

## Department of Electrical Engineering

EEL704, Robotics & Automation,

Major Test , 2006-2007/II.

Max. time : 2 hours, Max. marks: 80.

Marks: Q1: 10, Q2: 10, Q3: 12, Q4: 13, Q5: 10, Q6: 12, Q7: 13

➤ Write clearly each step of your calculation.

Q1 Suppose a perspective transformation of is given by  $T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\lambda} & 1 \end{bmatrix}$  with  $\lambda = 1.0$ .

Find the world coordinates whose image points are of the form  $[x_0 \ y_0 \ 0]^T$ . What is the expression of Z and explain reasons for any discrepancy in results?

Q2. (a) Suppose a transformation  $T = T_B^A$  where  $T = \begin{bmatrix} 0.866 & -0.5 & 0.0 & 10.0 \\ 0.5 & 0.866 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 5.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$

Draw the frame diagram which qualitatively shows their arrangement.

(b) Frames describing the base of a robot and an object are given relative to the universe

$$T_{obj}^U = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_R^U = \begin{bmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Find a transformation of the Robot configuration  $T_{obj}^R$  if the hand of the robot is to be placed on the object.

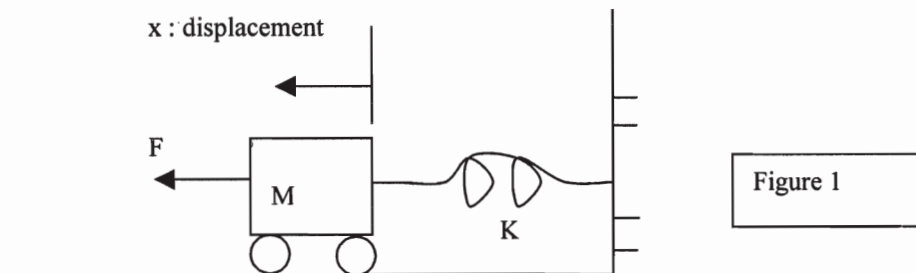
(c) Explain briefly the physical significance of Jacobian in the context of serial link manipulator.

Q3. Suppose  $\mathbf{R}$  is a rotation matrix and  $\mathbf{S}(\mathbf{a})$  is the skew symmetric matrix corresponding to vector  $\mathbf{a}$ . For any vector  $\mathbf{a}$  and  $\mathbf{b}$  belonging to  $\mathbb{R}^3$ , show that

(a)  $\mathbf{R} \mathbf{S}(\mathbf{a}) \mathbf{R}^T = \mathbf{S}(\mathbf{R}\mathbf{a})$

(b) Suppose  $\mathbf{a} = [1 \ -1 \ 2]^T$  and  $\mathbf{R} = \mathbf{R}_{x,90}$ . Compute  $\mathbf{S}(\mathbf{R}\mathbf{a})$ .

(c) Figure 1 shows a mass spring system. Derive the equation of motion using Euler-Lagrange equation.



Q4. The dynamics of a single link manipulator is given by

$$5\ddot{\theta} - 8 \sin\theta + 2 \dot{\theta}^2 = u$$

- (a) Suppose, a link is motionless at  $\theta=0^\circ$  and it is desired to move the joint to  $\theta=45^\circ$  in 3 secs with 0 velocity at final point. Find a cubic polynomial  $\theta_d(t)$  to define the path?
- (b) Design a trajectory following controller such that the system is always critically damped with closed loop stiffness = 10 to track  $\theta_d(t)$  (defined in part (a)).
- (c) Analyse the closed loop stability of the above system using Lyapunov stability theory.

Q5. Consider a n link manipulator described by

$$M(\theta)\ddot{\theta} + V(\theta, \dot{\theta})\dot{\theta} + G(\theta) = u$$

where  $M(\theta)$  is the inertia matrix,  $V(\theta, \dot{\theta})$  is the coriolis forces and  $G(\theta)$  is gravity term. To track a constant trajectory  $\theta_d$ , a control law is given as  $u = K_p E - K_d \dot{E} + G(\theta)$  where  $K_p$ ,  $K_d$  are positive definite diagonal matrices, error  $E = (\theta_d - \theta)$ . Show that the closed loop system is asymptotically stable using a suitable Lyapunov function.

Q6.(a) Explain briefly the different components of a fuzzy logic based inferencing system.

(b) Explain two methods for computing the intersection two fuzzy sets.

(c) Consider a fuzzy set  $A = 0.1/1 + 0.5/2 + 0.8/4 + 1/5 + .8/6 + 0.2/-2 + 0.1/-1$ .

Suppose a function  $f(x) = x^2$ . Find the fuzzy set  $f(A)$ ?

Q7. Figure 2 shows an image where 0 & 2 corresponds to background and object image respectively. each pixel of figure 2 is of unit length and the origin is assumed to be the top left-hand corner of the image.

- (a) Using sobel operators, find the gradient at pixel denoted as p.
- (b) Which type(s) of connectivity exist between the pixels p and q. Justify your answers.
- (c) Express the zero moment in terms of pixels?
- (d) What is the position and orientation of the connected component.

