Deptt. of Elec. Engg., IIT Delhi

EEL205, Signals & systems, Major Exam. M.M. 35, Dur: 2h

Prob. 1 (a) Define independence of two random variables, show how it affects their joint probability density function. (pdf)

(b) Let S = sample space = $\{h, t\}$ and $X(h) = 10$ & $X(t) = 20$. Find & plot $F_X(x)$ & $f_X(x)$.

Prob. 2 Let the signal $x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$, is passed through the system given in Fig. 2(b) (i) Find $y(t)$ & $Y(\omega)$.

(ii) Also design a system which takes $y(t)$ as input and outputs $x(t)$. (If possible)

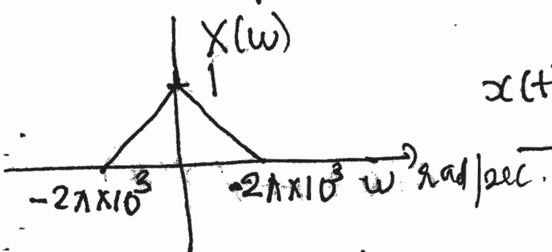


Fig. 2(a)

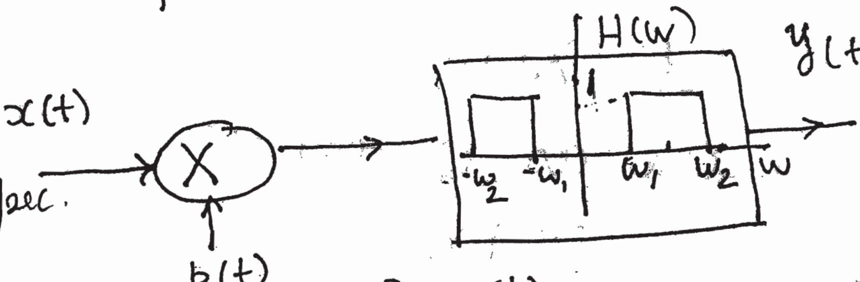
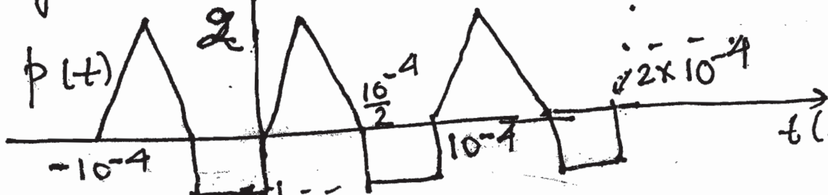


Fig. 2(b)

$\omega_1 = 18\pi \times 10^3$ ra
$\omega_2 = 22\pi \times 10^3$ rad/s

where periodic



Prob. 3 We have the following information about a discrete-time LTI system with input $x[n]$ and output $y[n]$

1. If $x[n] = (-2)^n \;\forall n$ then $y[n] = 0 \;\forall n$
2. If $x[n] = \left(\frac{1}{2}\right)^n u[n]$, then $y[n] = \delta[n] + a\left(\frac{1}{4}\right)^n u[n]$ where a is a constant

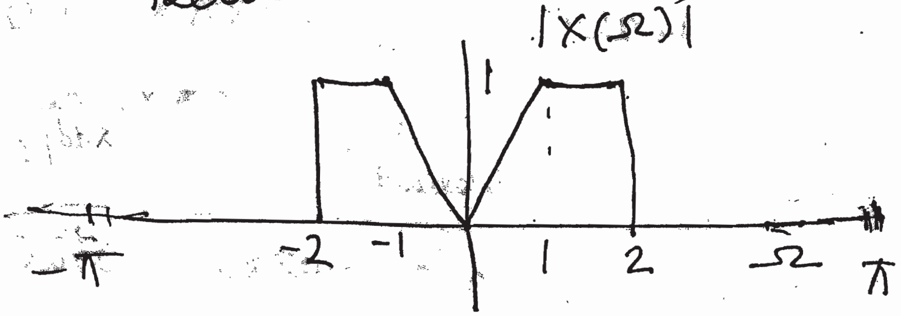
(a) Determine the value of 'a'

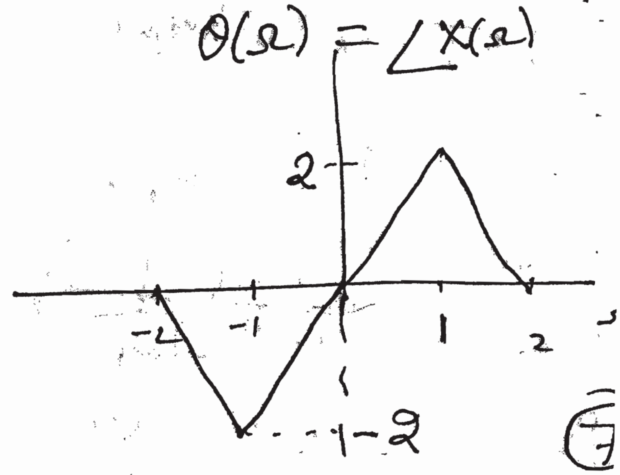
(b) Find $y[n]$ if $x[n] = 1 \;\forall n$.

Prob. 4 Suppose we are given the following

information about a causal and stable LTI system 'S' with impulse response $h(t) \overset{\mathcal{L}}{\longleftrightarrow} H(s)$:
① $H(1) = 0.2$; ② when input is $u(t)$ the output is absolutely integrable. ; ③ when the input is $tu(t)$ the output is not absolutely integrable. ④ The signal $\frac{d^2h(t)}{dt^2} + 2\frac{dh(t)}{dt} + 2h(t)$ is of finite duration. ⑤ $H(s)$ has exactly one zero at infinity. Determine $h(t)$, $H(s)$ and its ROC

Prob. 5 Let $x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(\Omega)$ be transform pair.
Further let $X(\Omega) = |X(\Omega)| e^{j\theta(\Omega)}$ where $|X(\Omega)|$ & $\theta(\Omega)$ are as given in Fig 5(a) & (b).
Determine $x[n]$





LIST OF FORMULA

CTFS $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$\omega_0 = \frac{2\pi}{T}$ $\quad a_k = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t} dt$

CTFT $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

DTFS $x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}$

Properties CTFT

1. $\alpha_1 x_1(t) + \alpha_2 x_2(t) \longleftrightarrow \alpha_1 X_1(\omega) + \alpha_2 X_2(\omega)$
2. $x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$
   $e^{j\omega_0 t} x(t) \longleftrightarrow X(\omega - \omega_0)$
3. $\overline{x(t)} \longleftrightarrow \overline{X(-\omega)}$
4. $x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$.
5. $\int |x(t)|^2 dt \longleftrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$
6. $\frac{dx(t)}{dt} \longleftrightarrow j\omega X(\omega)$

Properties DT

① $x[n-n_0] \longleftrightarrow e^{-j\Omega n_0}$
$e^{j\Omega_0 n} x[n] \longleftrightarrow X($
② $-jn x[n] \longleftrightarrow \frac{d}{d}$
③ Linearit
④ $\sum |x[n]|$
$= \frac{1}{2\pi}\int |X(\Omega)$
$x[n] y[n]$
$\longleftrightarrow \frac{1}{2\pi} X(\Omega)$