IIT DELHI

MAL 518 Applied Mathematics MAJOR TEST

Marks 45 30 APRIL 2010 TIME: 2 hrs

1. Solve the Laplace equation on a rectangle: $0 \le x \le 1$, $0 \le y \le 1$ with Boundary conditions:

(7) $u_x(0,y) = u_x(1,y) = u_y(x,0) = 0$ & $u_y(x,1) = f(x)$.

2. Solve $u_t = u_{xx} + \delta(x)\delta(t)$, $-\infty < x < \infty$ with BCs: $u(x,t)$ & $u_x(x,t)$ both vanish as $|x| \to \infty$,

(9) IC: $u(x,0) = \delta(x)$. Where δ is Dirac's delta function.

3. Let $K(s,t)$ be a symmetric kernel of an integral equation, λ be an eigenvalue and ϕ be its corresponding eigenfunction then the following are true or false? Justify your answers.

(12) (a) $\bar{\lambda} \neq \lambda$ (b) $\langle \phi_1, \phi_2 \rangle_{L^2} \neq 0$, (λ_1, ϕ_1), (λ_2, ϕ_2) are distinct eigenpairs.

(c) The set of eigenvalues of the second iterated kernel $K_2(s,t) = \int K(s,x) K(x,t)\,dx$ coincide with the set of squares of the eigenvalues of $K(s,t)$.

4. (a) Solve $g(s) = \sin s - \frac{s}{4} + \frac{1}{4}\int_0^{\pi/2} st\, g(t)\,dt$.

(b) State and prove Hilbert-Schmidt Theorem and using this solve the Poisson's integral equation:

$$f(s) = \int_0^{2\pi} K(s,t)\, h(t)\,dt \quad \text{where, for } 0 \le s \le 2\pi$$

(17)

$$K(s,t) = \frac{1-r^2}{2\pi}\left\{\frac{1}{1-2r\cos(s-t)+r^2}\right\},$$

$$(0 < r < 1)$$

and $h(t)$ is a given $L^2[0,2\pi)$ function.

———ITS FINISHED———→