MAL-526

Time: 2Hrs | Major Exam | Max marks:40

1. Solve the following cost minimizing transportation problem starting with the basic feasible solution $x_{12} = 30, x_{21} = 40, x_{32} = 20$ and $x_{43} = 60$

	D_1	D_2	D_3	
O_1	4	5	2	30
O_2	4	1	3	40
O_3	3	6	2	20
O_4	2	3	7	60
	40	50	60	

Write the dual of the above transportation problem and give its optimal solution and the optimal value.

2. Formulate the problem of determining a point $P : (x, y, z)$ which is in the first octant and lies on the plane $2x + 3y + 4z = 5$ such that its (Point P) distance from the origin is least. Solve the problem by appropriate technique and perform only one iteration of the same.

3. Consider the linear programming problem:

$$\max \quad x_1 + 6x_2$$
subject to,
$$\begin{aligned} x_1 + 3x_2 &\le 12 \\ 3x_1 + x_2 &\le 12 \\ x_1 + x_2 &= 13 \\ x_1, \ x_2 &\ge 0. \end{aligned}$$

and let following be its last tableau of a LPP which has been solved by Two phase method

X_b	a_1	a_2	a_3	a_4	a_5
3	0	1	3/8	−1/8	0
3	1	0	−1/8	3/8	0
7	0	0	−1/4	−1/4	1
−7	0	0	1/4	1/4	0

(a) What conclusion can you draw from the tableau?

(b) Update the table if b is changed to $(12, 12, 6)^T$, what do you infer now?

(c) Identify the redundant constraint (if any) and hence find the new optimal basic feasible solution of the reduced problem?

(d) Does the addition of the constraint $x_1 + x_2 = 4$ in the reduced problem effects the solution. If yes, then incorporate the constraint in the tableau(DON'T perform any iteration).

4. Consider the linear programming problem

$$\max \quad 5x_1 + 12x_2 + 4x_3$$
subject to,
$$\begin{aligned} x_1 + 2x_2 + x_3 &\le 10 \\ 2x_1 - x_2 + 3x_3 &= 8 \\ x_1, \ x_2, \ x_3 &\ge 0. \end{aligned}$$

Without using simplex algorithm find the dual optimal solution given that x_1, x_2 are strictly positive in the optimal solution.

5. Consider the following LPP

$$\begin{array}{llllll}
(P) & \text{Max} & 2x_1 + 3x_2 + 4x_3 + x_4 + 7x_5 + 5x_6 \\
& \text{subject to,} \\
\end{array}$$

$$\begin{array}{lllll}
x_1 & +x_2 & & \leq & 1 \\
x_3 & +x_4 & & \leq & 1 \\
x_5 & +x_6 & & \leq & 1 \\
x_1 & +x_3 & +x_5 & = & 1 \\
x_2 & +x_4 & +x_6 & = & 1 \\
& & x_i & \geq 0 & \forall i.
\end{array}$$

Obtain optimal solution and optimal value of the dual problem. (Hint: exploit the form of the coefficient matrix)

6. Consider the following optimal simplex tableu of the associated LPP for a given AILP (with maximization form, and x_3 and x_4 as slack variables)

	x_1	x_2	x_3	x_4
$x_1 = 11/2$	1	0	11/36	$-1/36$
$x_2 = 9/2$	0	1	$-1/12$	$-1/12$
$Z = 69/12$	0	0	7/12	3/12

Generate a Gomorey's cut constraint through x_1 and append the constraint in the above problem, perform one complete iteration of the dual simplex algorithm. Also show the actual cut constraint graphically.

7. Using KKT find the value of β for which $x_1 = 1, x_2 = 2$ will be optimal to the problem

$$\begin{array}{ll}
\text{Max} & 2x_1 + \beta x_2 \\
\text{subject to,} & \\
& x_1^2 + x_2^2 \leq 5, \\
& x_1 - x_2 \leq 2.
\end{array}$$

Verify your result by using a graphical procedure.

8. Are the following statement true? Give reasons. (No marks will be awarded if reason is not provided)

   - Let Z^* be the optimal value of a give LPP (say (LP1)). Let a constraint be dropped from $(LP1)$ to get a new LPP (say (LP2)). Let $\hat{Z}$ be the optimal value of (LP2). Then $\hat{Z} \leq Z^*$.
   - If all c_{ij} of an (4×4) assignment problem (AP) are increased by 5 then the optimal value will also increase by 5.
   - For the constraints $x_1 + x_2 \leq 8$, $2x_1 + x_2 \leq 10$, $x_2 - x_1 = 4$, the constraint qualification holds at the point (2,6).
   - Let $f : \mathbf{R} \to \mathbf{R}$ be a convex function with $f(x) > 0$ for all x. Let $\phi(x) = f(x) + (f(x))^3 + (f(x))^5, x \in \mathbf{R}$. Then ϕ is a convex function.
   - For the statement "Profit P is close to Rs 100" the membership function takes the value zero for all $P \neq 100$.