MEL 707 Applied Mathematics for Mechanical Engineers-- I Semester 2006-2007

Major

*Closed book, closed notes. You may use three (3) A-4 size sheets with your hand-written notes on it. Photocopied sheets are not be allowed. Use of programmable calculators is not permitted.*

Duration: 2 hrs 15 min  Full marks: 40

1. Evaluate the integral $\int_0^{\pi} e^{x}\cos(x)dx$ with a two-point Gauss quadrature formula. Then calculate the value of the integral with a four-segment application of the trapezoidal rule. Compute the true percent relative error in each case. Comment on the results. [8 marks]

2. For $f(x) = sin(x)$, a student is attempting to locate a value of the root between $x = 6$ and $x = 8$ ($x$ is measured in radians). She uses bisection method first and obtains the root. Next, she uses Newton-Raphson method with an initial guess of 7.86. Her results now are different and she faces difficulties in reconciling the results of one method with the other. She seeks your help. In order to help her, do the following:

(a) find the root using the bisection method. Comment on the accuracy of the value obtained.

(b) Use Newton-Raphson method with an initial guess of 7.86 and explain difficulties, if any, faced in the process. [8 marks]

3. Consider a slab of width $L$ at initial temperature $T_0$. At $x = 0$, heat transfer takes place by convection with the ambient at temperature $T_{\infty_1}$ with heat transfer coefficient $h_1$. At $x = L$, the ambient temperature is $T_{\infty_2}$ and the heat transfer coefficient is $h_2$. Determine the temperature distribution as a function of both $x$ and $t$.

Note that for a medium initially at temperature $T_0$ and exposed to a medium at zero temperature at both $x = 0$ and $x = L$, the transient temperature distribution is given by:

$T(x,t) = \sum_{m=1}^{\infty} e^{-\alpha\beta_m^2 t}(1/N(\beta_m))X(\beta_m, x)\int_0^L X(\beta_m, x')F(x')dx'$

You can use the above equation in the derivation of the desired expression for the temperature distribution. In the above,

$1/N(\beta_m) = 2[(\beta_m{}^2 + H_1{}^2)(L + \frac{H_2}{(\beta_m)^2 + H_2^2}) + H_1]^{-1}$

Where $H_i = h_i/k, i = 1,2$, with $k$ being the thermal conductivity of the slab and $h_1$ and $h_2$ are the heat transfer coefficients at the surfaces $x = 0$ and $x = L$ respectively. If there is a need, you have to *derive* the equation for $X(\beta_m, x)$ and also arrive at the equation for determination of $\beta_m$. [12 marks]

4. Consider a thermocouple junction, which may be approximated as a sphere, to be used for temperature measurement in a gas stream. The convection coefficient between the junction surface and the gas is $h = 400$ W/m$^2$K, and the junction thermophysical properties are $k = 20$ W/mK, $c = 400$ J/kgK, and $\rho = 8500$ kg/m$^3$. The junction diameter is 0.7 mm. If the junction is at 25°C and is placed in a gas stream that is at 200°C, how long will it take for the junction to reach 175°C? Determine the approximate time using 4$^{th}$ order Runge-Kutta method and compare with the exact analytical solution. Choose your step size. What will be the temperature ($T$) reached by the thermocouple at 2 s if it has an emissivity $\epsilon$ of 0.8 and receives radiation from a background surface at $T_b$ =600 K? The Stefan-Boltzman constant is $\sigma = 5.67\times10^{-8}$ W/m$^2$K$^4$. Set up the equation for the steady state temperature of the thermocouple when both convection and radiation are present and obtain the solution by a suitable numerical method. [12 marks]

Hint: The radiation heat flux received by the thermocouple junction is $\epsilon\sigma(T_b^4 - T^4)$.