

# MAJOR TEST

\* EPL 338: Nonlinear Phenomena in Physics & Engg.

Attempt 5 problems.

B. 5. 2008

Time: 2 hrs

M. Marks: 50

1. Obtain the values of period-2 fixed points  $x_1^*$ ,  $x_2^*$  for the maps i)  $x_{n+1} = A x_n (2 - x_n)$ , ii)  $x_{n+1} = 2 x_n \pmod{1}$ .
2. In an accelerator electron Lorentz factor  $\gamma$  varies as  $d\gamma/dz = A \cos \eta$ , where  $\eta = \omega t - k z$ ,  $\gamma^2 = 1 + \alpha A^2 + p_z^2/m^2 c^2$ ,  $p_z = \gamma m v_z$ ,  $m$  is the electron rest mass. Deduce the equation for the separatrix and estimate the maximum energy gain by a trapped electron.
3. A collisionless cold plasma has a wiggler magnetic field  $\vec{B}_w = B_0 (\hat{x} - i \hat{y}) e^{i k w z}$ . An em wave propagates through it with  $\vec{E} = A (\hat{x} + i \hat{y}) e^{i(\omega t - 0.9 \omega z/c)}$ . Obtain the value of  $k w$  required for resonant second harmonic generation. Obtain the amplitude of the second harmonic as a function of  $z$ .
4. The motion of a planet is governed by  $r^2 \ddot{\phi} = C$ ,  $\ddot{r} - r \dot{\phi}^2 = -K/r^2$ . Obtain the radius of circular orbit.

Small perturbation.

5. Obtain the growth rate of parametric oscillations governed by  $\ddot{x} + 2\Gamma \dot{x} + \omega_0^2 (1 + \mu \cos \Omega_0 t) x + \beta x^3 = 0$ , when  $\beta \neq 0$ ,  $\Omega_0 = 2\omega_0 + \Delta$ ,  $\Delta, \Gamma \ll \omega_0$ . Discuss the consequences of finite  $\Gamma$  and  $\Delta$ . If  $\beta \neq 0$ , what effect would you foresee physically?

OR

5. ~~Using the~~ A laser beam  $\vec{E} = \hat{x} A_0 e^{-i(\omega t - kz)}$ ,  $A_0|_{z=0} = e^{-i(\omega/c)r^2/2R} e^{-r^2/2r_0^2}$  is propagating in free space. Estimate the spot size at the focus. Plot it as a function of  $R$ .