Department of Physics. I.I.T., Delhi
Major Examination
PHL556: Statistical Mechanics

All questions are compulsary

Time: 2 hours
Full Marks: 50

Date: 06-05-2010

1. Consider a quantum ideal gas. Besides the discreteness of the energy levels, which other properties of the gas must be taken into account for the derivation of quantum distribution function? Consider the $k-th$ energy level of the system with n_k number of particles in it. This can be considered as a system with variable number of particles. Use the appropriate Gibbs distribution for the system to calculate $\langle n_k \rangle$ and derive the possible quantum distribution functions. Also, deduce the condition under which the quantum distribution function goes into the corresponding classical expression. **(20)**

2. Assume that you have a solid consisting of N atoms. Assume each atom is a 3D harmonic oscillator. That means that there are $3N$ degrees of freedom. Assume each oscillator has the same set of energy levels, $n\hbar\omega$, where $n = 1, 2, 3, ...$ Finally assume that the oscillators are not coupled. That is, each oscillator can oscillate independently. One can define a characteristic temperature, $\mathcal{T} = \frac{\hbar\omega}{k_B}$. Determine the energy and heat capacity of this system. Show that, for $T \gg \mathcal{T}$, the heat capacity agrees with the Dulong-Petit value. Also, show that, for $T \ll \mathcal{T}$, the heat capacity goes to zero exponentially. **(10)**

3. By explicit calculations show that the average energy per particle of an extreme relativistic ideal gas (total number of particles N) is twice the average energy per particle of the same gas in the nonrelativistic case. Note that the energy of a single particle in the nonrelativistic case is $\varepsilon = \vec{p}^2/2m$, while in the relativistic case it is $\varepsilon = cp$, where p is the momentum, m is the mass of a particle and c is the speed of light in free space. **(10)**

4. Consider a substance consisting of N atoms. Each atom has an intrinsic magnetic moment $\vec{\mu}$. In a constant magnetic field $\vec{B} = B\hat{z}$. the magnetic energy of an atom is given by

$$\varepsilon_m = -\mu B cos\alpha,$$

where α is the angle between $\vec{\mu}$ and $\vec{B}$. Compute (i) the classical partition function for a single atom, (ii) the average magnetic energy of the substance, (iii) the total magnetic moment, m, of the substance in the limiting cases of $\mu B/k_B T \gg 1$ and $\mu B/k_B T \ll 1$ (only upto first-order). **(10)**