

1. Prove the following:

Suppose  $\{p_i | i \in I\}$  are fuzzy sets in  $(X, \mathcal{F})$  and  $\{q_i | i \in I\}$  are fuzzy sets in  $(Y, \eta)$ . Then  $R_x^y := \inf_{i \in I} \{p_i(x) \xrightarrow{\&} q_i(y)\}$  and  $m_x^y := \sup_{i \in I} \{p_i(x) \& q_i(y)\}$  are fuzzy relations from  $(X, \mathcal{F})$  to  $(Y, \eta)$ .

Apply this to the following situation:

$R_i$  is the rule: If  $x$  is in the fuzzy singleton  $\sigma_i$  in  $(X, \mathcal{F})$  then  $y$  is in the fuzzy singleton  $\tau_i$  in  $(Y, \eta)$

Interpreting the 'if... then' connective as  $\xrightarrow{\&}$  as well as  $\&$  in two different ways, justify these two interpretations and assuming that a fuzzy control map from  $(X, \mathcal{F})$  to  $(Y, \eta)$  is to be a fuzzy partial function so that  $\varphi^\#(\sigma_i)$  is a singleton in  $(Y, \eta)$  if  $\sigma_i$  is a fuzzy singleton in  $(X, \mathcal{F})$  show that  $M_x^y := \sup_{i \in I} [\sigma_i(x) \& \tau_i(y)] \leq \inf_{i \in I} [\sigma_i(x) \xrightarrow{\&} \tau_i(y)] =: R_x^y$  can be deduced.

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2. Explain the compositional rule of inference and deduce the Generalized Modus Ponens from it. Provide one instance of the use of generalized modus ponens.

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3. Explain the Zadeh Extension Principle and use it to define addition and division of 'fuzzy quantities', i.e. functions  $\mathbb{R} \rightarrow [0, 1]$ . Prove that every real number can be regarded as a fuzzy quantity and explain the correct meaning of the statement "division by zero is possible in the arithmetic of fuzzy quantities".

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