Department of Electrical Engineering, IIT Delhi

EEL806 Computer Vision: Major Examination

(Closed book/Closed Notes) Time: 2 hours Maximum Marks: 25

"Thou shalt not covet thy neighbour's answers"

Useful Formulae and Results:
The KLT: $\mathbf{r_i} = \mathbf{U}^T \mathbf{p_i}$. Here $\mathbf{p_i}$ are Type-I normalised $k \times 1$ patterns (n of which can be stacked to get the Type-I normalised pattern matrix $\mathbf{P}$). $\mathbf{U}$ is a matrix of eigenvectors of the covariance matrix $\mathbf{A} = \frac{1}{n}\mathbf{P}\mathbf{P}^T$. There are k eigenvectors $\mathbf{u_i}$ corresponding to eigenvalues λ_i. Λ is a diagonal matrix having eigenvalues λ_i along the main diagonal.
Useful Result 1: Eigenvectors of a symmetric matrix are orthonormal.
Useful Result 2: Diagonalisation of a square matrix $\mathbf{B}$: $\mathbf{B} = \mathbf{U}\Lambda\mathbf{U}^{-1}$.
The SVD: $\mathbf{P} = \mathbf{U}\ \Sigma\ \mathbf{V}^T$, where this $\mathbf{U}$ is the $k \times k$ matrix of orthonormal basis vectors, Σ is a $k \times n$ matrix having singular values σ_i along the main diagonal (the other values are all zero), and $\mathbf{V}$ is an $n \times n$ matrix of the eigenvectors of $\mathbf{A}' = \mathbf{P}^T\mathbf{P}$. These eigenvectors $\mathbf{v_i}$ correspond to eigenvalues λ_i, and we define $\sigma_i = \sqrt{\lambda_i}$, and $\mathbf{u_i} = \frac{1}{\sigma_i}\mathbf{P}\mathbf{v_i}$.

1. **The long and short of it...**

   (a) Explain the concept of compression with regard to the KLT. Use suitable mathematical expressions and explanations. What are the properties of the compression, which are similar to the original KLT?

   (b) Explain the concept of compression in the SVD. *((2+2)+2 marks)*

2. **Reconstruction constriction** Given a pattern $\mathbf{p}$, we project it onto a set of orthonormal basis vectors, and consider a reconstruction $\hat{\mathbf{p}}$ in terms of a linear combination of the basis vectors $\hat{\mathbf{p}} = \sum_j c_j \mathbf{u_j}$.

   (a) Consider both the KLT and SVD without any compression. Show the following result for both cases, with suitable limits on the summation, and appropriate mathematical expressions: If the input vector $\mathbf{p}$ is one of the pattern vectors $\mathbf{p_i}$ in $\mathbf{P}$, show what $\hat{\mathbf{p}} = \mathbf{p_i}$. *(3+3 marks)*

   (b) If a certain set of linear combination coefficients c_j multiplying a set of basis vectors $\mathbf{u_i}$ gives a result $\hat{\mathbf{p}}$ - that set of coefficients is unique. Give a simple proof of the same. *(2 marks)*

3. **A cranky rank question** What happens in the KLT if the covariance matrix is rank-deficient? Explain in words, with the help of mathematical expressions. You can use suitable examples. *(2 marks)*

4. **Prediction, Updating** Using the basic laws of probability, prove the *Prediction Equation* and the *Update Equation*, below:

$$P(\mathbf{X}_t|\mathbf{Z}_{1:t-1}) = \int P(\mathbf{X}_t|\mathbf{X}_{t-1})P(\mathbf{X}_{t-1}|\mathbf{Z}_{1:t-1})d\mathbf{X}_{t-1} \quad (1)$$

$$P(\mathbf{X}_t|\mathbf{Z}_{1:t}) \propto P(\mathbf{Z}_t|\mathbf{X}_t)P(\mathbf{X}_t|\mathbf{Z}_{1:t-1}) \quad (2)$$

   Use the discrete case in the proofs. Clearly mention the point where you needed to make assumptions, the assumptions themselves, and some justification for the assumptions. *(2+2 marks)*

Please turn over...

5. **Tracker Cracker** Explain how a particle filter can track multiple objects of the same kind, in respect of the following points. Mention how this is different for a particle filter tracking a single object.

   (a) The INITIALISATION step

   (b) The SELECT step

   (c) The PREDICT step

   (d) The MEASURE step

   (e) The OUTPUT step *(5 marks)*