Department of Chemical Engineering
India Institute of Technology-Delhi

CHL 110: Transport Phenomena: (Group 11+12)
Major Test

02 May 2008, Duration: 2 Hours, Max. Marks: 30, Closed book/notes

Notes:
1. Solution steps also carry the marks.
2. Use of only the Appendix B & C of BSL is allowed.
3. Question paper is also to be returned along with the answer book

1. Viscous heat generation in a narrow slit

An incompressible, Newtonian and viscous liquid is in laminar flow through a narrow slit as shown in Figure 1. It is understood that B<<W<<L, so that "edge effects" are unimportant. As the fluid flows, the layers of fluid rub against each other and this friction between the adjacent layers produces heat ("viscous dissipation"). The walls at x=+B and x=-B are kept at the temperature of T_W and k is the thermal conductivity of the fluid. The variation of k with temperature can be ignored.

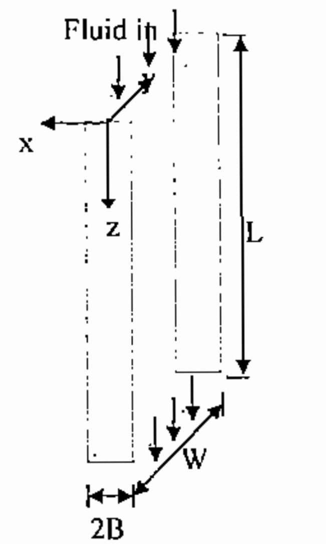

Figure 1: Viseous heating in a narrow slit

(a) Using the equation of motion, obtain an expression for non-zero velocity component in terms of the max. velocity (e.g. $V_{z,max}$)(the pressures at z=o and z=L can be taken to be p_o and p_L). (4 marks)

(b) Using the equation of temperature and velocity expression obtained in (a) above, derive an expression of T(x). Before solving the differential equation, convert it into dimensionless form using appropriate variables (tip: use the Brinkman number defined as $Br = \mu V_{z,max}^2 / kT_W$).

(6 marks)

(some of the standard forms of differential equations and there solutions are given ot the end of question paper)

2. Using the basic definitions of concentrations, velocities and fluxes,

(a) Show that $\dfrac{j_A}{\rho \omega_A \omega_B} = \dfrac{J_A^*}{c x_A x_B}$ (2 marks)

(b) Show that the basic expression for combined molar flux $N_\alpha = J_\alpha^* + c_\alpha v^*$ can also be written as $N_A = -cD_{AB}\nabla x_A + x_A(N_A + N_B)$ for a binary system. (2 marks)

3. Diffusion and Chemical Reaction inside a porous catalyst

Consider a spherical porous catalyst particle of radius R kept in a gas stream containing reactant A and product B (see Figure 2). The concentration of A at the surface of the spherical particle is known to be C_{AR} (moles of A per unit volume). The species A diffuse through the micro-pores in the spherical catalyst particle and is converted to B on the catalyst surface with a first order reaction (as shown in the inset). In this case the effective diffusivity for species A in the porous medium (D_A) is to be used. The rate of the reaction is given by $R_A=k''_1 aC_A$ where "a" is the available catalyst surface area per unit volume. For such a porous catalyst, the consumption of A by the catalytic reaction is to be modeled as a volumetric source (where "a", which is the catalyst surface area per unit volume, is known).

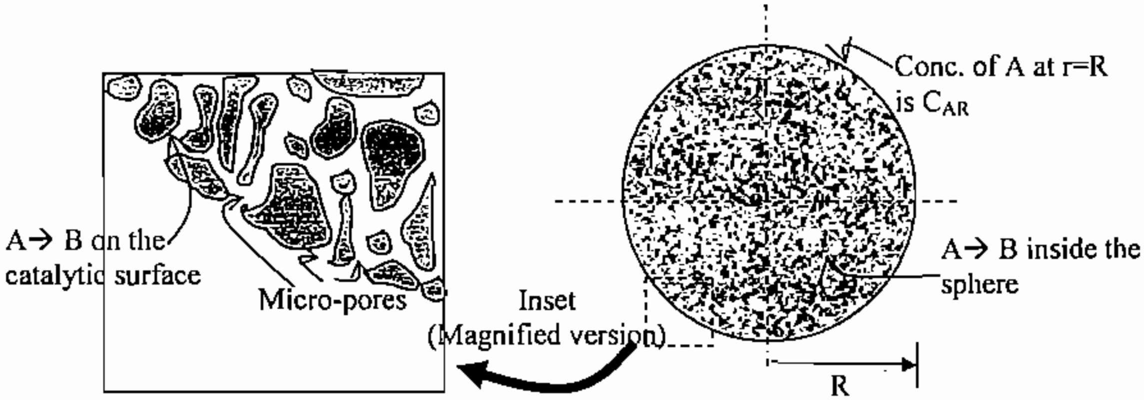

Figure 2: Porous catalyst sphere and magnified view of the micro-pores in the catalyst in which diffusion and chemical reaction occurs

Using shell balance, derive an expression for:

(a) The C_A distribution inside the sphere. (6 marks)

(b) The molar rate of conversion (moles/sec) of A at the surface r=R. (2 marks)

(c) If all the catalyst surface area ("a") was exposed to the gas stream with $C_A=C_{AR}$, then the species would not have to diffuse through the pores and then react. In this case, what would be the molar rate of conversion of A? (2 marks)

(some of the standard forms of differential equations and there solutions are given at the end of question paper)

4. Formulation of differential equations and boundary conditions for a forced convection mass transfer problem (6 marks)

A dilute solution of solute A in solvent S (with $C_A=C_{A0}$) enters the catalytic tubular reactor shown in Figure 3 and is in fully developed laminar flow in the region z<0. When it encounters the catalytic wall in the region $0 \le z \le L$, solute A is instantaneously and irreversibly converted to B. Using the appropriate equations of change (given in Appendix B), formulate the differential equation and boundary conditions to obtain the concentration distribution of solute A in the region $0 \le z \le L$. **DO NOT SOLVE THE DIFFERENTIAL EQUATION.** However, make all possible simplifications, substitute the expressions for the required terms in it and reduce it to final form. Consider the following simplifications/assumptions:

- Flow is steady, isothermal and neglect the presence of B.
- Consider ρD_{AS} to be constant and ignore the small amount of B produced by the reaction.
- The axial diffusion of A can be neglected with respect to the axial convection of A.

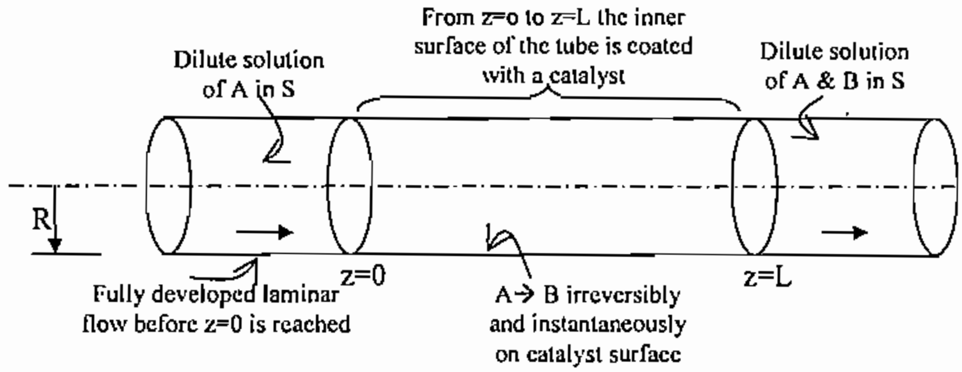

Figure 3: Catalytic Tubular Reactor

Equation	Solution
(1) $\frac{d^2 y}{dx^2} = f(x)$	$y = \int_0^x \int_0^{\bar{x}} f(\bar{\bar{x}}) d\bar{\bar{x}}\, d\bar{x} + C_1 x + C_2$
(2) $\frac{1}{x^2}\frac{d}{dx}\left(x^2 \frac{dy}{dx}\right) - a^2 y = 0$	$y = \frac{C_1}{x}e^{+ax} + \frac{C_2}{x}e^{-ax}$