Department of Mathematics
MAL 717: Fuzzy Sets and Applications; Major Exam

Maximum Marks: 35 **Time: 2 hr**

Each question carries equal marks.

Q1. Let $\langle L, \leq, \wedge, \vee, \odot, \rightarrow\rangle$ be a generalized residuated lattice. Using the associativity of $\odot$, show that

$$x \odot y \rightarrow z = x \rightarrow (y \rightarrow z), \quad \forall\, x, y, z \in L.$$

Q2. Let $s : [0,1] \times [0,1] \rightarrow [0,1]$ be defined as

$$s(x,y) = \frac{x + y - 2xy}{1 - xy}, \quad x, y \in [0,1].$$

Is $s(\cdot,\cdot)$ a t-conorm? If so, construct its associated t-norm such that (t, s) forms a dual pair.

Q3. Let $\langle L, \leq, \wedge, \vee, \mu, f, g, 0, 1\rangle$ be a complete generalized residuated lattice, and $S \subset V \times W, T \subset U \times W$ be binary fuzzy relations with values in L. Suppose $I : U \times V \rightarrow L$ is an identical fuzzy relation defined as

$$I(u,v) = \begin{cases} 1, & \text{if } u = v \\ 0, & \text{otherwise}. \end{cases}$$

Prove that the fuzzy relation equation

$$X \mu S = T$$

has a solution for each fuzzy $T \subset U \times W$ if and only if $H_1(S,T)\mu S = I$, where $H_1(\cdot,\cdot)$ and μ (operator) have their standard meaning.

Q4. We wish to develop a controller to regulate the temperature of a room. The logic is that 'when the temperature is HOT then cool the room down by turning the fan FAST'. Fuzzy sets for temperature and fan speed are given by:

$$\tilde{H} = \text{HOT} = \{0/60 + 0.1/70 + 0.6/80 + 0.9/90 + 1/100\} \text{ in } {}^\circ F;$$
$$\tilde{F} = \text{FAST} = \{0.2/1 + 0.5/2 + 0.7/3 + 0.9/4 + 1/5\} \text{ in } 1000 \text{ rpm}.$$

(a) Use the Mamdami implication to find the relation matrix depicting the IF-THEN relation.

(b) Suppose a new rule uses 'EXTREMELY HOT' defined by

$$\tilde{EH} = \{0/60 + 0.2/70 + 0.8/80 + 1/90 + 1/100\} \text{ in}^\circ F.$$

Using the max-product composition find the resulting fuzzy fan speed.

PTO

Q5. A patient is diagnosed by two doctors. The diagnosis of doctor 1 is that the evidence support 0.01 brain tumor (BT) and 0.99 is meningitis (M). The second doctor is of the opinion that the evidence support 0.01 brain tumor and 0.99 concussion (C). Using Dempster's rule of combination, combine the two evidences and conclude about the patient disease. Is your conclusion compatible with the two doctors theories? Justify.

Q6. The following table shows the intensity values (for gray scale 255) of 20 pixels.

(a) Use the fuzzy image enhancement technique once to enhance the image.

220	30	15	250
205	230	$\boxed{0}$	230
225	20	239	220
217	$\boxed{255}$	10	215
220	25	$\boxed{255}$	235

(b) If the boxed blocks are "noise", use the smoothing algorithm with $a_0 = \frac{1}{2}$, $a_1 = \frac{1}{8}$, $N = 4$, to remove the noise. Perform only one iteration.

Q7. "It is given that an optimal solution of the linear programming problem lies at an extreme (corner) point of the feasible set."

Using the Werner's model construct an equivalent crisp LPP of the following fuzzy linear program.

$$\tilde{\max}\ z = 2x_1 + x_2$$

subject to

$$\begin{aligned} x_1 &\lesssim 3 \\ x_1 + x_2 &\lesssim 4 \\ 3x_1 + 2x_2 &\gtrsim 6 \\ x_1, x_2 &\geq 0. \end{aligned}$$

The tolerance vector is $(3, 4, 2)$.