# Advanced Quantum Mechanics (PHL 744)
## (Major Test)

Max. Marks: 50

1. Write down the relativistic wave equation (Klein-Gordon equation) starting from the relativitistic energy-momentum relation.
   Derive the equation of continuity and discuss the problems encountered with this relativistic equation.

   [10]

2. Examine the influence of the charge conjugation ($\psi_c = C\gamma^0\psi^*$) on an electron at rest with positive energies (in both spin up and spin down states).

   [4]

3. Starting from the Maxwell's equations
   $\partial_\nu F^{\mu\nu} = j^\mu$ and, $\partial^\lambda F^{\mu\nu} + \partial^\mu F^{\nu\lambda} + \partial^\nu F^{\lambda\mu}=0$, with $F^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu$, $j^\mu = (\rho, \vec{j})$
   derive the two Maxwell's equations corresponding to the divergence
   and the curl of the magnetic field, $\vec{B}$.
   Electric and magnetic fields:
   $E^i = F^{0i}$, $B^i = \frac{1}{2}\epsilon^{ijk}F^{jk}$.

   [8]

4. Under a Lorentz transformation ($x'^\mu = a^\mu{}_\nu x^\nu$), Dirac wave function transforms as $\psi(x) \rightarrow \psi'(x') = S(a)\psi(x)$. From the form invariance of the Dirac equation, find the contraint the matrix $S(a)$ must satisfy.

   [6]

5. Dirac equation in the presence of an external electromagnetic field is given as
   $i\hbar\frac{\partial\psi}{\partial t} = \left(c\vec{\alpha}\cdot(\vec{p} - \frac{e}{c}\vec{A}) + \beta mc^2 + e\phi\right)\psi$.
   Show that in the non-relativistic limit, this reduces to the Pauli equation.

   [10]

6. For a complex scalar field, $\phi$, derive an expression for the Feynmann propagator,
   $$i\Delta_F(x' - x) = \langle 0|T(\phi(x')\phi^\dagger(x))|0\rangle,$$
   by using the field operator expansions for $\phi$ and $\phi^\dagger$ and the commutation relations of the creation and annihilation operators. Show that the above obtained experssion is equivalent to the integral
   $$i\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m^2 + i\epsilon}e^{-ik\cdot(x'-x)},$$
   where $\epsilon \rightarrow 0_+$.

   [12]