PHYSICS DEPARTMENT, IIT DELHI

MAJOR-TEST, PHL-110, Nov. 30-2006

Max. Marks: 50 Time: 2 hour

Note: Attempt all questions.

1.Consider a long hollow dielectric cylinder of inner and outer radii R_1 and R_2 with $\varepsilon=\varepsilon_0$ for $r < R_1$ and $\varepsilon=\varepsilon_0\alpha/r$ for $R_1 < r < R_2$, where α is a constant. A thin wire of linear charge density λ is placed along the axis of the cylinder. Estimate the bound volume charge density ρ_b and the surface charge density σ_b at $r = R_1$ and R_2 so produced. <u>6</u>

2. A long cylindrical conductor of radius R has a cylindrieal hole of radius a which is drilled out such that thc axis of the hole is parallel to the axis of the cylinder. If b is the distance between the two axcs and if I_0 is the current passing uniformly through the remaining solid cylinder, show that the magnitude of the magnetic field vector $\vec{B}$ inside the hole is constant throughout and is given by <u>6</u>

$$\frac{\mu_0 I_0 b}{2\pi(R^2-a^2)}$$

3(a) At the interface between two linear dielectric media, the electric field lines bend. If the dielectrie eonstants of the two media are ε_1 *and* ε_2 respectively, obtain a relation between angles θ_1 *and* θ_2 (Fig.1). It is given that there is no free charge at the boundary. <u>3</u>

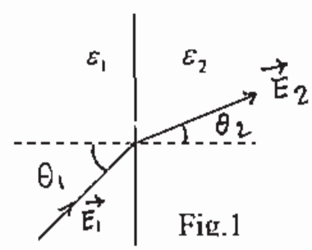


Fig.1

(b) If a static magnetie field in free space is given by-- $\bar{B}=\hat{z}(1+\alpha_1 x+\alpha_2 y+\alpha_3 z)$, Estimate α_1, α_2, and α_3. <u>2</u>

(c) If the static magnetic field is given by $\vec{B}=\hat{y}\dfrac{B_0}{a}x$,Obtain the current density in the region of $\vec{B}$ producing such a magnetic field. <u>2</u>

4. A plane electromagnetic wave is incident on a dielectric interface separating two dielectric media of dielectrie constants ε_1 *and* ε_2 respeetively with an ineident angle θ_1 (from ε_1 side). Write expressions for the ineident, reflected and transmitted electric field and obtain the Snell's law by applying the boundary conditions <u>5</u>

5(a) Obtain the Fraunhoffer amplitude diffraetion pattern of the aperture shown in Fig. 2. <u>4</u>

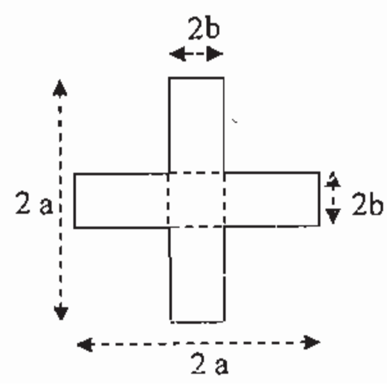
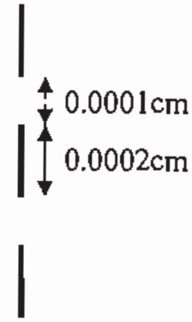


Fig.2

(b) Consider a diffraction grating consisting of transparent portion of width 0.0001cm each separated by an opaque portion of 0.0002 em each as shown in the adjacent figure. How many orders will be observed at $\lambda = 0.9$ μm. Will there be any missing orders, if yes which ones? 3



6. Show that in Compton Scattering, the kinetic energy K of the recoiled electron and the energy E of the incident photon are related by

$$K = E\,\frac{2p\sin^2\phi/2}{1+2p\sin^2\phi/2},$$

where $p = \dfrac{h\nu}{m_0c^2}$ and ϕ is the photon scattering angle and various symbols have their usual meanings. 6

7(a) A blackbody radiator is in the shape of a cube with 2 cm. sides, and is maintained at a temperature of 1500 K. calculate the number of modes of vibration per unit volume in the cavity in the wavelength range 4995 Å to 5005 Å. 2

(b) Show that <xp> - <px> = i h, 3

(c) The probability of a particle of energy E tunnels through a potential barrier of height U (E < U) and width L, is 2%. Estimate the probability of its tunneling in the width of the barrier is doubled. 2

8. Consider a harmonic oscillator characterized by mass m and the potential energy U(x) $= \frac{1}{2}kx^2$. If the wave-function in an energy state is given by,

$$\psi = Bx\exp\{-\alpha x^2\} \quad where \quad \alpha = \frac{\sqrt{mk}}{2\hbar}$$

Obtain the value of constants B and the total energy of the particle in the above state. 6

Useful relations: $\int_{-\infty}^{\infty} e^{-x^2}dx = \Gamma\tfrac{1}{2} = \sqrt{\pi}$

In Cylindrical co-ordinates:

Gradient. $$\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}$$

Divergence. $$\nabla\cdot\mathbf{v} = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl. $$\nabla\times\mathbf{v} = \left[\frac{1}{r}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{r} + \left[\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right]\hat{\phi} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rv_\phi) - \frac{\partial v_r}{\partial \phi}\right]\hat{z}$$