EEL 316 Major Test Semester II 2008-2009

Answer all questions (Q.1: 30 marks, Q.2: 30 marks, Q.3: 10 marks) Full Marks: 70

1. Consider the case of BFSK in which signals $s_1(t)$ and $s_2(t)$ (corresponding to symbols '0' and '1', respectively), given by

$$s_1(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c t) , \quad s_2(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi [f_c + f_d] t) ,$$

are *transmitted with equal a priori probabilities* over an AWGN channel with noise p.s.d. of $N_0/2$ over a symbol interval of $0 \le t < T_s$, where $f_c \gg \frac{1}{T_s}$ and $\underline{\frac{1}{2T_s} \le f_d \le \frac{1}{T_s}}$. Let an orthonormal basis for the signal space be $\{\phi_1(t), \phi_2(t)\}$, where

$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) , \quad 0 \le t < T_s.$$

   (a) Find $\phi_2(t)$ using Gram-Schmidt orthogonalization. [6]

   (b) Find the signal vectors $\underline{s}_1$ and $\underline{s}_2$ and draw the signal constellation, labeling the relevant portions. [8]

   (c) If coherent detection with a MAP receiver is performed, then find the SEP $P_{e,coh}$ in terms of E_s, T_s, N_0, f_d. Find the value of $f_d T_s$ for which $P_{e,coh}$ is a minimum. Calculate $P_{e,coh,min}$ (the minimum value of $P_{e,coh}$) when $\frac{N_0}{2} = \frac{E_s}{40}$. [10]

   (d) What is the minimum value of $f_d T_s$ for the signaling to be orthogonal? If noncoherent detection is performed with orthogonal signaling, then calculate the SNR E_s/N_0 in dB so that the symbol error probability $P_{e,noncoh}$ equals $P_{e,coh,min}$ found in (b). [6]

2. (a) The SEP of a 16-QAM system operating at $E_{av}/N_0 = 20$ dB is always less than or equal to the SEP of an M-PAM system operating at $E_{av}/N_0 = 30$ dB. Find M_{min} (a power of 2), the minimum value of M for the M-PAM system. For $M = M_{min}$, calculate the bandwidth efficiency ratio $\rho_{M_{min}\text{-PAM}}/\rho_{16\text{-QAM}}$. [10]

   (b) The SEP of an M-PSK system operating at $E_s/N_0 = 20$ dB is always less than or equal to the SEP of an M_{min}-PSK (M_{min} as in (a)) system operating at $E_s/N_0 = 30$ dB. Find M_{max} (a power of 2), the maximum value of M for the M-PSK system operating at $E_s/N_0 = 20$ dB. [6]

   (c) The bit error probability of a coherent orthogonal 4-FSK system operating at $E_s/N_0 = 20$ dB is the same as the SEP of a noncoherent orthogonal M_{max}-FSK (M_{max} as in (b)). Using appropriate approximations, calculate the E_s/N_0 (in dB) at which the noncoherent orthogonal M_{max}-FSK system operates. [8]

   (d) A noncoherent orthogonal BFSK system and a coherent orthogonal 16-FSK system operating at the same E_s/N_0 have the same SEP. Calculate the E_s/N_0 in dB. [6]

3. An analog signal is sampled, quantized, and encoded into a binary PCM wave. The number of representation levels used is 16. Two synchronizing pulses are added at the end of each codeword representing a sample of the analog signal. The resulting PCM wave is transmitted over a channel of bandwidth 30 kHz using a quaternary PAM system with raised cosine spectrum. The rolloff factor is 1/3.

(a) Find the rate (in bps) at which information is transmitted through the channel. [6]

(b) Find the rate at which the analog signal is sampled. What is the maximum possible value for the highest frequency component of the analog signal? [4]

Some Formulae

- $\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \text{if } |t| \leq \frac{T}{2}, \\ 0 & \text{if } |t| > \frac{T}{2}, \end{cases}$ $\qquad \text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$

- In the range $0 < x < 2$, $\text{sinc}(x)$ is minimum at $x = 1.43$, and $\text{sinc}(1.43) = -0.2172$.

- Fourier Transform pairs:

$$\text{rect}\left(\frac{t}{T}\right) \leftrightarrow T\,\text{sinc}\,(fT)\,, \qquad \exp(\jmath 2\pi f_0 t) \leftrightarrow \delta(f - f_0)\,, \qquad G(t) \leftrightarrow g(-f)$$

- MAP receiver: $\hat{i} = \arg\left\{\max_i \; -\|\underline{x} - \underline{s}_i\|^2 + N_0 \ln p_i\right\}$

- If $X \sim \mathcal{N}(0,1)$, then its p.d.f.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,, \quad -\infty < x < \infty, \qquad \text{and} \quad \Pr[X > x] = \int_x^\infty f_X(y)dy = Q(x) = 1 - Q(-x)$$

- Use the approximation $Q(x) \approx \dfrac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,, \quad x \geq 2\,,$ wherever applicable.

- PSK: $P_e = \dfrac{1}{\pi} \displaystyle\int_0^{\frac{\pi(M-1)}{M}} \exp\left(-\frac{E_s}{N_0} \frac{\sin^2\left(\frac{\pi}{M}\right)}{\sin^2\phi}\right) d\phi$

- PSK: $P_e \approx 2Q\left(\sqrt{\dfrac{2E_s}{N_0}} \sin\left(\dfrac{\pi}{M}\right)\right)$ for large SNR, $M \geq 4$

- PAM: $P_e = \dfrac{2(M-1)}{M} Q\left(\sqrt{\dfrac{6E_{av}}{(M^2-1)N_0}}\right)$

- coherent orthogonal FSK union bound: $P_e \leq (M-1)Q\left(\sqrt{\dfrac{E_s}{N_0}}\right)$ (bound tight for large SNR)

- noncoherent orthogonal FSK: $P_e = \displaystyle\sum_{k=1}^{M-1} \frac{(-1)^{k+1}}{(k+1)} \binom{M-1}{k} e^{-\frac{k}{(k+1)}\frac{E_s}{N_0}}$

- $\rho_{\text{PSK}} = \log_2 M$, $\rho_{\text{PAM}} = 2\log_2 M$, $\rho_{\text{FSK}} = \frac{2\log_2 M}{M}$