m.m. 100

AML 410/AML 811
Advanced CFD

6/5/2007
Major Examination

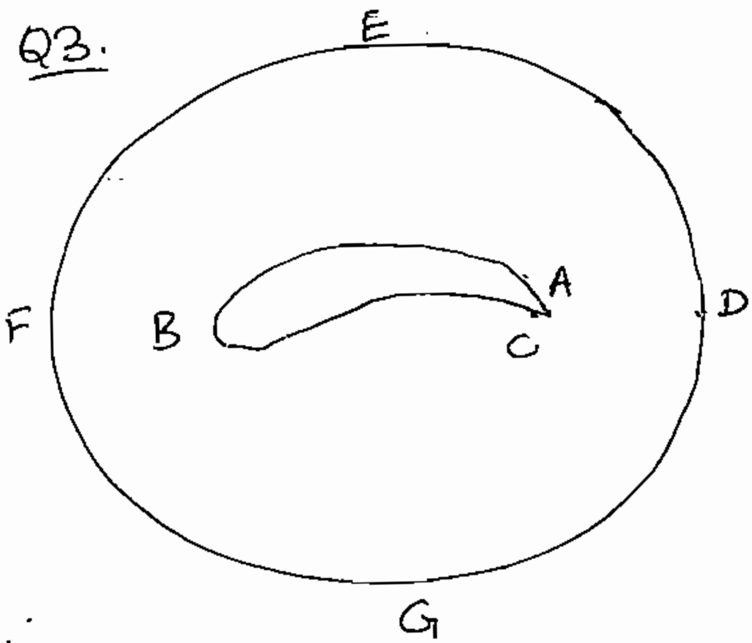
Q1. Clearly explain the difference between a staggered grid and a collocated grid with the help of a figure. Why is the staggered grid used? (10)

Q2. Consider a physical and computational plane where the grid transformations are given by

$$\xi = xy \; ; \quad \eta = \ln(y+1)$$

For incompressible flow, transform the continuity equation $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\right)$ in the computational coordinates (ξ, η). (15)

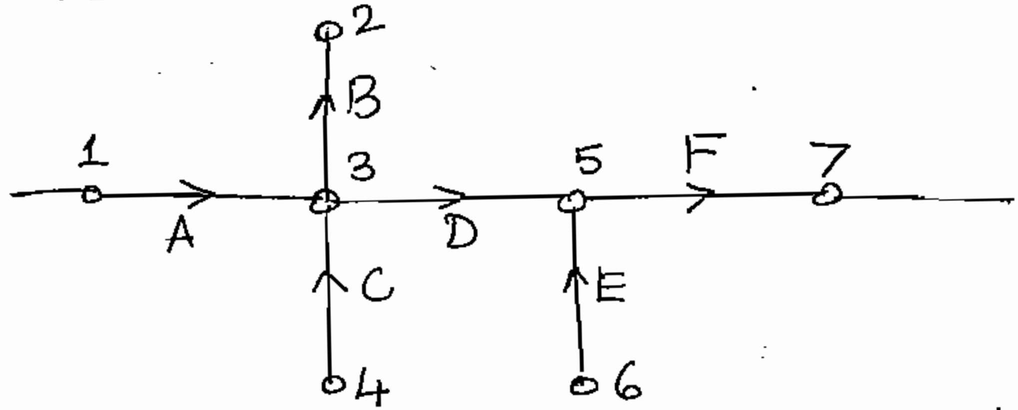
Q3.



We wish to solve the Navier Stokes equations for flow past an aerofoil in the domain shown.
We wish to carry out computations in a rectangular (ξ, η) plane as shown.

Mark the points ABCDEFG (approximately) on the rectangular domain shown. (10)

Q4. A portion of a water supply system is shown. The flow rate in the pipe is given by $Q = C\Delta p$, where Δp is the pressure drop over the length of the pipe, and for this problem $C = 1.0$ in appropriate units.



Given: $p_1 = 275$ $\quad p_2 = 270$ $\quad p_4 = 0$ $\quad p_6 = 40$

$Q_F = 20$

We wish to find $p_3, p_5, Q_A, Q_B, Q_C, Q_D$ and Q_E by the following method:

a) Guess p_3 and p_5 (Start by taking them as 0)

b) Obtain Q_i^* based on guessed pressure values.

c) Construct pressure correction equation and solve for p_3' and p_5'. (Use common sense).

d) Correct the guessed pressures

e) Find new values of Q_i^*.

f) Do you need to iterate? Why

25 ~~30~~

Q5: The reason why most codes did not converge was that most of you found p', u', v'. Then you under-relaxed u', v' and the new u^* was taken as $u^*_{old} + \alpha_u u'$ etc. This was the error as the only thing you could under-relax at this stage was p.

a) Why u and v cannot be underrelaxed at this part?

b) Where (in the whole code) do you under-relax u and

Q6

Consider the following set of ODEs.

$dX / dt = -PX + PY$

$dY / dt = -Y + rX + XZ$

$dZ / dt = -Z + XY$

where the dependent variables X, Y and Z are functions of the independent variable time (t). P and r are parameters (like Re in the Navier Stokes equations).
We wish to carry out the averaging of these equations and so we define
$X = \langle X\rangle + x$, $Y = \langle Y\rangle + y$, and $Z = \langle Z\rangle + z$.

a) Derive the equations for $\langle X\rangle$, $\langle Y\rangle$ and $\langle Z\rangle$.
b) Is the set of equations derived in a) above closed? If yes why? If no, then identify the unclosed terms.

(10)

Q7

For some specific flows, the Reynolds stress term is written as:

$$\langle u_1 u_2\rangle = -\nu_t \frac{\partial \langle U_1\rangle}{\partial x_2}$$

However, the general way of specifying eddy viscosity in incompressible flows is

$$\langle u_i u_j\rangle - \frac{2}{3} k \delta_{ij} = -\nu_T \left(\frac{\partial \langle U_i\rangle}{\partial x_j} + \frac{\partial \langle U_j\rangle}{\partial x_i} \right)$$

Comment on the role of the extra terms in the second equation as compared to the first one.

(5)

Q8

In solving a problem using the k-ε model the mean velocity, continuity, k and ε equations are given as below:

$$\frac{\partial \langle U_j\rangle}{\partial x_j} = 0 ; \quad \frac{\partial \langle U_j\rangle}{\partial t} + \langle U_i\rangle \frac{\partial \langle U_j\rangle}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \langle P\rangle}{\partial x_j} + \nu \frac{\partial^2 \langle U_j\rangle}{\partial x_i \partial x_i} - \frac{\partial}{\partial x_i}\langle u_i u_j\rangle$$

$$\frac{\partial k}{\partial t} + \langle U_i\rangle \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left\{\left(-\frac{\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_i}\right\} - \langle u_i u_j\rangle \frac{\partial \langle U_i\rangle}{\partial x_j} + \nu \frac{\partial^2 k}{\partial x_i \partial x_i} - \varepsilon$$

$$\frac{\partial \varepsilon}{\partial t} + \langle U_i\rangle \frac{\partial \varepsilon}{\partial x_i} \frac{\partial}{\partial x_i}\left\{\frac{\nu_T}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right\} + C_{\varepsilon 1}\frac{\varepsilon}{k}\langle u_i u_j\rangle \frac{\partial \langle U_i\rangle}{\partial x_j} - C_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

a) Given a turbulent flow where $\langle U_i\rangle = (\langle U\rangle(y), 0, 0)$, $\langle P\rangle = \langle P\rangle(x,y)$, all other statistical quantities are functions of y only, $\langle u_1 u_3\rangle = 0$, and $\langle u_2 u_3\rangle = 0$.
Simplify the equations for all velocity components (including the continuity equation) and the k and ε equations.

b) Consider the above equations for a case where $\langle U_i\rangle = (\langle U\rangle, \langle V\rangle, \langle W\rangle)$ and all statistical quantities are functions of (x, y and z). then

i) If the mixing length model is used, list the quantities for which you will be solving PDE's. (Do not write the equations).

ii) If the k-ε model is used to solve this flow in list the quantities for which you will be solving PDE's (Do not write the equations) (15+5)

(6)