Wide binaries distributions

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1 Wide binaries - Distributions

1.1 Parameters

- Eccentricity:
$$e := \text{Thermal distribution} \rightarrow p_e(e) = 2Ke\Big|_0^1$$

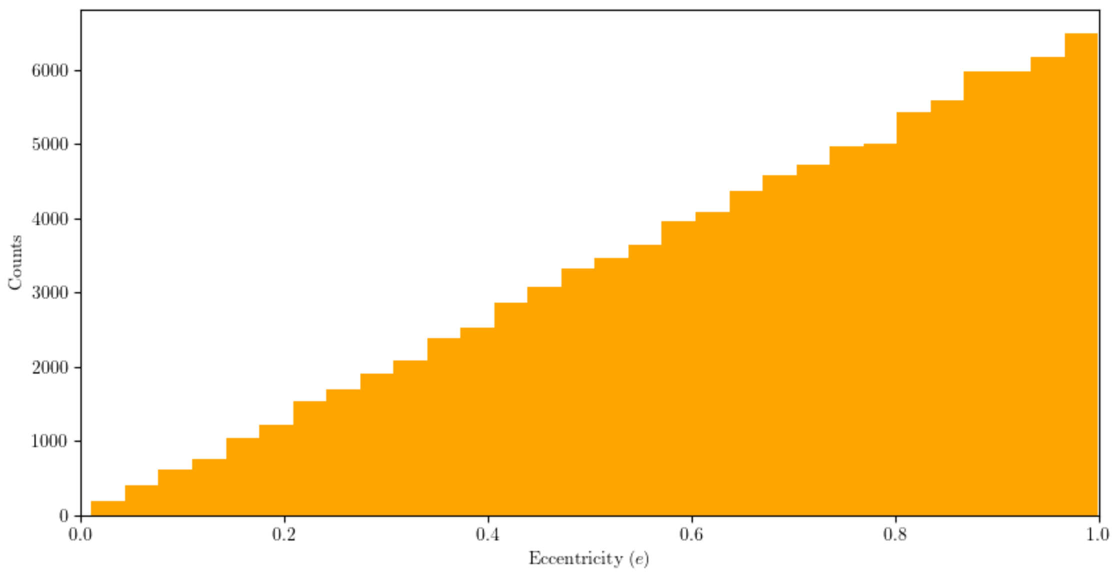

Figure 1.1: Eccentricity, normalized thermal distribution - 100,000 samples.

- Eccentricity squared:

$$e^2 := \text{Uniform distribution} \rightarrow p_{e^2}(e^2) = \mathcal{U}\{0,1\}$$

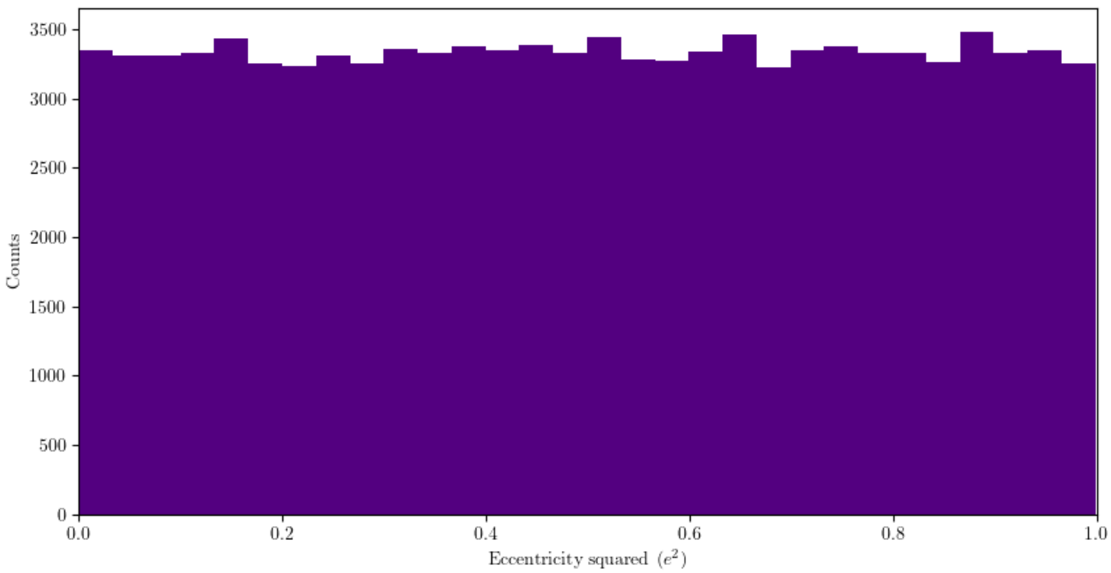

Figure 1.2: Eccentricity squared, normalized uniform distribution - 100,000 samples. This parameter is not relevant, since we already have the eccentricity. Nevertheless, it was computed using the e-thermal distribution, in order to prove that we obtain a uniform distribution for the squared, as predicted.

- Initial phase angle:

$$\phi_0 := \text{Uniform distribution} \rightarrow p_{\phi_0}(\phi_0) = \mathcal{U}\{0,2\pi\}$$

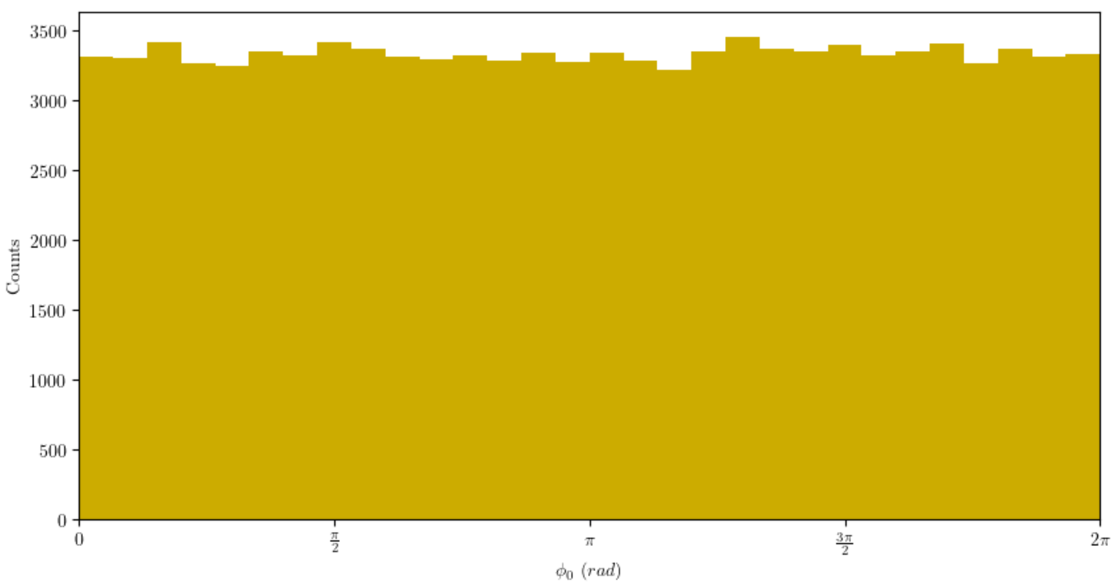

Figure 1.3: Initial phase angle, normalized uniform distribution - 100,000 samples.

- Phase angle:

$$phi := \text{Phase angle distribution} \rightarrow p_\phi(\phi|e) = \left.\frac{(1-e^2)^(3/2)}{2\pi(1+e\cos\phi)}\right|_0^{2\pi}$$

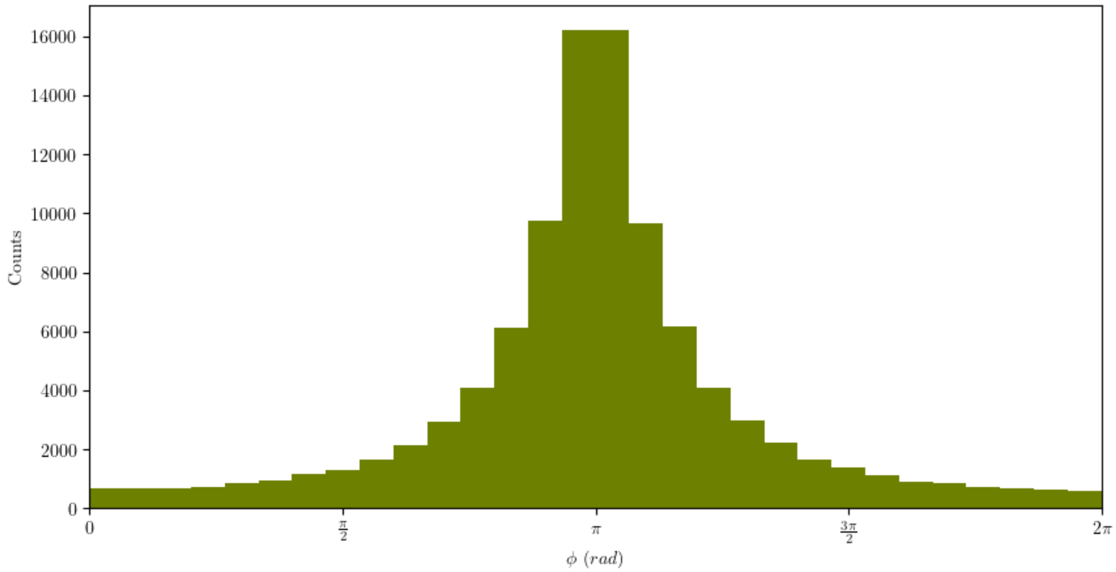

Figure 1.4: Phase angle, already normalized distribution - 100,000 samples

- Orbit semi-major axis:

$$a := \text{Power law distribution} \rightarrow p_a(a) = \left.\frac{K}{a}\right|_{200UA}^{0.6pc}$$

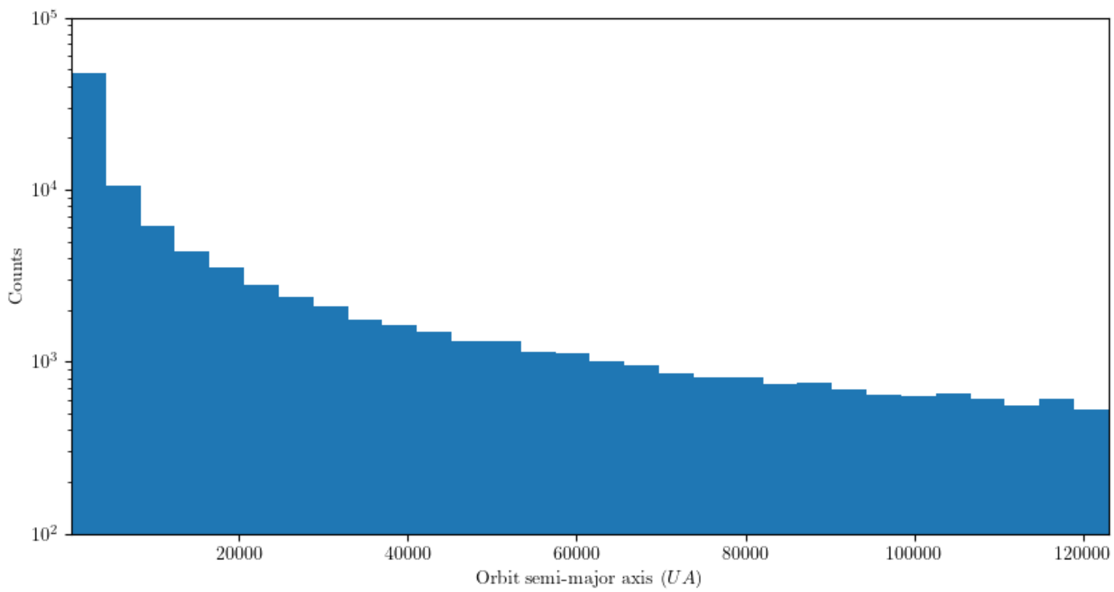

Figure 1.5: Orbit semi-major axis, normalized power-law distribution - 100,000 samples.

- Orbit angle:

$$i := \text{Orbit angle distribution} \rightarrow p_i(i) = \mathcal{U}\left\{0, \pi\right\}$$

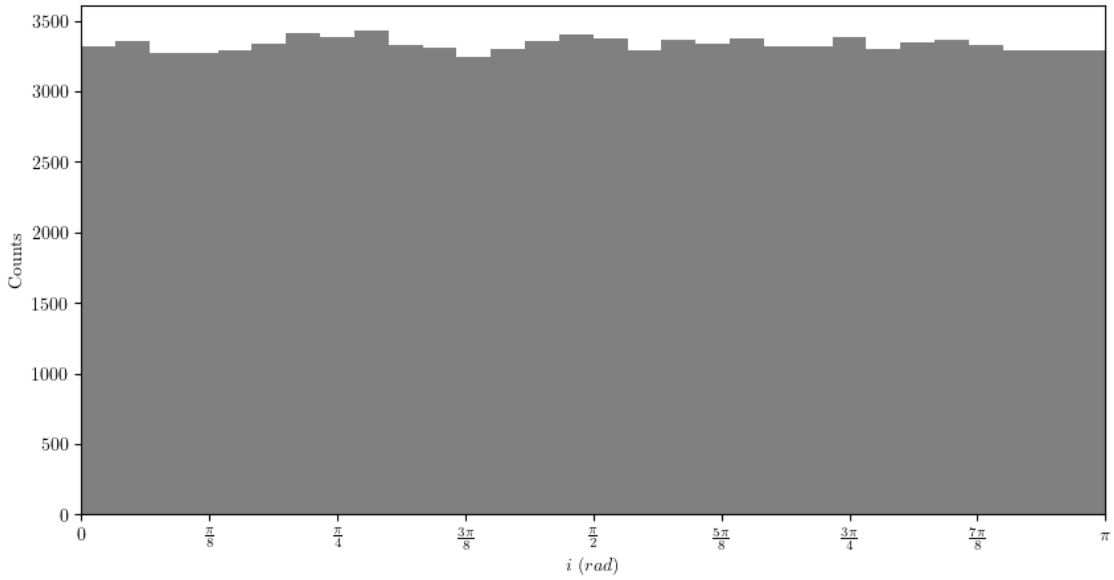

Figure 1.6: Orbit angle, normalized uniform distribution - 100,000 samples

- Angle between $\vec{v}$ and $\vec{r}$:

$$\alpha := \widehat{\vec{v}\vec{r}} \text{ distribution} \rightarrow \alpha(\phi, e) = \sin^{-1}\left(\frac{1 + e\cos\phi}{\sqrt{1 + e^2 + 2e\cos\phi}}\right)$$

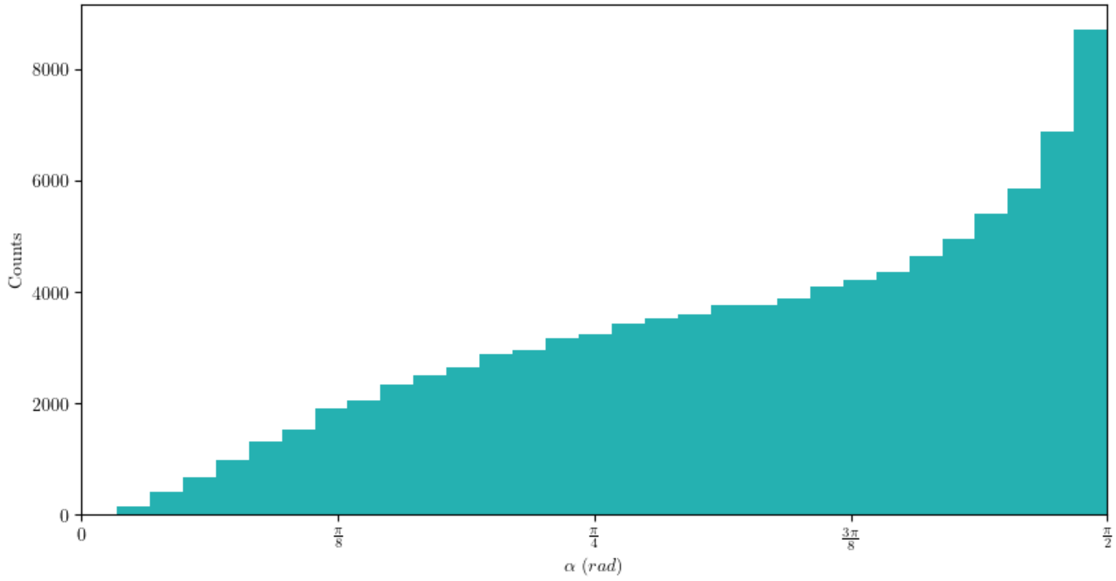

Figure 1.7: $\widehat{\vec{v}\vec{r}}$ angle distribution - 100,00 samples.

1.2 Projected distance r_{2D}

$$r_{2D}(a, \phi, \phi_0, i) = \frac{a(1-e^2)}{1+e\cos\phi}\sqrt{1-\sin^2 i\cos^2(\phi-\phi_0)}$$

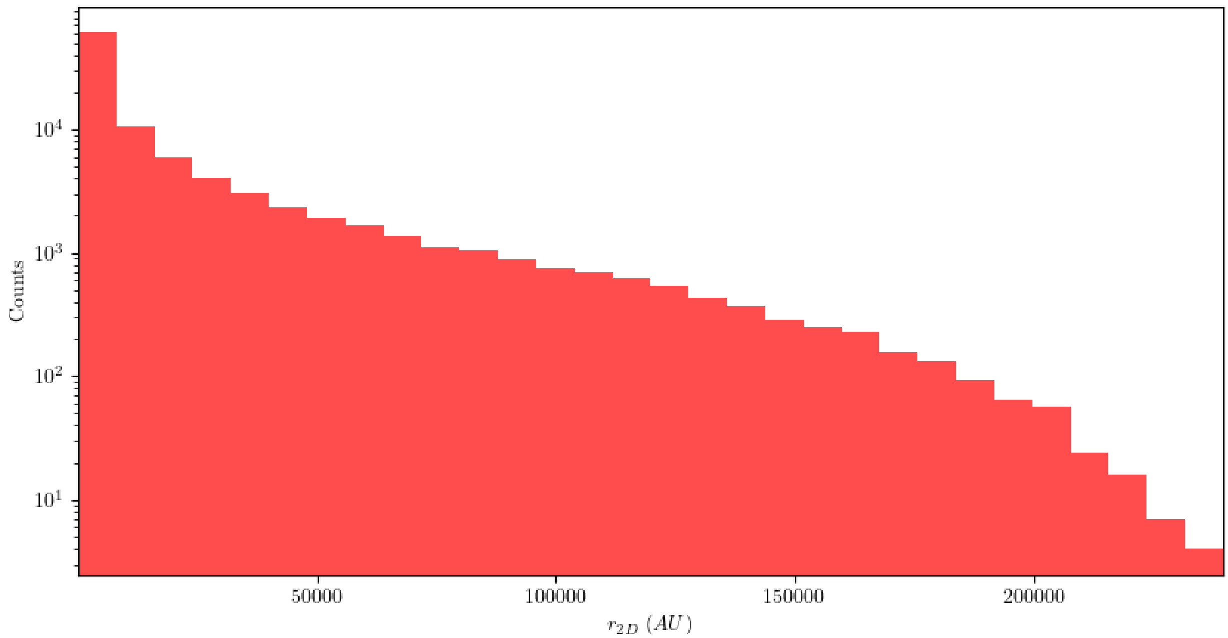

Figure 1.8: Estimated distribution for the projected distance - 100,000 samples.

1.3 Projected velocity v_{2D}

$$v_{2D}(\phi,\phi_0,i)=\sqrt{\frac{GM(1+e^2+2e\cos\phi)}{a(1-e^2)}}\sqrt{1-sin^2 i\cos^2(\alpha+\phi-\phi_0)}$$

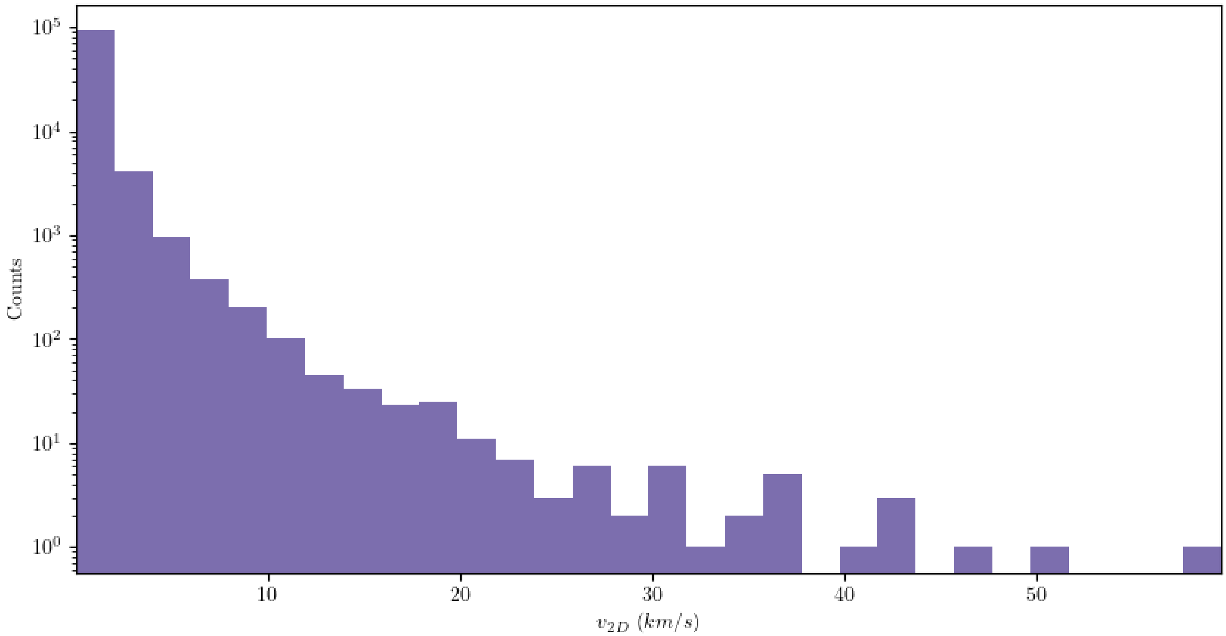

Figure 1.9: Estimated distribution for the projected velocity - 100,000 samples.

The stellar mass follows a uniform distribution $p_M(M)=\mathcal{U}\left\{0.5,1.5\right\}M_\odot$:

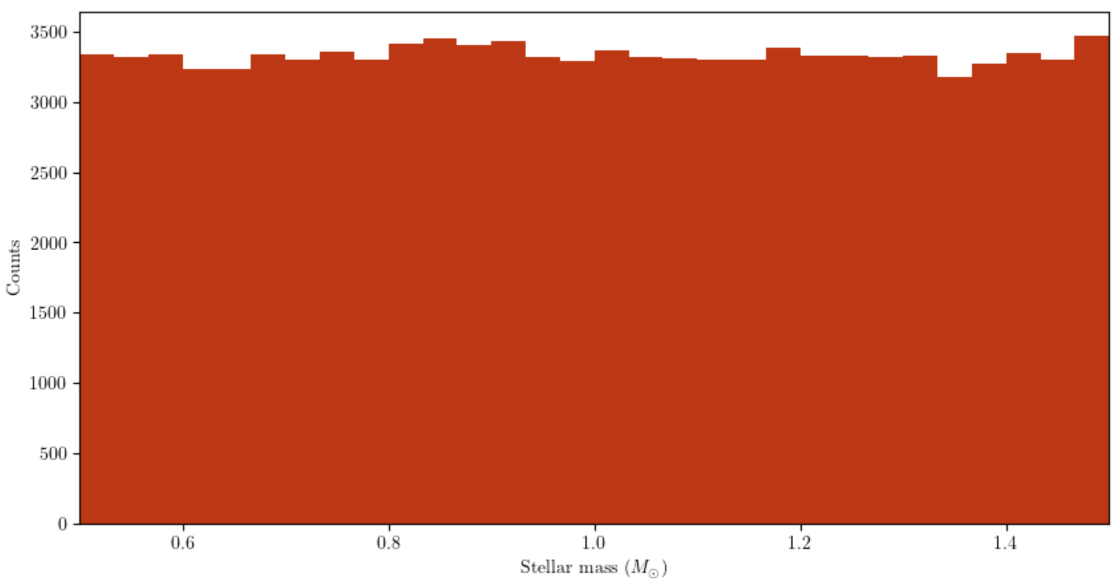

Figure 1.10: Stellar mass, normalized uniform distribution - 100,000 samples.