Department of Electrical Engg.
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EEL772, Optimal Control Theory

Major test, Max. time: 2 hrs, Max marks: 80, II Sem (2007-2008)

* write each step of your calculations.

Q1: 15 Q2: 15 Q3: 15 Q4: 15 Q5: 10 Q6: 10

Q1. Consider a System

$$\dot{x} = Ax + Bu, \quad y = cx, \quad x(0) = x_0.$$

(a) Show that the design of a dynamic output feedback Controller is equivalent to the design of a Static output feedback Controller of a augmented System.

(b) suppose a performance function J is given as

$$J = \frac{1}{2}\int_0^\infty (x^T x + u^T u)\, dt$$

Derive a Static output feedback Controller That minimize J.

How to make the controller gain independent of $x(0)$.

Q2. Consider a Single input System $\dot{x} = Ax + bu$.

(a) Determine the time-optimal Control input using Pontryagin prina

(b) Briefly explain the concept Singular and non-Singular of time optimal Control.

Q3. Consider a nonlinear uncertain System

$$\dot{x} = A(x) + \Delta A(x) + Bu$$

where $\Delta A(x)$ is the System uncertainty.

(a) What is robust Stability problem in this case?

(b) Briefly explain the concept of matched & unmatched uncertainty

(c) Show that the Solution of a robust Stability problem can be obtained by Solving an optimal Control problem

4. (a) What is the principle of optimality?

(b) Suppose $x(k+1) = Ax(k) + Bu(k)$ k

$$J = \frac{1}{2} x^T(N) \mathcal{H}(N) + \frac{1}{2} \sum_{k=0}^{N-1} \left( x^T(k) x^*(k) + u^T(k) u(k) \right).$$

Find the control sequence $u(k)$ y $k = 0, 1, \cdots N-1$ that minize $J$. Derive the results using principle of optimality.

5. Consider a fixed end problem where performance functional.

$$J = \int_{to}^{tf} g(x, \dot{x}, t)\, dt$$

(a) Derive the necessary condition to extremize $J$.

(b) Explain how two point boundary value problem arises in this context.

6. Consider a system $\dot{x} = Ax + Bu$, $x(0) = x_0$ and

$$J = \int_0^\infty e^{2t} \left( x^T x + u^T u \right) dt$$

(a) Suppose a state feedback controller is designed by minimizing $J$.

(a) Show that this problem can be converted into a standared LQR problem.

(b) Show that the real part of all closed loop poles will be less than $-1$.