

**Department of Electrical Engineering**  
**EEL771, Random process in control and estimation,**  
**Major Test II, 2008-2009/I.**  
*Max. time : Two hours, Max. marks: 95.*

**Marks: Q1: 15, Q2: 15, Q3: 15, Q4: 10, Q5: 10, Q6: 10, Q7: 10, Q8:10**

➤ **Write clearly each step of your calculation.**

- Q1. (a) Given an arbitrary positive function  $S(w)$ . Find a process  $X(t)$  with power spectrum  $S(w)$ ?  
 (b) Suppose  $X(t)=a \cos wt + b \sin wt$ . Show that this process is WSS if  $a$  &  $b$  are uncorrelated with equal variance?  
 (c) Define the Ergodic process. What is the first and second order statistics of a random process.

- Q2. (a) Let  $X$  and  $Y$  be two random process. Explain briefly the following : (i) Orthogonality of  $X$  and  $Y$ . (ii) Independence of  $X$  and  $Y$  (iii) Power spectral density function.  
 (b) Let  $X(t)=A \exp(j2\pi ft)$  with  $f$ , a known real constant and  $A$  be a real random variable. Find the restriction on  $A$  such that  $X(t)$  will be a WSS process.

- Q3. (a) Let  $Z=X_1+X_2$  with  $f_{x_1}(x)$ ,  $f_{x_2}(x)$  and  $f_z(z)$  denoting the pdf's of  $X_1$ ,  $X_2$  and  $Z$  respectively. Show that  $\Phi_z(w) = \Phi_{x_1}(w) \Phi_{x_2}(w)$  where  $\Phi_z(w)$ ,  $\Phi_{x_1}(w)$  and  $\Phi_{x_2}(w)$  are the characteristic functions of  $Z$ ,  $X_1$  and  $X_2$  respectively.  
 (b) Suppose  $X$  and  $Y$  are two independent r.v., Show that  $g(X)$  and  $h(Y)$  are also independent where  $g(x)$  and  $h(x)$  are two functions.?

- Q4 (a) Let  $X$  be an r.v. with p.d.f  $N(0,1)$  and  $y = x^2$ . Determine the p.d.f of  $y$ ?  
 (b) Let  $X$  and  $Y$  be i.i.d. r.v.'s with  $X = N(0,1)$ . What is the p.d.f of  $Z=X^2 + Y^2$

5(a) Given  $f_{xy}(x, y) = \begin{cases} k, 0 < x < y < 1 \\ 0, \text{otherwise} \end{cases}$

- (a) Determine  $k$  (b) Also determine also conditional densities  $f(x|y)$  and  $f(y|x)$ ?

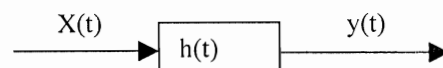
Q6 Consider two rv  $x$  &  $y$  whose joint pdf is

$$f_{xy}(x, y) = \begin{cases} 0, x < -a, y < -a \\ \frac{1}{4a^2}, -a \leq x < a, -a \leq y < a \\ 0, x \geq a, y \geq a \end{cases}$$

- (a) Show that random variables are independent and uncorrelated  
 (b) Also find correlation coefficient and covariance function.

- Q7(a) Let  $X$  be a poisson distributed random variable. Suppose  $B=\{X \text{ is even}\}=\{X=0,2, \dots\}$ . Find  $P(k|X \text{ is even})$ ? Also find the conditional p.d.f.

Q8. Suppose a linear stable with impulse response  $h(t)$  is excited by a wss process  $x(t)$ .



- (a) Determine mean of  $y(t)$  and  $R_{yx}(t)$ ,  $R_{yy}(t)$ .  
 (b) Suppose the transfer function of this system is  $H(s) = 1/(s+1)^2$ . Express the power spectral density function of the output in terms of  $S_{xx}(w)$ .