EEL 731 MAJOR/SCDR/02-05-08

Full marks : 100 ; Time : 120 minutes

All questions do not carry equal marks

Q. 1 Give a lattice realization of the two transfer functions $H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3}$ and $G(z) = 1 - 2z^{-1} + 3z^{-2} - 4z^{-3}$ by the procedure given in the class. Show that the realization is virtually multiplierless. [13+2=15]

Q. 2 An ideal analog differentiator has the transfer function $H_a(j\omega) = j\omega$. For an ideal digital linear phase differentiator, the transfer function will be of the form

$$H_d(e^{j\omega}) = j\omega e^{-j\omega\tau}, \; 0 \le |\omega| \le \pi.$$

Find an expression for the impulse response $h_d(n)$ of the differentiator. Design an FIR differentiator of length 5 with a rectangular window and give a rough sketch of the magnitude response by computing its values at $\omega = \pi/6, \pi/4, \pi/3, \pi/2, 2\pi/3$ and π. [8+4+8=20]

Q. 3 A second order IIR digital bandpass filter has a maximum response of unity at $\omega = \pi/3$ and a 3 dB bandwidth of 0.2213 radian. This filter is cascaded with an IIR digital bandstop filter whose rejection frequency is $\omega = \pi/3$ and whose difference between the two 3 dB frequencies is 0.2213 radian. Give a rough sketch of the magnitude response of the overall filter and find the frequencies at which the maximum response occurs. What is the value of the maximum response ? [6+8+1=15]

Q. 4 An analog normalized lowpass filter has the magnitude squared function

$$|H_a(j\omega)|^2 = (1+\varepsilon^2)/\{[1+\varepsilon^2 C_n^2(\omega)][1+\varepsilon^2 C_{n-1}^2(\omega)]\},$$

where $C_n(\omega)$ is the Chebyshev polynomial of order n.

(a) What is the order of the resulting filter ?

(b) What are the magnitude values at $\omega = 0$ and $\omega =$ infinity ?

(c) Show that the magnitude satisfies the following inequality in the passband :

$$A \le |H_a(j\omega)| < B.$$

Find A and B. Why is the equality sign missing in the upper bound ?

(d) Show that $dC_n(\omega)/d\omega|_{\omega=1} = n^2$.

(e) Hence find the cutoff slope of the filter. [2+4+5+5+9=25]

Q. 5 (a) Show that $A_1(z) = (z^{-1} - \alpha^*)/(1 - \alpha z^{-1})$ is an all-pass function irrespective of whether α is real or complex.

(b) Show that

$$1 - |A_1(z)|^2 = (|z|^2 - 1)(1 - |\alpha|^2) / |z - \alpha|^2.$$

(c) Hence show that $|A_1(z)|$ is >, = or < 1 for $|z|$ <, = or > 1.

(d) Finally, argue that in part (c), $A_1(z)$ can be replaced by any all-pass function. [8+9+4+4=25]