IIT Delhi

EEL 718 Statistical Signal Processing
(Major Exam – 2007-08/II)

- **Time allowed: 2 hours.**
- **Maximum marks: 90; Q1: 8; Q2: 10, Q3: 14, Q4: 14, Q5: 12, Q6: 18, Q7:14.**
- **Show all steps of your calculations. Any necessary assumptions must be clearly stated.**

1. Derive the PSD $S_y(z)$ of the output of LTI system in terms of its transfer function H(z) and the PSD of its input process $S_x(z)$.

2. Consider the process $x[n] = \cos(0.2\pi n + \Phi_1) + 2\sin(0.2n + \Phi_2)$ where Φ_1 and Φ_2 are IID random variables uniformly distributed in the interval $[0, 2\pi)$.
(i) Obtain the autocorrelation function.
(ii) Is the process wide-sense stationary? Is it ergodic? Explain.

3. Consider a realization of an ARMA (2, 2) random process $x[n]$ = 1, 0, -1, 1, 0, 2, 1, 2, -1, 1, 1 for n = 0, ..., 10. Obtain the optimum parameters of the model.

4. Express the minimum variance spectral estimator in terms of the Fourier transform of the eigenvectors and associated eigenvalues of the autocorrelation matrix. How is the estimator related to the MUSIC pseudo-spectrum?

5. For the realization of the process in 3. above, obtain the Pisaranko pseudo-spectrum for M=3 (P=2).

6. Consider a data matrix **X** whose SVD is given below.

$$U = \begin{Bmatrix} -0.25 & 0.35 & 0.25 & 0.87 \\ -0.70 & 0.28 & -0.64 & -0.13 \\ -0.67 & -0.47 & 0.55 & -0.16 \\ 0.04 & -0.76 & -0.47 & 0.45 \end{Bmatrix}$$

$$S = \begin{Bmatrix} 3.09 & 0 \\ 0 & 1.86 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}$$

$$V = \begin{Bmatrix} -0.76 & 0.65 \\ -0.65 & -0.76 \end{Bmatrix}$$

(a) Obtain $\mathbf{XX}^H$ without computing **X**, and doing the minimum number of computations. Show all steps of your computations and explain the simplifications done.

(b) Obtain $\mathbf{X}^+$, the pseudo-inverse of **X** without computing **X**.

(c) What do you understand by the products $\mathbf{XX}^+$ and $\mathbf{X}^+\mathbf{X}$?

7. Consider an AR(1) process $x[n] = ax[n-1] + w[n]$, where $w[n]$ is white Gaussian noise with zero mean and variance σ_w^2.

(a) Determine the autocorrelation $r_x(k)$, the optimum first order linear predictor and the corresponding MMSE.

(b) We wish to design a one-step first order linear predictor using the LMS algorithm:

$$\hat{x}[n] = \hat{a}[n-1]x[n-1]$$
$$e[n] = x[n] - \hat{x}[n]$$
$$\hat{a}[n] = \hat{a}[n-1] + 2\mu e[n]x[n-1]$$

where μ is the adaptation step size. Using independence assumption, first determine and then solve the difference equation for $E\{\hat{a}[n]\}$.