DEPARTEMENT OF CIVIL ENGINEERING: IIT DELHI
SEMESTER II: SESSION 2006-07
CEL 722: SOLID MECHANICS IN STRUCTURAL ENGINEERING

MAJOR TEST

Max. Marks = 40 Time allowed = 2hrs

All questions compulsory

I The loading function for an elastic-work-hardening plastic solid is stated as

$$f(\sigma_{ij}, k) = \sqrt{J_2} + \alpha I_1 - k = 0$$

where k is the hardening functions of the hardening parameter p.

a) Assuming the validity of associative flow rule, derive the incremental constitutive equation with stresses as the primary variables.
(No need to derive the expression for plastic modulus, Hp).

b) Check whether this elasto-plastic solid exhibits stress-induced anisotropy under general and hydrostatic states of stress.

8+4=12 marks

II A solid circular prismatic bar is fixed at one end and is under the action of a torque T and an axial force P. The displacement vector field is specified as follows:

$$u_1 = -\alpha X_2 X_3 \qquad u_2 = \alpha X_1 X_3 \qquad u_3 = 0$$

The X_3-axis is oriented along the centroidal axis of the bar.

a) Obtain the Almansi strain tensor field e_{ij}

b) If the constitutive equation is stated as

$$\sigma_{ij} = \alpha_0 \delta_{ij} + \alpha_1 e_{ij} + \alpha_2 e_{ik} e_{kj}$$

Obtain the explicit expressions for the components of the stress tensor in the bar.

7+6=13 marks

III The strain energy function W for a linear hyperelastic solid in stated as

$$W = A(\varepsilon^2_{11} + \varepsilon^2_{22}) + B\,\varepsilon^2_{33} + C\,\varepsilon_{11}\varepsilon_{22} + D\,\varepsilon_{33}(\varepsilon_{11} + \varepsilon_{22}) + E(\varepsilon^2_{23} + \varepsilon^2_{31}) + (2A - C)\,\varepsilon^2_{12}$$

where A, B, C, D and E are material constants.

a) Obtain the expressions for the stress tensor components σ_i as well as the elasticity tensor components C_{ij} in $\sigma_i = C_{ij}\,\varepsilon_j$ i, j= 1, 2, ...6.

b) Identify the type of the anisotropy exhibited by the solid.

6+3=9 marks

IV a) Identify any ONE class of solids which does NOT belong to NOLL's Class of SIMPLE SOLIDS. Justify the choice made.

b) It is contended that the stiffness matrix of a conservative linear elastic structure is always symmetric. Prove the above contention.