

**Department of Mathematics**  
**MAL 250 (Probability and Stochastic Processes)**  
**Major Test**

**Maximum marks: 50**

**Time: 2 hrs**

**Note: There are 10 questions in all**

1. Let  $X$  be a random variable such that  $P(X = 2) = \frac{1}{4}$  and its distribution function is given by

$$F_X(x) = \begin{cases} 0, & x < -3 \\ \alpha(x+3), & -3 \leq x < 2 \\ 3/4, & 2 \leq x < 4 \\ \beta x^2, & 4 \leq x < \sqrt{3/8} \cdot 8/\sqrt{3} \\ 1, & x \geq \sqrt{3/8} \cdot 8/\sqrt{3} \end{cases}$$

Find  $\alpha, \beta$  if 2 is the only jump discontinuity of  $F$ .

Compute  $P(X < 3/X \geq 2)$

(2 + 2 marks)

2. On an average a production process produces one defective among 300 items manufactured. Find the probability that the 5th defective will appear sometimes after 1000 and 10th defective sometimes after 2000 items have been manufactured. Find also the approximate probability of this event using Poisson distribution. (4marks)
3. We begin recording new call attempts at a telephone switch at time 0. Let  $X$  and  $Y$  be the arrival times of 1st call and 2nd call respectively. Suppose  $(X, Y)$  has joint pdf

$$f_{X,Y}(x, y) = \begin{cases} \lambda^2 e^{-\lambda y}, & 0 \leq x < y \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional distribution of interarrival time  $Y - X$  given  $X + Y$ . (5 marks)

4. Let  $X_1, X_2, \dots, X_n$  be independent and  $\ln(X_i)$  has normal-distribution  $N(2i, 1)$ ,  $i = 1, 2, \dots, n$ . Let

$$W = X_1^\alpha X_2^{2\alpha} \dots X_n^{n\alpha}, \alpha > 0 \text{ a constant}$$

Determine  $E(W)$ ,  $Var(W)$  and the probability distribution of  $W$ . (6 marks)

5. A digital camera needs three batteries to run. You buy a pack of 6 batteries, install three of these batteries into the camera. Whenever a battery is drained, you immediately replace the drained battery with the one new battery from the available stock. Assume that each battery lasts for an amount of time that is exponentially distributed with mean  $1/\mu$ , independent of all other batteries. Eventually camera stops running, only two batteries will be left out in the camera that are not drained. Find the expected time that your camera will be able to run with the pack of batteries bought. (5 marks) **P.T.O**