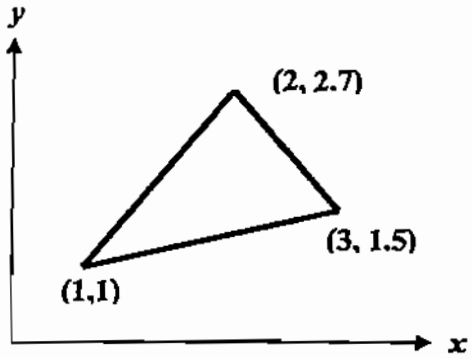
Department of Mechanical Engineering
IIT Delhi

MEL 735 Major S. Mukherjee/A Chawla, 2006

One A4 size sheet in your own handwriting is allowed in the examination. Section A and Section B to be answered in separate answer books.

SECTION A

Q1. If ϕ_1 is the linear interpolation function associated with the node at (1,1) for the triangular element shown in the figure below, evaluate $\int(\phi_1)^2\, dA$. (5)



Q2. Consider the one dimensional problem with governing equation:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{for } 0 < x < 1 \text{ with BC } u(0,t) = 0 \text{ and } \frac{\partial u}{\partial x}(1,t) = 0 \text{ and IC } u(x,0) = 1$$

a) Write the expression for the simplest shape functions that gives the required continuity?
b) Derive expressions for the elemental K, C and Q matrices of the form :
$$[K]\{\phi\} + [C]\{\dot{\phi}\} = \{Q\}$$
c) What is the C matrix when lumped using the row summing method?
d) Using three equal elements over the domain, assemble the FE equation.
e) Estimate the value of u at $x = 2/3$ and $t = 0.25$ using the *forward difference* method using a single time step of 0.25.
f) If you continue with this time step, will the solution be stable and/or oscillate?

(1 + 3 + 1+ 1 + 2 + 2)

Q3. A 3D parametric cubic curve starts from (0,0,0) and ends at (1,0,0) and the tangent vectors at the two locations are (0, 0, 2) and (0, 0.5, 0).

a) Sketch the projection of the curve on the xy plane.
b) Subdivide the curve into three segments with joints at $u = 1/3$ and 2/3. Reparametrise the middle segment and compute the geometric coefficients, **p**'s.

(2 + 3)

MAJOR EXAM **MEL 735**

November 30, 2006

SECTION B

Max marks: 15

Answer on a separate booklet

Q 1. 1.5 X 5 = 7.5 marks

a) Derive suitable transformation matrix to rotate a point $(x_0\ y_0)$ about a line given by the equation $y = m\ x + c$.

b) In order to transform a given Bezier surface through a transformation T, is it sufficient to transform all the control points? Proove your answer.

c) What are twist vectors? Why are they needed as input if four boundary curves are given for a bicubic surface?

d) For a cubic B-spline curve, suggest a method to get the control points of an equivalent Bezier curve for each curve segment.

e) You are given bi-cubic surfaces 'p' and 'q'. You are required to make a bi-cubic surface 'r' which is G1 continuous with one edge of 'p' and with one edge of 'q'. What constraints are imposed on 'r' and how many degrees of freedom does it have?

Q 2. For a bicubic surface, what is an auxiliary curve? For a surface given by $P = UAW^T$, proove that, an auxiliary curve follows a cubic relation. Derive the same for, $u = 0$. 4 marks

Q 3. Derive the blending functions for a non-periodic cubic B-spline curve with 6 control points. 3.5 marks