*IITD, CHL: 830: **Major Exam 2009*** (100 Marks)
Open Notes, Open Handouts, No photocopied notes, No human aid.

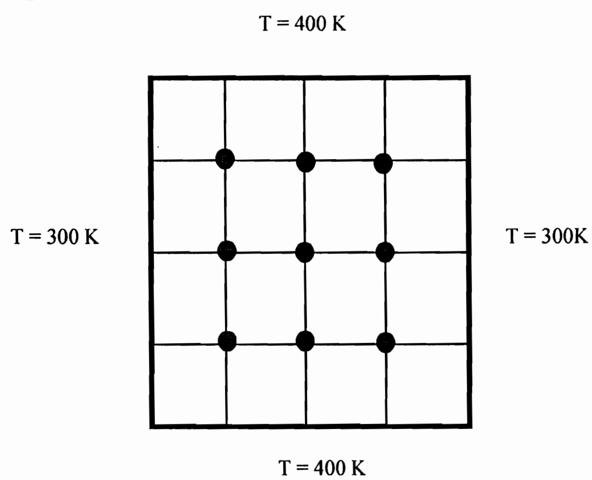
Q1. Take the differential equation $u,_{xx} + u,_{x} + f = 0$ in the domain 0 to 1

BC-1: $u(0) = g$ BC-2: $-u,_{x}(1) = h$

1.1 Write the weak form of the problem.
1.2 Identify the nature of the boundary conditions given in the problem.
1.3 Write down the respective Sobolev Spaces for the test and the trail functions.
1.4 Write the Galerkian form of the problem.
1.5 Write the general matrix form of the problem ($\underline{K}\,\underline{d} = \underline{F}$).
1.6 Is the stiffness matrix ($\underline{K}$) symmetric? Explain.
1.7 For a single element, what will be the necessary shape functions?
1.8 Solve the problem for 1 Degree of Freedom when f is constant
1.9 Solve the same problem with Finite Difference Method using both the boundary conditions (Find $u(1)$).

Marks : 40 (8×4+8)

Q2. The walls of a slab are maintained at constant temperatures, shown in the figure. Inside the domain heat is conducted according to Lap lace equation. You need to find the temperatures at the points mentioned. Take into consideration:



2.1 The figure is supposed to depict Δx = Δy.
2.2 Use the Alternating Direction Implicit Method (ADI).
2.3 To improve convergence use the parameter p such that the amount of the diagonal term kept in the implicit term is -(2+p) and the term kept on the R.H.S is (2-p).
2.4 The value of p can range between $0 \leq p \leq 2$. Take p=1 to solve the problem.
2.5 Sweep in the horizontal direction for a particular value of j.
2.6 Repeat step 2.5 for j=j+1 until j=Ny-1
2.7 Sweep in the vertical direction for a particular value of i.
2.8 Repeat step 2.7 for i=i+1 until i=Nx-1
2.9 At the end of it check you have swiped in both X and Y direction once.
2.10 To ease your calculations at each stage make a diagram with updated temperatures.
2.11 Take the unknown temperatures as 350K (= (400+300)/2) as the initial guess.

Marks: 40 (Formulation: 4+18×2)
