons from a single photon stream without "Alice" and 'Bob" becoming aware of it, she can add photons into the stream to prevent them exchanging a key without their becoming aware of this intruder. (b) it is not enough for Alice and Bob to exchange their sequence of results to generate the key but the times of transmission are also required.

6. **Josephson Junction**(a) As it is the lattice which causes the pairing, the junction between the superconductors must contain an insulator and cannot be a vacuum. (b) If both sides of a Josephson junction are identical, no Josephson current will be observed if they are at the same voltage.

7. **Single Electron Transistor**(a) As a band structure is required, the device will not work if the gate is separated from the rest of the device by a vacuum. (b) As the blockade is a capacitive effect, the characteristics are unchanged in the presence of a magnetic field.

8. **Squeezed states**(a) Measurement of the unsqueezed variable in a squeezed state will destroy the squeezing.(b) Squeezing is due to the uncertainty principle and can occur in any system.