

# GRAPH-R1: TOWARDS AGENTIC GRAPHRAG FRAMEWORK VIA END-TO-END REINFORCEMENT LEARNING

Haoran Luo<sup>1,2</sup>, Haihong E<sup>1\*</sup>, Guanting Chen<sup>1</sup>, Qika Lin<sup>3</sup>, Yikai Guo<sup>4</sup>, Fangzhi Xu<sup>2</sup>, Zemin Kuang<sup>5</sup>, Meina Song<sup>1</sup>, Xiaobao Wu<sup>2</sup>, Yifan Zhu<sup>1</sup>, Luu Anh Tuan<sup>2</sup>

<sup>1</sup>Beijing University of Posts and Telecommunications <sup>2</sup>Nanyang Technological University

<sup>3</sup>National University of Singapore <sup>4</sup>Beijing Institute of Computer Technology and Application

<sup>5</sup>Beijing Anzhen Hospital, Capital Medical University

{luohaoran, ehaihong, yifan\_zhu}@bupt.edu.cn, anhtuan.luu@ntu.edu.sg

## ABSTRACT

Retrieval-Augmented Generation (RAG) mitigates hallucination in LLMs by incorporating external knowledge, but relies on chunk-based retrieval that lacks structural semantics. GraphRAG methods improve RAG by modeling knowledge as entity-relation graphs, but still face challenges in high construction cost, fixed one-time retrieval, and reliance on long-context reasoning and prompt design. To address these challenges, we propose **Graph-R1**, an agentic GraphRAG framework via end-to-end reinforcement learning (RL). It introduces lightweight knowledge hypergraph construction, models retrieval as a multi-turn agent-environment interaction, and optimizes the agent process via an end-to-end reward mechanism. Experiments on standard RAG datasets show that Graph-R1 outperforms traditional GraphRAG and RL-enhanced RAG methods in reasoning accuracy, retrieval efficiency, and generation quality. Our code is publicly available<sup>1</sup>.

## 1 INTRODUCTION

Large Language Models (LLMs) (Zhao et al., 2025) have achieved widespread success in NLP tasks. However, when applied to knowledge-intensive or proprietary knowledge-dependent applications, they still suffer from the hallucination problem (Zhang et al., 2023), generating inaccurate content. To improve credibility and factual consistency, Retrieval-Augmented Generation (RAG) (Lewis et al., 2020) introduces external knowledge sources as references, alleviating the knowledge bottleneck of pure language modeling. Nevertheless, existing RAG methods mostly rely on chunk-based text blocks (Gao et al., 2024), which makes it difficult to capture complex knowledge structures among entities. To address this, GraphRAG methods (Edge et al., 2025; Guo et al., 2025; Luo et al., 2025a) represent knowledge as entity-relation graphs, enhancing retrieval efficiency and generation quality.

Generally, GraphRAG methods consist of three processes: *knowledge graph construction*, *graph retrieval*, and *answer generation*. First, knowledge graphs are typically constructed by LLMs to extract entities and relations from text, forming a graph structure (Xu et al., 2024). Second, the retrieval process queries relevant subgraphs or paths through subgraph retrieval or path pruning strategies (Chen et al., 2025; Gutiérrez et al., 2025). Finally, the generation process prompts LLMs to generate answers based on the retrieved graph-based knowledge (Xiao et al., 2025).

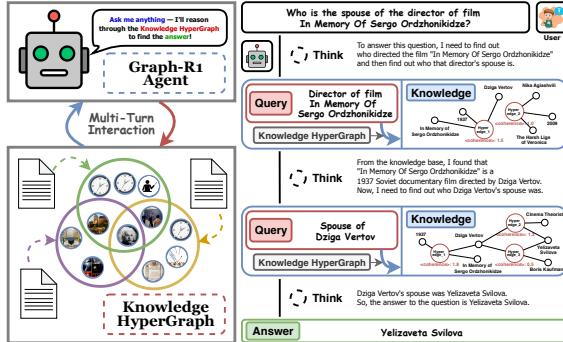


Figure 1: An illustration of Graph-R1.

\* Corresponding author.

<sup>1</sup><https://github.com/LHRLAB/Graph-R1>

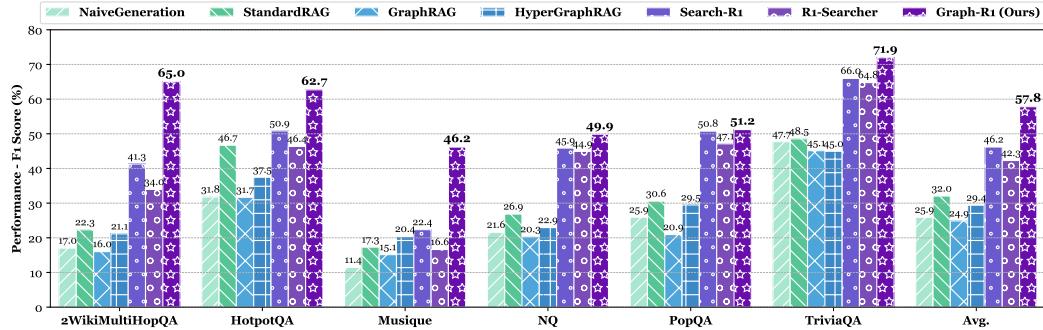


Figure 2: Comparison of F1 scores across RAG benchmarks. Using a graph as the knowledge environment enables RL to achieve a higher performance ceiling compared to chunk-based knowledge.

However, current GraphRAG methods still face three key challenges: **(i) High cost and semantic loss in knowledge construction process.** Compared to standard RAG, GraphRAG methods convert natural language knowledge into graph structures using LLMs, which results in high cost and often causes semantic loss relative to the original content (Luo et al., 2024; 2025a). **(ii) Fixed retrieval process with only one-time interaction in graph retrieval process.** Although existing GraphRAG methods design various retrieval strategies to improve efficiency, they all aim to gather sufficient knowledge in a single fixed retrieval (Chen et al., 2025), which limits performance in complex queries. **(iii) Dependence on large LLMs for long-context analysis and prompt quality in answer generation process.** Generation based on retrieved graph-structured knowledge often requires strong long-context reasoning ability, making the output quality highly dependent on the LLM’s parameter size and prompt design (Guo et al., 2025), resulting in unstable reasoning and generation.

To address these challenges, we propose **Graph-R1**, as illustrated in Figure 1, an agentic GraphRAG framework enhanced by end-to-end reinforcement learning (RL), inspired by DeepSeek-R1 (DeepSeek-AI et al., 2025). First, we propose a lightweight knowledge hypergraph construction method to establish a standard agent environment for the query action space. Moreover, we model the retrieval process as a multi-turn agentic interaction process, enabling LLMs to repeatedly perform the reasoning loop of “think-retrieve-rethink-generate” within the knowledge hypergraph environment. Furthermore, we design an end-to-end reward mechanism that integrates generation quality, retrieval relevance, and structural reliability of graph paths into a unified optimization objective. Using RL, the agent learns a generalizable graph reasoning strategy and achieves tighter alignment between structured graph-based knowledge and language generation.

We perform experiments on various standard RAG datasets (Jin et al., 2025b). Experimental results demonstrate that Graph-R1 outperforms traditional GraphRAG methods and RAG combined with RL methods (Jin et al., 2025a; Song et al., 2025) in reasoning accuracy, retrieval efficiency, and generation quality. As shown in Figure 2, the end-to-end RL strategy guides the agent through multiple turns of interaction and goal-driven exploration in the graph, effectively bridging the gap between knowledge representation and language generation. This work lays a foundation for building the next generation of knowledge-driven and strategy-optimized agent-based generation systems.

## 2 RELATED WORK

**RAG and GraphRAG.** Retrieval-Augmented Generation (RAG) (Lewis et al., 2020) improves LLM factuality by retrieving external knowledge, but suffers from data silos and limited structural understanding. GraphRAG (Edge et al., 2025) addresses these limitations by leveraging graph-structured knowledge to enhance retrieval and reasoning. Based on this, enterprise-oriented systems (Wu et al., 2024; Liang et al., 2025; Wang et al., 2025a) and efficient variants like LightRAG (Guo et al., 2025) are proposed. Recent efforts further extend representation power via hypergraphs, causal graphs, or heterogeneous graphs (Luo et al., 2025a; Feng et al., 2025b; Wang et al., 2025b; Xu et al., 2025), while retrieval is optimized using path-based exploration and pruning techniques (Chen et al., 2025; Gutiérrez et al., 2025; Liu et al., 2025; Wang, 2025). Our work introduces Graph-R1, the first agentic GraphRAG framework with end-to-end reinforcement learning.

**Reinforcement Learning for LLMs.** Reinforcement learning (RL) is increasingly adopted to enhance LLM reasoning (Wu, 2025; Luo et al., 2025b), as demonstrated by OpenAI’s o1/o3/o4 (OpenAI et al., 2024b). DeepSeek-R1 (DeepSeek-AI et al., 2025) achieves comparable capabilities and further introduces the Group Relative Policy Optimization (GRPO) (Shao et al., 2024) for scalable end-to-end training. GRPO-based reasoning has been extended to tasks such as visual understanding (Shen et al., 2025), logical reasoning (Xie et al., 2025), and program synthesis (Ma et al., 2025). RL-enhanced agents have also shown strong performance in multi-turn interaction (Lu et al., 2025; Feng et al., 2025a) and open-domain retrieval (Jin et al., 2025a; Song et al., 2025; Zheng et al., 2025; Sun et al., 2025), highlighting RL’s potential in agentic GraphRAG frameworks (Gao et al., 2025).

### 3 PRELIMINARIES

We formalize the GraphRAG pipeline into three stages as detailed below:

**(a) Knowledge Graph Construction.** This stage extracts structured relational facts from raw text. Given a knowledge collection  $K = \{d_1, d_2, \dots, d_N\}$ , the goal is to extract facts  $f_d$  from each semantic unit  $d \in K$  and aggregate them into a unified graph  $\mathcal{G}_K$ :

$$\mathcal{G}_K \sim \sum_{d \in K} \pi_{\text{ext}}(f_d \mid d), \quad (1)$$

where  $\pi_{\text{ext}}$  denotes an LLM-based extractor that parses each  $d$  into a set of relation-entity pairs  $f_d = \{(r_i, \mathcal{V}_{r_i})\}$ , with  $r_i$  as the relation and  $\mathcal{V}_{r_i} = \{v_1, \dots, v_n\}$  the participating entities.

**(b) Graph Retrieval.** Graph retrieval is formulated as a two-step process over  $\mathcal{G}_K$ : (1) retrieving candidate reasoning paths and (2) pruning irrelevant ones. Conditioned on a query  $q$ , the model first retrieves a candidate set  $\mathcal{X}_q = \{x_1, \dots, x_m\}$  and then selects a relevant subset  $\mathcal{Z}_q \subseteq \mathcal{X}_q$ . The overall objective is to maximize the expected joint likelihood of the two steps:

$$\max_{\theta} \mathbb{E}_{\mathcal{Z}_q \sim P(\mathcal{Z}_q \mid q, \mathcal{G}_K)} \left[ \prod_{t=1}^{T_x} P_{\theta}(x_t \mid x_{<t}, q, \mathcal{G}_K) \cdot \prod_{t=1}^{T_z} P_{\theta}(z_t \mid z_{<t}, \mathcal{X}_q, q) \right], \quad (2)$$

where  $T_x$  and  $T_z$  denote the number of retrieved and selected paths, respectively.

**(c) Answer Generation.** Given a query  $q$  and selected paths  $\mathcal{Z}_q$ , answer generation produces a natural language answer  $y$  grounded in graph-based evidence, formulated as:

$$P(y \mid q, \mathcal{G}_K) = \sum_{\mathcal{Z}_q \subseteq \mathcal{X}_q} P(y \mid q, \mathcal{Z}_q) \cdot P(\mathcal{Z}_q \mid q, \mathcal{G}_K), \quad (3)$$

where  $P(y \mid q, \mathcal{Z}_q)$  is generation likelihood and  $P(\mathcal{Z}_q \mid q, \mathcal{G}_K)$  is retrieval-pruning distribution.

### 4 METHODOLOGY: GRAPH-R1

In this section, as illustrated in Figure 3, we introduce Graph-R1, including agent initialization, multi-turn graph interaction, and outcome-directed end-to-end reinforcement learning.

#### 4.1 KNOWLEDGE CONSTRUCTION AND AGENT INITIALIZATION

Graph-R1 adopts an LLM-driven agent, initialized with a knowledge hypergraph environment  $\mathcal{G}_H$ , the action space  $\mathcal{A}$ , the state space  $\mathcal{S}$ , and the answer target  $y_q$  for the given query  $q$ .

**Graph Environment  $\mathcal{G}_H$ .** To support agentic reasoning, we propose a lightweight method for constructing a knowledge hypergraph  $\mathcal{G}_H$  from given domain knowledge  $K = \{d_1, d_2, \dots, d_N\}$ . For each chunk unit  $d \in K$ , an LLM-based extractor  $\pi_{\text{ext}}$  identifies  $m$  n-ary relational facts, where each comprises a semantic segment  $h_i$  and a set of participating entities  $\mathcal{V}_{h_i} = \{v_1, \dots, v_n\}$ . A shared encoder  $\phi(\cdot)$  is then used to generate semantic embeddings for both entities and relations:

$$\mathcal{G}_H = (V, E_H, \phi), \quad \text{where } \pi_{\text{ext}}(d) \rightarrow \{(h_i, \mathcal{V}_{h_i})\}_{i=1}^m, \quad \phi(v) = \text{Enc}(v), \quad \phi(h_i) = \text{Enc}(h_i), \quad (4)$$

where each  $h_i$  defines a hyperedge  $h_i \in E_H$  connecting its associated entities  $\mathcal{V}_{h_i}$  as  $v \in V$ . The resulting hypergraph  $\mathcal{G}_H$  encodes high-order relational structures with rich semantic grounding.

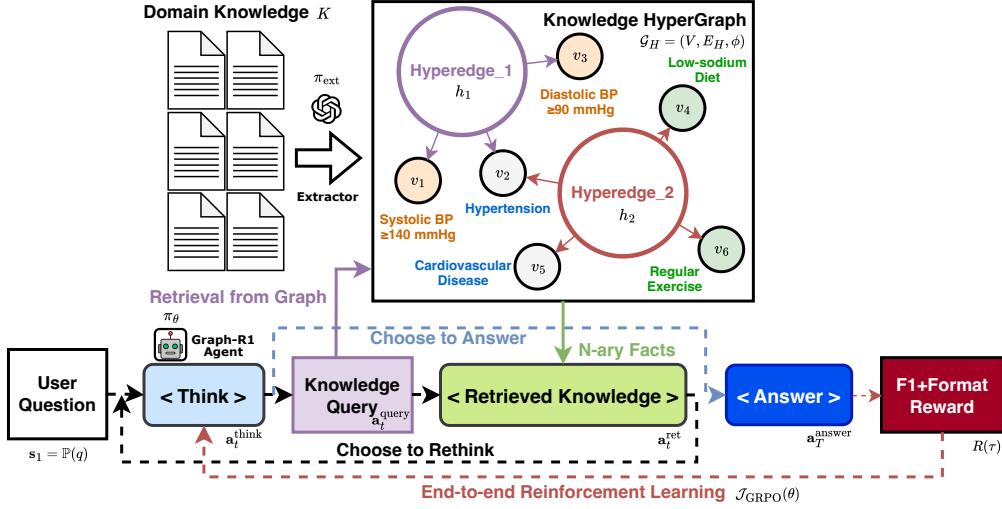


Figure 3: Overview of the Graph-R1 framework: an RL-enhanced reasoning trajectory over knowledge hypergraph, where the agent iteratively decides to think, query, retrieve knowledge, and answer.

**The Agent Action Space  $\mathcal{A}$ .** In Graph-R1, each agent action  $\mathbf{a}_t \in \mathcal{A}$  comprises four sub-actions: *Thinking*  $\mathbf{a}_t^{\text{think}}$ , which decides whether to continue or terminate reasoning; *Query Generation*  $\mathbf{a}_t^{\text{query}}$ , which formulates a retrieval query; *Graph Retrieval*  $\mathbf{a}_t^{\text{ret}}$ , which extracts relevant knowledge from the hypergraph; and *Answering*  $\mathbf{a}_t^{\text{ans}}$ , which produces a final response if reasoning ends. The agent action  $\mathbf{a}_t$  has two compositional forms, and the joint action log-likelihood is defined as:

$$\log \pi(\mathbf{a}_t | \mathbf{s}_t) = \begin{cases} \log \mathcal{G}_H(\mathbf{a}_t^{\text{ret}} | \mathbf{s}_t, \mathbf{a}_t^{\text{think}}, \mathbf{a}_t^{\text{query}}) + \log \pi(\mathbf{a}_t^{\text{query}} | \mathbf{s}_t, \mathbf{a}_t^{\text{think}}) + \log \pi(\mathbf{a}_t^{\text{ret}} | \mathbf{s}_t, \mathbf{a}_t^{\text{think}}) & \text{if } \mathbf{a}_t^{\text{think}} \rightarrow \text{continue,} \\ \log \pi(\mathbf{a}_t^{\text{ans}} | \mathbf{s}_t, \mathbf{a}_t^{\text{think}}) + \log \pi(\mathbf{a}_t^{\text{think}} | \mathbf{s}_t), & \text{if } \mathbf{a}_t^{\text{think}} \rightarrow \text{terminate,} \end{cases} \quad (5)$$

where, at each step, the agent first performs *Thinking*, and then conditionally chooses between continuing reasoning (*Query Generation* and *Graph Retrieval*) or terminating via *Answering*.

**The Agent State Space  $\mathcal{S}$  and Target  $y_q$ .** At each step  $t$ , the state  $\mathbf{s}_t \in \mathcal{S}$  is defined as  $\mathbf{s}_t = (\mathbf{s}_1, \mathbf{a}_1, \dots, \mathbf{a}_{t-1})$ , with  $\mathbf{s}_1$  initialized from the input query  $q$ . Once a termination action  $\mathbf{a}_T$  is issued, the agent reaches final state  $\mathbf{s}_T$ , where  $T$  is the total number of reasoning steps, and an answer  $y_q \sim \mathbf{a}_T^{\text{ans}}$  is produced to address  $q$ .

**Proposition 1.** *Graph-structured knowledge boosts agent accuracy by richer representation.*

*Proof.* We provide experimental results in Section 5.2 and theoretical proofs in Appendix B.1.  $\square$

## 4.2 KNOWLEDGE REASONING VIA MULTI-TURN GRAPH INTERACTION

We model reasoning as a multi-turn interaction between an agent  $\pi_\theta$  and a hypergraph  $\mathcal{G}_H$ . We first define the step-wise policy  $\pi_\theta(\cdot | \mathbf{s}_t)$  prompted by Table 1, then describe how to retrieve knowledge  $\mathcal{G}_H(\mathbf{a}_t^{\text{ret}} | \cdot, \mathbf{a}_t^{\text{query}})$  based on  $\mathbf{a}_t^{\text{query}}$  in each step, and finally present the objective to optimize  $P(y_q | \cdot)$ .

**Modeling the Step-wise Reasoning Policy.** At each reasoning step  $t$ , the LLM governs the agent's behavior by generating a structured output consisting of: (i) a thinking reflection  $\mathbf{a}_t^{\text{think}}$  that summarizes the current state and highlights potential knowledge gaps; (ii) a composition indicator  $\alpha_t \in \mathcal{A}_{\text{type}} = \{(\text{query}, \text{retrieve}), (\text{answer})\}$  that determines the sub-action structure; and (iii) a content output  $\mathbf{a}_t^{\text{out}} \in \mathcal{A}_{\text{content}}$ , representing either a retrieval query or a final answer. We model this decision-making process as a hierarchical policy conditioned on the agent state  $\mathbf{s}_t \in \mathcal{S}$ , which encodes the history of prior actions and retrieved information. The policy is factorized as:

$$\pi_\theta(\mathbf{a}_t^{\text{think}}, \alpha_t, \mathbf{a}_t^{\text{out}} | \mathbf{s}_t) = \pi_\theta(\mathbf{a}_t^{\text{out}} | \alpha_t, \mathbf{a}_t^{\text{think}}, \mathbf{s}_t) \cdot \pi_\theta(\alpha_t | \mathbf{a}_t^{\text{think}}, \mathbf{s}_t) \cdot \pi_\theta(\mathbf{a}_t^{\text{think}} | \mathbf{s}_t), \quad (6)$$

where  $\pi_\theta$  denotes the LLM-parameterized policy, which encourages three aligned behaviors: generating reflections  $\mathbf{a}_t^{\text{think}}$  that assess knowledge sufficiency, selecting  $\alpha_t$  to balance exploration and termination, and producing  $\mathbf{a}_t^{\text{out}}$  that advances retrieval  $\mathbf{a}_t^{\text{query}}$  or yields a direct answer  $\mathbf{a}_t^{\text{ans}}$ .

---

You are a helpful assistant. Answer the given question. You can query from knowledge base provided to you to answer the question. You can query knowledge as many times as you want. You must first conduct reasoning inside `<think>...</think>`. If you need to query knowledge, you can set a query statement between `<query>...</query>` to query from knowledge base after `<think>...</think>`. When you have the final answer, you can output the answer inside `<answer>...</answer>`. Question: **question**. Assistant:

---

Table 1: Template for Graph-R1. **question** will be replaced with the specific user query. Note that the knowledge retrieved is placed within `<knowledge>...</knowledge>` after `<query>`.

**Knowledge Interaction via Hypergraph Retrieval.** Given a query  $\mathbf{a}_t^{\text{query}}$  generated by the reasoning LLM, we retrieve relevant knowledge  $\mathbf{a}_t^{\text{ret}}$  from the hypergraph  $\mathcal{G}_H = (V, E_H)$  through a dual-path interaction process: entity-based retrieval and direct hyperedge retrieval. The resulting n-ary relational facts are then aggregated via rank-based fusion to support downstream reasoning.

(i) *Entity-based Hyperedge Retrieval.* We first identify a set of top-ranked entities based on their similarity to the extracted entities  $V_{\mathbf{a}_t^{\text{query}}}$ , and collect hyperedges that connect to any retrieved entity:

$$\mathcal{R}_V(\mathbf{a}_t^{\text{query}}) = \underset{v \in V}{\text{argmax}} \left( \text{sim}(\phi(V_{\mathbf{a}_t^{\text{query}}}), \phi(v)) \right), \quad \mathcal{F}_V^* = \bigcup_{v_i \in \mathcal{R}_V} \{(e_H, V_{e_H}) \mid v_i \in V_{e_H}, e_H \in E_H\}, \quad (7)$$

where  $\phi(V_{\mathbf{a}_t^{\text{query}}})$  is the aggregated embedding of entities extracted from  $\mathbf{a}_t^{\text{query}}$ ,  $\phi(v)$  is the entity embedding,  $k_V$  is the number of retrieved entities, and  $V_{e_H}$  denotes the entity set of hyperedge  $e_H$ .

(ii) *Direct Hyperedge Retrieval.* In parallel, we directly retrieve hyperedges based on query-hyperedge similarity, and collect their associated relational facts:

$$\mathcal{R}_H(\mathbf{a}_t^{\text{query}}) = \underset{e_H \in E_H}{\text{argmax}} \left( \text{sim}(\phi(\mathbf{a}_t^{\text{query}}), \phi(e_H)) \right), \quad \mathcal{F}_H^* = \bigcup_{e_i \in \mathcal{R}_H} \{(e_i, V_{e_i}) \mid V_{e_i} \subseteq V\}, \quad (8)$$

where  $\phi(\mathbf{a}_t^{\text{query}})$  is the query embedding,  $\phi(e_H)$  is the hyperedge embedding,  $k_H$  is the number of retrieved hyperedges, and  $V_{e_i}$  denotes the entity set of hyperedge  $e_i$ .

(iii) *Fusion via Reciprocal Rank Aggregation.* To produce the final knowledge set, we merge results from both retrieval paths using reciprocal rank aggregation over hyperedges:

$$\mathbf{a}_t^{\text{ret}} = \mathcal{F}_{\mathbf{a}_t^{\text{query}}}^* = \text{Top-}k \left( \mathcal{F}_V^* \cup \mathcal{F}_H^*, \text{RankScore}(f) = \frac{1}{r_V} + \frac{1}{r_H} \right)_{\mathbf{a}_t^{\text{query}}}, \quad (9)$$

where  $r_V$  and  $r_H$  are the ranks of n-ary relational fact  $f$  in  $\mathcal{F}_V^*$  and  $\mathcal{F}_H^*$  respectively (set to  $\infty$  if absent), and  $k$  is the number of retrieved facts  $\mathbf{a}_t^{\text{ret}}$  returned to the agent.

**Optimization Objective for Agent Trajectories.** The agent aims to learn a reasoning trajectory  $\tau \in \mathcal{T}_q$  that yields a faithful and contextually grounded answer  $y_q$ . Each trajectory  $\tau = ((\mathbf{s}_1, \mathbf{a}_1), (\mathbf{s}_2, \mathbf{a}_2), \dots, (\mathbf{s}_T, \mathbf{a}_T))$  comprises a sequence of actions executed over  $\mathcal{G}_H$ , defined as:

$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_{\theta}(\mathcal{T}_q | q; \mathcal{G}_H)} [\log P(y_q \mid \tau)], \quad (10)$$

where  $P(y_q \mid \tau)$  denotes the likelihood of the correct answer  $y_q \sim \mathbf{a}_t^{\text{ans}}$  under trajectory  $\tau$ , guiding  $\pi_{\theta}$  toward answer-consistent reasoning.

**Proposition 2.** *Multi-turn interaction with the graph environment improves retrieval efficiency.*

*Proof.* We provide experimental results in Section 5.5 and theoretical proofs in Appendix B.2.  $\square$

#### 4.3 OUTCOME-DIRECTED END-TO-END REINFORCEMENT LEARNING

To optimize the reasoning policy  $\pi_{\theta}$  toward generating faithful and well-structured answers, we adopt an end-to-end reinforcement learning objective based on Group Relative Policy Optimization (GRPO) (Shao et al., 2024)  $\mathcal{J}_{\text{GRPO}}(\theta)$  and design an outcome-directed reward function  $R(\tau)$ .

**End-to-end RL Objective  $\mathcal{J}_{\text{GRPO}}(\theta)$ .** Given a dataset question  $q \in \mathcal{D}_Q$ , the agent interacts with the knowledge hypergraph  $\mathcal{G}_H$  to generate a group of multi-turn reasoning trajectories  $\{\tau_i\}_{i=1}^N \subseteq \mathcal{T}_q$ ,

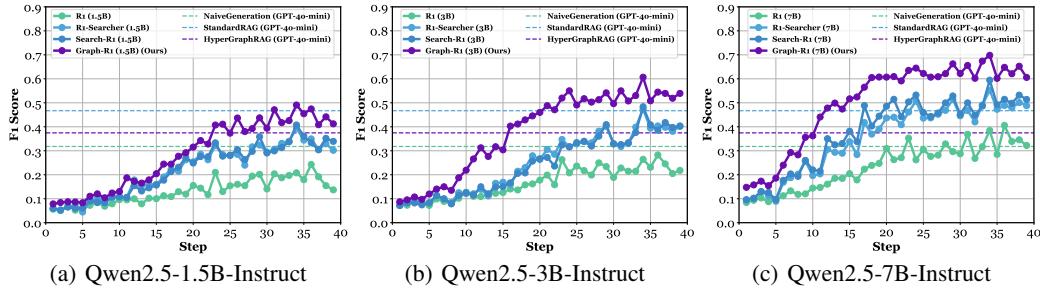


Figure 4: Step-wise F1 score on HotpotQA based on Qwen2.5 (1.5B, 3B, 7B), where Graph-R1 outperforms baselines and GPT-4o-mini variants (NaiveGeneration, StandardRAG, HyperGraphRAG).

where each  $\tau_i = ((\mathbf{s}_1^{(i)}, \mathbf{a}_1^{(i)}), \dots, (\mathbf{s}_T^{(i)}, \mathbf{a}_T^{(i)}))$  denotes a sequence of state-action pairs sampled from the environment. We optimize the policy  $\pi_\theta$  using the GRPO-based objective, which is defined as:

$$\mathcal{J}_{\text{GRPO}}(\theta) = \mathbb{E}_{[\mathbf{s}_1 \sim \{\mathbb{P}(q) | q \in \mathcal{D}_Q\}, \{\tau_i\}_{i=1}^N \sim \pi_{\theta_{\text{old}}}(\mathcal{T}_q | \mathbf{s}_1; \mathcal{G}_H)]} \left[ \frac{1}{N} \sum_{i=1}^N \frac{1}{|\tau_i|} \sum_{t=1}^{|\tau_i|} \min \left( \rho_\theta(\mathbf{a}_t^{(i)}) \hat{A}(\tau_i), \text{clip} \left( \rho_\theta(\mathbf{a}_t^{(i)}), 1 \pm \epsilon \right) \hat{A}(\tau_i) \right) - \beta \mathbb{D}_{\text{KL}}(\pi_\theta \| \pi_{\text{ref}}) \right], \quad (11)$$

$$\text{where } \rho_\theta(\mathbf{a}_t^{(i)}) = \frac{\pi_\theta(\mathbf{a}_t^{(i)} | \mathbf{s}_{t-1}^{(i)}; \mathcal{G}_H)}{\pi_{\theta_{\text{old}}}(\mathbf{a}_t^{(i)} | \mathbf{s}_{t-1}^{(i)}; \mathcal{G}_H)}, \text{ and } \hat{A}(\tau_i) = \frac{R(\tau_i) - \text{mean}(\{R(\tau_j)\}_{j=1}^N)}{F_{\text{norm}}(\{R(\tau_j)\}_{j=1}^N)}. \quad (12)$$

Here,  $\pi_\theta$  is the current policy, and  $\pi_{\theta_{\text{old}}}$  is the behavior policy used for sampling. The importance ratio  $\rho_\theta(\mathbf{a}_t^{(i)})$  adjusts for distribution shift, while the advantage  $\hat{A}(\tau_i)$  normalizes the reward using a scaling function  $F_{\text{norm}}(\cdot)$  (e.g., standard deviation). The  $\text{clip}(\cdot)$  operator stabilizes updates by constraining policy shifts. A KL term  $\mathbb{D}_{\text{KL}}(\pi_\theta \| \pi_{\text{ref}})$  regularizes toward a reference policy  $\pi_{\text{ref}}$ , with  $\beta$  controlling its strength. This objective encourages high-reward, stable reasoning over  $\mathcal{G}_H$ .

**Outcome-directed Reward Function  $R(\tau)$ .** To meet outcome requirements, we define a reward function  $R(\tau)$  composed of two parts: a *format reward*  $R_{\text{format}}(\tau)$  and an *answer reward*  $R_{\text{answer}}(\mathbf{a}_T^{\text{ans}})$ , promoting both thoughtful retrieval and accurate answer generation.

(i) *Format Reward.* The format reward  $R_{\text{format}}(\tau)$  encourages the agent to follow the intended reasoning structure. At each step  $(\mathbf{s}_t, \mathbf{a}_t)$ , we check whether the output includes a well-formed block  $(\mathbf{a}_t^{\text{think}}, \alpha_t, \mathbf{a}_t^{\text{out}})$ . Each valid step receives 0.5 reward, capped at 1.0 overall:

$$R_{\text{format}}(\tau) = \min \left( 1.0, 0.5 \cdot \sum_{t=1}^T \mathbb{I} \{ (\mathbf{a}_t^{\text{think}}, \alpha_t, \mathbf{a}_t^{\text{out}}) \text{ is well-formed} \} \right), \quad (13)$$

where  $\mathbb{I}\{\cdot\}$  is an indicator function that returns 1 if the step output matches the expected format.

(ii) *Answer Reward.* The answer reward  $R_{\text{answer}}(\mathbf{a}_T^{\text{ans}})$  measures the semantic correctness of the generated answer  $\mathbf{a}_T^{\text{ans}}$  by comparing it with the ground-truth answer  $y_q^*$  using a token-level F1 score:

$$R_{\text{answer}}(\mathbf{a}_T^{\text{ans}}) = \frac{2 \cdot |\text{tokens}(\mathbf{a}_T^{\text{ans}}) \cap \text{tokens}(y_q^*)|}{|\text{tokens}(\mathbf{a}_T^{\text{ans}})| + |\text{tokens}(y_q^*)|}, \quad (14)$$

where  $|\cdot|$  denotes multiset cardinality. The function  $\text{tokens}(\cdot)$  applies standard preprocessing including lowercasing and whitespace-based tokenization.

(iii) *Overall Outcome Reward.* The total reward for a reasoning trajectory  $\tau$  is defined as:

$$R(\tau) = -1.0 + R_{\text{format}}(\tau) + \mathbb{I}\{R_{\text{format}}(\tau) = 1.0\} \cdot R_{\text{answer}}(\mathbf{a}_T^{\text{ans}}), \text{ where } \mathbf{a}_T^{\text{ans}} \in \tau, \quad (15)$$

ensuring that answer correctness is only rewarded when the format is structurally valid. With the outcome-directed reward  $R(\tau)$ , high answer quality  $\mathbf{a}_T^{\text{ans}}$  is attainable through structurally coherent and reasoning-complete trajectories  $\tau$  with multi-turn iteration with knowledge hypergraph  $\mathcal{G}_H$ .

**Proposition 3.** *End-to-end RL bridges the gap between graph-based knowledge and language.*

*Proof.* We provide experimental results in Section 5.6 and theoretical proofs in Appendix B.3.  $\square$

Method	2Wiki.		HotpotQA		Musique		NQ		PopQA		TriviaQA		Avg.			
	F1	G-E	EM	F1	R-S	G-E										
<i>GPT-4o-mini</i>																
∅ ⊖ NaiveGeneration	17.03	74.86	31.79	78.48	11.45	76.61	21.59	84.64	25.95	<b>72.75</b>	47.73	83.33	11.36	25.92	-	78.45
∅ ⊕ StandardRAG	<b>22.31</b>	73.02	<b>46.70</b>	<b>81.88</b>	17.31	74.93	<b>26.85</b>	84.55	<b>30.58</b>	69.42	<b>48.55</b>	84.63	<b>18.10</b>	<b>32.05</b>	52.68	78.07
∅ ⊕ GraphRAG	16.02	72.81	31.67	77.37	15.14	74.43	20.31	82.36	20.92	65.88	45.13	82.76	12.50	24.87	32.48	75.94
∅ ⊕ LightRAG	16.59	71.94	30.70	73.42	14.39	73.75	19.09	80.20	20.47	67.76	40.18	81.60	9.77	23.57	47.42	74.78
∅ ⊕ PathRAG	12.42	67.19	23.12	71.81	11.49	69.94	20.01	81.99	15.65	60.58	37.44	80.94	7.03	20.02	46.71	72.08
∅ ⊕ HippoRAG2	16.27	68.78	31.78	76.43	12.37	73.05	24.56	<b>84.65</b>	21.10	63.31	46.86	83.55	13.80	25.49	36.41	74.96
∅ ⊕ HyperGraphRAG	21.14	<b>76.76</b>	37.46	80.50	<b>20.40</b>	<b>79.29</b>	22.95	81.22	29.48	70.55	44.95	<b>85.20</b>	13.15	29.40	<b>61.82</b>	<b>78.92</b>
<i>Qwen2.5-1.5B-Instruct</i>																
∅ ⊖ NaiveGeneration	7.78	49.13	4.27	45.77	2.35	46.63	6.03	46.74	10.06	42.67	8.10	52.92	1.17	6.43	-	47.31
∅ ⊕ StandardRAG	11.46	55.38	9.93	52.91	3.18	39.46	11.39	59.73	13.08	50.29	17.43	60.52	5.73	11.08	52.84	53.05
∅ ⊖ SFT	13.26	34.72	13.61	38.93	5.14	28.50	11.56	46.61	15.61	31.35	26.18	46.66	9.83	14.23	-	37.80
∅ ⊖ R1	26.28	47.48	20.07	44.43	4.84	39.12	16.75	45.95	21.36	44.50	34.78	48.59	14.19	20.68	-	45.01
∅ ⊕ Search-R1	28.43	60.61	39.99	64.16	4.69	39.32	20.26	59.93	39.63	58.19	44.16	63.01	23.18	29.53	50.45	57.54
∅ ⊕ R1-Searcher	28.01	58.81	<b>41.50</b>	61.54	6.26	38.31	<b>36.86</b>	<b>60.79</b>	38.37	56.02	42.57	61.24	23.70	32.26	50.68	56.12
∅ ⊕ Graph-R1 (ours)	<b>35.13</b>	<b>65.73</b>	40.62	<b>65.30</b>	<b>28.28</b>	<b>58.82</b>	35.62	59.13	<b>43.55</b>	<b>66.46</b>	<b>57.36</b>	<b>70.83</b>	<b>31.90</b>	<b>40.09</b>	<b>59.35</b>	<b>64.38</b>
<i>Qwen2.5-3B-Instruct</i>																
∅ ⊖ NaiveGeneration	7.59	55.00	11.16	53.75	3.67	54.00	8.90	57.18	10.89	49.08	10.89	48.16	3.26	8.85	-	52.86
∅ ⊕ StandardRAG	12.52	60.01	15.41	62.51	2.92	50.40	10.69	65.13	14.70	57.25	21.92	68.43	3.39	13.03	52.69	60.62
∅ ⊖ SFT	12.40	52.31	16.48	51.35	5.04	51.31	11.23	58.20	16.95	46.42	33.02	59.98	9.64	15.85	-	53.26
∅ ⊖ R1	28.45	56.92	25.33	55.38	8.07	47.53	21.51	55.11	27.11	48.65	47.91	60.74	19.66	26.40	-	54.06
∅ ⊕ Search-R1	38.04	54.39	43.84	69.32	7.65	46.43	37.96	52.90	38.67	63.74	47.99	60.37	28.65	35.69	49.99	57.86
∅ ⊕ R1-Searcher	23.50	55.86	42.44	64.60	12.81	50.07	36.53	63.33	40.18	66.23	54.00	60.52	27.08	34.91	49.98	60.10
∅ ⊕ Graph-R1 (ours)	<b>57.56</b>	<b>76.45</b>	<b>56.75</b>	<b>77.46</b>	<b>40.51</b>	<b>67.84</b>	<b>44.75</b>	<b>69.92</b>	<b>45.65</b>	<b>71.27</b>	<b>62.31</b>	<b>75.01</b>	<b>42.45</b>	<b>51.26</b>	<b>60.19</b>	<b>72.99</b>
<i>Qwen2.5-7B-Instruct</i>																
∅ ⊖ NaiveGeneration	12.25	66.75	16.58	65.31	4.06	65.47	13.00	69.56	12.82	60.50	24.51	72.65	3.12	13.87	-	66.71
∅ ⊕ StandardRAG	12.75	60.06	21.10	66.13	4.53	59.84	15.97	70.49	16.10	60.86	24.90	73.71	5.34	15.89	52.67	65.18
∅ ⊖ SFT	20.28	63.85	27.59	65.65	10.02	63.50	19.02	68.19	27.93	56.31	39.21	70.25	15.57	24.01	-	64.63
∅ ⊖ R1	30.99	59.19	37.05	60.12	14.53	49.39	28.45	57.63	30.35	53.38	57.33	66.73	25.91	33.12	-	57.74
∅ ⊕ Search-R1	41.29	70.26	50.85	73.85	22.35	57.68	45.88	67.58	50.76	66.08	65.98	76.15	38.54	46.19	51.60	68.60
∅ ⊕ R1-Searcher	33.96	69.61	46.36	74.56	16.63	59.05	44.93	68.54	47.12	66.74	64.76	75.95	34.51	42.29	51.26	69.08
∅ ⊕ Graph-R1 (ours)	<b>65.04</b>	<b>82.42</b>	<b>62.69</b>	<b>80.03</b>	<b>46.17</b>	<b>71.42</b>	<b>49.87</b>	<b>70.97</b>	<b>51.22</b>	<b>73.43</b>	<b>71.93</b>	<b>79.11</b>	<b>48.57</b>	<b>57.82</b>	<b>60.40</b>	<b>76.23</b>

Table 2: Main results with best in **bold**.  $\varnothing$  means prompt engineering,  $\bullet$  means training,  $\ominus$  means no knowledge interaction,  $\oplus$  means chunk-based knowledge, and  $\otimes$  means graph-based knowledge.

## 5 EXPERIMENTS

This section presents the experimental setup, main results, and analysis. We answer the following research questions (RQs): **RQ1**: Does Graph-R1 outperform other methods? **RQ2**: Does the main component of Graph-R1 work, and how is its comparative analysis? **RQ3-6**: How are construction cost, retrieval efficiency, generation quality, and generalizability of Graph-R1, respectively?

### 5.1 EXPERIMENTAL SETUP

**Datasets.** To evaluate the performance of Graph-R1, we conduct experiments across six standard RAG datasets (Jin et al., 2025b): 2WikiMultiHopQA (**2Wiki.**) (Ho et al., 2020), **HotpotQA** (Yang et al., 2018), **Musique** (Trivedi et al., 2022), Natural Questions (**NQ**) (Kwiatkowski et al., 2019), **PopQA** (Mallen et al., 2023), and **TriviaQA** (Joshi et al., 2017). More details are in Appendix D.

**Baselines.** We mainly compare Graph-R1 with **NaiveGeneration**, **StandardRAG** (Lewis et al., 2020), **SFT** (Zheng et al., 2024), **R1** (Shao et al., 2024), **Search-R1** (Jin et al., 2025a), and **R1-Searcher** (Song et al., 2025) at three Qwen2.5 (Qwen et al., 2025) scales: 1.5 B, 3 B, and 7 B. We also compare **GraphRAG** (Edge et al., 2025), **LightRAG** (Guo et al., 2025), **PathRAG** (Chen et al., 2025), **HippoRAG2** (Gutiérrez et al., 2025), and **HyperGraphRAG** (Luo et al., 2025a) based on GPT-4o-mini (OpenAI et al., 2024a) as a reference. More details are in Appendix E.

**Evaluation Metrics.** We evaluate Graph-R1 and baselines with four metrics: Exact Match (**EM**), **F1**, Retrieval Similarity (**R-S**), and Generation Evaluation (**G-E**). More details are in Appendix F.

**Implementation Details.** We use GPT-4o-mini for knowledge construction in Graph-R1 and GraphRAG baselines. For retrieval, we use bge-large-en-v1.5 (Chen et al., 2023) in all variants. All experiments are done on 4 NVIDIA A100 GPUs (80GB). More details are in Appendix G.

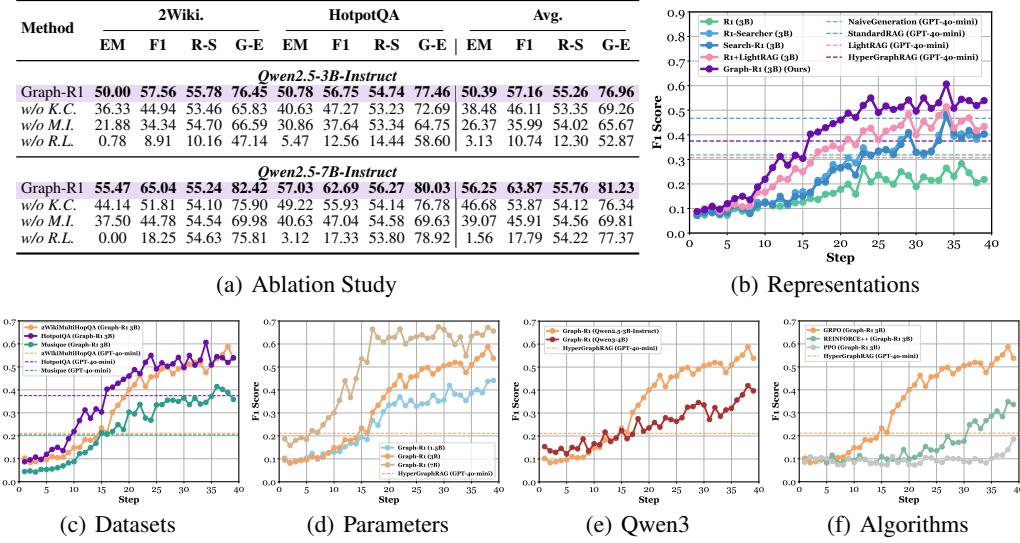


Figure 5: (a) Ablation study of Graph-R1. (b-f) Performance comparison across different kinds of knowledge representations, RAG datasets, model parameters, Qwen versions, and RL algorithms.

## 5.2 MAIN RESULTS (RQ1)

As shown in Table 2, we compare Graph-R1 with baselines across different base models, and observe that Graph-R1 consistently outperforms all baselines. In addition, we have two key observations.

**RL Unlocks the Power of Graph Representations.** Prompt-only GraphRAG methods often underperform StandardRAG, showing that graph structures alone are not sufficient. Graph-R1, with multi-turn RL optimization, fully exploits structural signals, achieving 57.28 F1 under Qwen2.5-7B-Instruct, surpassing StandardRAG (32.05), HyperGraphRAG (29.40) and Search-R1 (46.19).

**Larger Base Model Further Enhances Performance.** As base model size increases from 1.5B to 3B and 7B, Graph-R1 achieves steadily higher F1 scores: 40.09, 51.26, and 57.82. Moreover, its gap over other RL-enhanced baselines such as Search-R1 and R1-Searcher becomes increasingly evident. This shows that larger models better exploit the synergy between graph structures and RL.

## 5.3 ABLATION STUDY AND COMPARATIVE ANALYSIS (RQ2)

As shown in Figures 5, we conduct an ablation study and comparative analysis on Graph-R1.

**Ablation Study.** We remove three core components of Graph-R1: knowledge construction (K.C.), multi-turn interaction (M.I.), and reinforcement learning (R.L.), to assess their individual contributions. As shown in Figure 5(a), removing any module leads to performance degradation.

**Comparison with Different Knowledge Representations.** As shown in Figures 4 and 5(b), models without external knowledge (green) perform the worst. Chunk-based knowledge with RL (blue) performs better, but is still inferior to graph-based methods using binary relations (pink), while hypergraph-based knowledge with RL (red) achieves the highest ceiling. This demonstrates that, when combined with RL, stronger knowledge representations yield higher performance potential.

**Comparison across Datasets and Base Models.** As shown in Figures 5(c) and 5(d), Graph-R1 consistently outperforms baselines across different datasets and parameter sizes, showcasing strong scalability. Interestingly, Figure 5(e) shows that when Graph-R1 is trained on Qwen3 (4B) (Yang et al., 2025), which is already well trained by RL, the model tends to over-rely on its own internal reasoning. Despite a stronger starting point, its overall performance ceiling appears slightly lower.

**Comparison with Different RL Algorithms.** Figure 5(f) compares different RL strategies. GRPO significantly outperforms REINFORCE++ (Hu et al., 2025) and PPO (Schulman et al., 2017), achieving the highest F1. This confirms that GRPO facilitates more stable training and stronger multi-turn graph reasoning, making it a favorable choice for training agentic GraphRAG models.

#### 5.4 ANALYSIS OF GRAPH-R1'S CONSTRUCTION COST (RQ3)

As shown in Table 3, we utilize metrics: time per 1K tokens (TP1KT), cost per 1M tokens (CP1MT), number of nodes & edges, time per query (TPQ), cost per 1K queries (CP1KQ), and final F1 score.

**Construction Cost.** Graph-R1 requires only 5.69 seconds and \$2.81 per 1K tokens for knowledge construction, lower than GraphRAG (8.04s, \$3.35) and HyperGraphRAG (6.76s, \$4.14). Generating over 120K nodes and 98K edges, Graph-R1 maintains a semantically rich structure.

**Generation Cost.** By leveraging end-to-end RL and localized knowledge retrieval, Graph-R1 achieves not only the best F1 but also a response time of 7.0s per query and a generation cost of \$0, outperforming baselines such as HyperGraphRAG (9.6s, \$8.76), highlighting its superior potential for real-world deployment.

#### 5.5 ANALYSIS OF GRAPH-R1'S RETRIEVAL EFFICIENCY (RQ4)

As shown in Figure 6, to evaluate Graph-R1's retrieval efficiency, we analyze it from (a) response length, (b) number of interaction turns, and (c) performance with average retrieval content lengths.

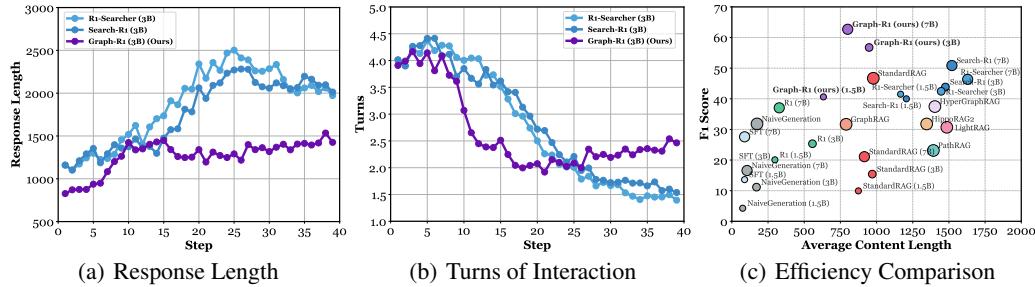


Figure 6: Step-wise response length & turns of interaction, and efficiency comparison on HotpotQA.

**Tendency toward More Concise Thinking and Adequate Interaction.** As shown in Figures 6(a) and 6(b), Graph-R1 generates shorter responses and conducts more interaction turns, averaging around 1200-1500 tokens and 2.3-2.5 turns, leading to more stable and accurate retrieval.

**Balancing Performance and Retrieved Content Length.** As shown in Figure 6(c), Graph-R1 achieves the highest F1 scores with a moderate amount of average retrieved content compared to other methods, balancing input length and performance through its multi-turn interaction strategy.

#### 5.6 ANALYSIS OF GRAPH-R1'S GENERATION QUALITY (RQ5)

As shown in Figure 7, we evaluate the generation quality in seven dimensions and present a case study in Table 4.

**High-Quality Generation Performance.** Graph-R1 outperforms all RL-based baselines and achieves generation quality comparable to GPT-4o-mini-based methods like HyperGraphRAG, with strong results in Correctness (86.9), Relevance (95.2), and Logical Coherence (88.5).

**RL Bridges the Gap Between Graph & Language.** HyperGraphRAG performs similarly to StandardRAG, indicating limited gains from graph structure alone. In contrast, Graph-R1 achieves a much higher Overall score (82.4 vs. 70.3) than Search-R1, showing that graph-based reasoning becomes truly effective when combined with RL.

Table 3: Time & Cost Comparisons on 2Wiki.

Method	Knowledge Construction				Retrieval & Generation			
	TP1KT	CP1MT	#Node	#Edge	TPQ	CP1KQ	F1	
NaiveGeneration	0 s	0 \$	-	-	3.7 s	0.16 \$	17.0	
StandardRAG	0 s	0 \$	-	-	4.1 s	1.35 \$	22.3	
GraphRAG	8.04 s	3.35 \$	7,771	4,863	7.4 s	3.97 \$	16.0	
LightRAG	6.84 s	4.07 \$	59,197	24,596	12.2 s	8.11 \$	16.6	
PathRAG	6.84 s	4.07 \$	59,197	24,596	15.8 s	8.28 \$	12.4	
HippoRAG2	3.25 s	1.26 \$	11,819	40,654	8.8 s	7.68 \$	16.3	
HyperGraphRAG	6.76 s	4.14 \$	173,575	114,426	9.6 s	8.76 \$	21.1	
Graph-R1 (7B) (ours)	5.69 s	2.81 \$	120,499	98,073	7.0 s	0 \$	65.0	

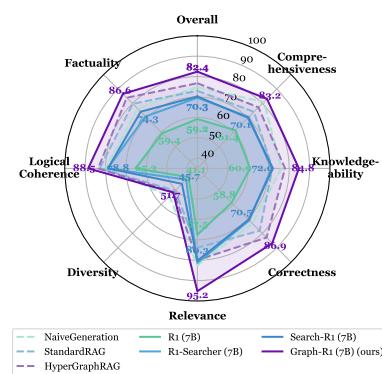


Figure 7: Generation Evaluations.

Table 4: Case study on generation quality under a query, comparing NaiveGeneration, Standard-RAG, HyperGraphRAG based on GPT-4o-mini, with R1 (7B), Search-R1 (7B), and Graph-R1 (7B).

**Case Study Analysis.** As shown in Table 4, NaiveGeneration and R1 fail to provide the correct answer, and both StandardRAG and HyperGraphRAG also fall short despite using structured prompts. Search-R1, though RL-enhanced, shows limited improvement due to weak retrieval grounding. In contrast, Graph-R1 accurately identifies both the director and birthdate, achieves the highest G-E score (88.57), and demonstrates that RL is more effective with graph-based knowledge interaction.

## 5.7 ANALYSIS OF GRAPH-R1'S GENERALIZABILITY ON O.O.D. SETTINGS (RQ6)

As shown in Figure 8, to verify generalization, we conduct O.O.D. cross-validation for Search-R1 (3B) & Graph-R1 (3B) across six datasets: (a-b) F1 comparison, and (c-d) O.O.D.-to-I.I.D. ratios.

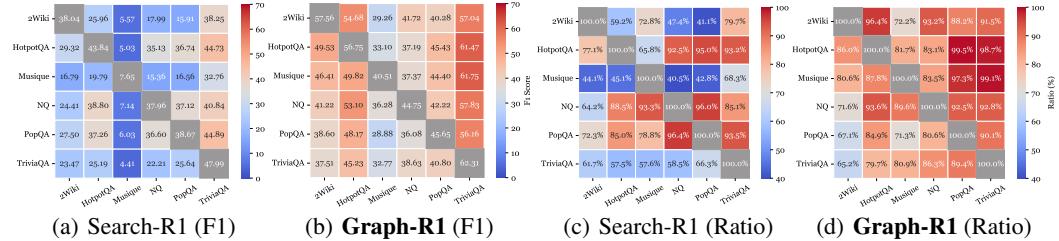


Figure 8: F1 comparison and performance ratios across six datasets under O.O.D. cross-validation.

**F1 Performance Across Datasets.** Figures 8(a) and 8(b) show that Graph-R1 outperforms Search-R1 on six datasets in O.O.D. validation, with notable gains on NQ and TriviaQA. Its multi-turn interaction with hypergraph retrieval ensures more stable performance under distribution shifts.

**Robust Generalization Ability.** Figures 8(c) and 8(d) show that Graph-R1 achieves higher O.O.D.-to-I.I.D. ratios than Search-R1, often above 85% and exceeding 90% in some cases, reflecting its strong robustness and cross-domain generalizability via end-to-end RL over knowledge hypergraph.

## 6 CONCLUSION

In this work, we introduce Graph-R1, an agentic GraphRAG framework powered by end-to-end RL. By introducing lightweight knowledge hypergraph construction and modeling retrieval as a multi-turn interaction process, Graph-R1 bridges graph-structured knowledge with natural language generation. A unified reward mechanism enables outcome-directed reasoning that outperforms prior GraphRAG methods and RL-enhanced baselines. Experiments across six benchmarks demonstrate Graph-R1’s superiority in accuracy, retrieval efficiency, generation quality, and generalizability.

## REFERENCES

- Boyu Chen, Zirui Guo, Zidan Yang, Yuluo Chen, Junze Chen, Zhenghao Liu, Chuan Shi, and Cheng Yang. Pathrag: Pruning graph-based retrieval augmented generation with relational paths, 2025. URL <https://arxiv.org/abs/2502.14902>.
- Jianlv Chen, Shitao Xiao, Peitian Zhang, Kun Luo, Defu Lian, and Zheng Liu. Bge m3-embedding: Multi-lingual, multi-functionality, multi-granularity text embeddings through self-knowledge distillation, 2023.
- DeepSeek-AI, Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, Xiaokang Zhang, Xingkai Yu, Yu Wu, Z. F. Wu, Zhibin Gou, Zhihong Shao, Zhuoshu Li, Ziyi Gao, Aixin Liu, Bing Xue, Bingxuan Wang, Bochao Wu, Bei Feng, Chengda Lu, Chenggang Zhao, Chengqi Deng, Chenyu Zhang, Chong Ruan, Damai Dai, Deli Chen, Dongjie Ji, Erhang Li, Fangyun Lin, Fucong Dai, Fuli Luo, Guangbo Hao, Guanting Chen, Guowei Li, H. Zhang, Han Bao, et al. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning, 2025. URL <https://arxiv.org/abs/2501.12948>.
- Darren Edge, Ha Trinh, Newman Cheng, Joshua Bradley, Alex Chao, Apurva Mody, Steven Truitt, Dasha Metropolitansky, Robert Osazuwa Ness, and Jonathan Larson. From local to global: A graph rag approach to query-focused summarization, 2025. URL <https://arxiv.org/abs/2404.16130>.
- Lang Feng, Zhenghai Xue, Tingcong Liu, and Bo An. Group-in-group policy optimization for llm agent training, 2025a. URL <https://arxiv.org/abs/2505.10978>.
- Yifan Feng, Hao Hu, Xingliang Hou, Shiquan Liu, Shihui Ying, Shaoyi Du, Han Hu, and Yue Gao. Hyper-rag: Combating llm hallucinations using hypergraph-driven retrieval-augmented generation, 2025b. URL <https://arxiv.org/abs/2504.08758>.
- Yunfan Gao, Yun Xiong, Xinyu Gao, Kangxiang Jia, Jinliu Pan, Yuxi Bi, Yi Dai, Jiawei Sun, Meng Wang, and Haofen Wang. Retrieval-augmented generation for large language models: A survey, 2024. URL <https://arxiv.org/abs/2312.10997>.
- Yunfan Gao, Yun Xiong, Yijie Zhong, Yuxi Bi, Ming Xue, and Haofen Wang. Synergizing rag and reasoning: A systematic review, 2025. URL <https://arxiv.org/abs/2504.15909>.
- Zirui Guo, Lianghao Xia, Yanhua Yu, Tu Ao, and Chao Huang. Lightrag: Simple and fast retrieval-augmented generation, 2025. URL <https://arxiv.org/abs/2410.05779>.
- Bernal Jiménez Gutiérrez, Yiheng Shu, Weijian Qi, Sizhe Zhou, and Yu Su. From rag to memory: Non-parametric continual learning for large language models, 2025. URL <https://arxiv.org/abs/2502.14802>.
- Xanh Ho, Anh-Khoa Duong Nguyen, Saku Sugawara, and Akiko Aizawa. Constructing a multi-hop QA dataset for comprehensive evaluation of reasoning steps. In Donia Scott, Nuria Bel, and Chengqing Zong (eds.), *Proceedings of the 28th International Conference on Computational Linguistics*, pp. 6609–6625, Barcelona, Spain (Online), December 2020. International Committee on Computational Linguistics. doi: 10.18653/v1/2020.coling-main.580. URL <https://aclanthology.org/2020.coling-main.580/>.
- Jian Hu, Jason Klein Liu, and Wei Shen. Reinforce++: An efficient rlhf algorithm with robustness to both prompt and reward models, 2025. URL <https://arxiv.org/abs/2501.03262>.
- Bowen Jin, Hansi Zeng, Zhenrui Yue, Jinsung Yoon, Sercan Arik, Dong Wang, Hamed Zamani, and Jiawei Han. Search-r1: Training llms to reason and leverage search engines with reinforcement learning, 2025a. URL <https://arxiv.org/abs/2503.09516>.
- Jiajie Jin, Yutao Zhu, Zhicheng Dou, Guanting Dong, Xinyu Yang, Chenghao Zhang, Tong Zhao, Zhao Yang, and Ji-Rong Wen. Flashrag: A modular toolkit for efficient retrieval-augmented generation research. In *Companion Proceedings of the ACM on Web Conference 2025*, WWW '25, pp. 737–740, New York, NY, USA, 2025b. Association for Computing Machinery. ISBN 9798400713316. doi: 10.1145/3701716.3715313. URL <https://doi.org/10.1145/3701716.3715313>.

Mandar Joshi, Eunsol Choi, Daniel Weld, and Luke Zettlemoyer. TriviaQA: A large scale distantly supervised challenge dataset for reading comprehension. In Regina Barzilay and Min-Yen Kan (eds.), *Proceedings of the 55th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 1601–1611, Vancouver, Canada, July 2017. Association for Computational Linguistics. doi: 10.18653/v1/P17-1147. URL <https://aclanthology.org/P17-1147/>.

Tom Kwiatkowski, Jennimaria Palomaki, Olivia Redfield, Michael Collins, Ankur Parikh, Chris Alberti, Danielle Epstein, Illia Polosukhin, Jacob Devlin, Kenton Lee, Kristina Toutanova, Llion Jones, Matthew Kelcey, Ming-Wei Chang, Andrew M. Dai, Jakob Uszkoreit, Quoc Le, and Slav Petrov. Natural questions: A benchmark for question answering research. *Transactions of the Association for Computational Linguistics*, 7:452–466, 2019. doi: 10.1162/tacl\_a\_00276. URL <https://aclanthology.org/Q19-1026/>.

Patrick Lewis, Ethan Perez, Aleksandra Piktus, Fabio Petroni, Vladimir Karpukhin, Naman Goyal, Heinrich Küttler, Mike Lewis, Wen-tau Yih, Tim Rocktäschel, Sebastian Riedel, and Douwe Kiela. Retrieval-augmented generation for knowledge-intensive nlp tasks. In H. Larochelle, M. Ranzato, R. Hadsell, M.F. Balcan, and H. Lin (eds.), *Advances in Neural Information Processing Systems*, volume 33, pp. 9459–9474. Curran Associates, Inc., 2020. URL [https://proceedings.neurips.cc/paper\\_files/paper/2020/file/6b493230205f780e1bc26945df7481e5-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2020/file/6b493230205f780e1bc26945df7481e5-Paper.pdf).

Lei Liang, Zhongpu Bo, Zhengke Gui, Zhongshu Zhu, Ling Zhong, Peilong Zhao, Mengshu Sun, Zhiqiang Zhang, Jun Zhou, Wenguang Chen, Wen Zhang, and Huajun Chen. Kag: Boosting llms in professional domains via knowledge augmented generation. In *Companion Proceedings of the ACM on Web Conference 2025*, WWW ’25, pp. 334–343, New York, NY, USA, 2025. Association for Computing Machinery. ISBN 9798400713316. doi: 10.1145/3701716.3715240. URL <https://doi.org/10.1145/3701716.3715240>.

Hao Liu, Zhengren Wang, Xi Chen, Zhiyu Li, Feiyu Xiong, Qinhuan Yu, and Wentao Zhang. Hoprag: Multi-hop reasoning for logic-aware retrieval-augmented generation, 2025. URL <https://arxiv.org/abs/2502.12442>.

Zhengxi Lu, Yuxiang Chai, Yaxuan Guo, Xi Yin, Liang Liu, Hao Wang, Han Xiao, Shuai Ren, Guanjing Xiong, and Hongsheng Li. Ui-rl: Enhancing efficient action prediction of gui agents by reinforcement learning, 2025. URL <https://arxiv.org/abs/2503.21620>.

Haoran Luo, Haihong E, Yuhao Yang, Tianyu Yao, Yikai Guo, Zichen Tang, Wentai Zhang, Shiyao Peng, Kaiyang Wan, Meina Song, Wei Lin, Yifan Zhu, and Anh Tuan Luu. Text2nkg: Fine-grained n-ary relation extraction for n-ary relational knowledge graph construction. In A. Globerson, L. Mackey, D. Belgrave, A. Fan, U. Paquet, J. Tomczak, and C. Zhang (eds.), *Advances in Neural Information Processing Systems*, volume 37, pp. 27417–27439. Curran Associates, Inc., 2024. URL [https://proceedings.neurips.cc/paper\\_files/paper/2024/file/305b2288122d46bf0641bdd86c9a7921-Paper-Conference.pdf](https://proceedings.neurips.cc/paper_files/paper/2024/file/305b2288122d46bf0641bdd86c9a7921-Paper-Conference.pdf).

Haoran Luo, Haihong E, Guanting Chen, Yandan Zheng, Xiaobao Wu, Yikai Guo, Qika Lin, Yu Feng, Zemin Kuang, Meina Song, Yifan Zhu, and Luu Anh Tuan. Hypergraphrag: Retrieval-augmented generation via hypergraph-structured knowledge representation, 2025a. URL <https://arxiv.org/abs/2503.21322>.

Haoran Luo, Haihong E, Yikai Guo, Qika Lin, Xiaobao Wu, Xinyu Mu, Wenhao Liu, Meina Song, Yifan Zhu, and Anh Tuan Luu. KBQA-o1: Agentic knowledge base question answering with monte carlo tree search. In *Forty-second International Conference on Machine Learning*, 2025b. URL <https://openreview.net/forum?id=QuecSemZIy>.

Peixian Ma, Xialie Zhuang, Chengjin Xu, Xuhui Jiang, Ran Chen, and Jian Guo. Sql-r1: Training natural language to sql reasoning model by reinforcement learning, 2025. URL <https://arxiv.org/abs/2504.08600>.

Alex Mallen, Akari Asai, Victor Zhong, Rajarshi Das, Daniel Khashabi, and Hannaneh Hajishirzi. When not to trust language models: Investigating effectiveness of parametric and non-parametric memories. In Anna Rogers, Jordan Boyd-Graber, and Naoaki Okazaki (eds.), *Proceedings of*

*the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 9802–9822, Toronto, Canada, July 2023. Association for Computational Linguistics. doi: 10.18653/v1/2023.acl-long.546. URL <https://aclanthology.org/2023.acl-long.546/>.

OpenAI, :, Aaron Hurst, Adam Lerer, Adam P. Goucher, Adam Perelman, Aditya Ramesh, Aidan Clark, AJ Ostrow, Akila Welihinda, Alan Hayes, Alec Radford, Aleksander Mądry, Alex Baker-Whitcomb, Alex Beutel, Alex Borzunov, Alex Carney, Alex Chow, Alex Kirillov, Alex Nichol, Alex Paino, Alex Renzin, Alex Tachard Passos, Alexander Kirillov, Alexi Christakis, Alexis Conneau, Ali Kamali, Allan Jabri, Allison Moyer, Allison Tam, Amadou Crookes, Amin Tootoochian, Amin Tootoonchian, Ananya Kumar, Andrea Vallone, Andrej Karpathy, Andrew Braunstein, Andrew Cann, Andrew Codispoti, et al. Gpt-4o system card, 2024a. URL <https://arxiv.org/abs/2410.21276>.

OpenAI, :, Aaron Jaech, Adam Kalai, Adam Lerer, Adam Richardson, Ahmed El-Kishky, Aiden Low, Alec Helyar, Aleksander Madry, Alex Beutel, Alex Carney, Alex Iftimie, Alex Karpenko, Alex Tachard Passos, Alexander Neitz, Alexander Prokofiev, Alexander Wei, Allison Tam, Ally Bennett, Ananya Kumar, Andre Saraiva, Andrea Vallone, Andrew Duberstein, Andrew Kondrich, Andrey Mishchenko, Andy Applebaum, Angela Jiang, Ashvin Nair, Barret Zoph, Behrooz Ghorbani, Ben Rossen, Benjamin Sokolowsky, Boaz Barak, Bob McGrew, Borys Minaiev, Botaao Hao, et al. Openai o1 system card, 2024b. URL <https://arxiv.org/abs/2412.16720>.

Qwen, :, An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan Li, Dayiheng Liu, Fei Huang, Haoran Wei, Huan Lin, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Yang, Jiaxi Yang, Jingren Zhou, Junyang Lin, Kai Dang, Keming Lu, Keqin Bao, Kexin Yang, Le Yu, Mei Li, Mingfeng Xue, Pei Zhang, Qin Zhu, Rui Men, Runji Lin, Tianhao Li, Tianyi Tang, Tingyu Xia, Xingzhang Ren, Xuancheng Ren, Yang Fan, Yang Su, Yichang Zhang, Yu Wan, Yuqiong Liu, Zeyu Cui, Zhenru Zhang, and Zihan Qiu. Qwen2.5 technical report, 2025. URL <https://arxiv.org/abs/2412.15115>.

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms, 2017. URL <https://arxiv.org/abs/1707.06347>.

Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, Y. K. Li, Y. Wu, and Daya Guo. Deepseekmath: Pushing the limits of mathematical reasoning in open language models, 2024. URL <https://arxiv.org/abs/2402.03300>.

Haozhan Shen, Peng Liu, Jingcheng Li, Chunxin Fang, Yibo Ma, Jiajia Liao, Qiaoli Shen, Zilun Zhang, Kangjia Zhao, Qianqian Zhang, Ruochen Xu, and Tiancheng Zhao. Vlm-r1: A stable and generalizable r1-style large vision-language model, 2025. URL <https://arxiv.org/abs/2504.07615>.

Huatong Song, Jinhao Jiang, Yingqian Min, Jie Chen, Zhipeng Chen, Wayne Xin Zhao, Lei Fang, and Ji-Rong Wen. R1-searcher: Incentivizing the search capability in llms via reinforcement learning, 2025. URL <https://arxiv.org/abs/2503.05592>.

Hao Sun, Zile Qiao, Jiayan Guo, Xuanbo Fan, Yingyan Hou, Yong Jiang, Pengjun Xie, Yan Zhang, Fei Huang, and Jingren Zhou. Zerosearch: Incentivize the search capability of llms without searching, 2025. URL <https://arxiv.org/abs/2505.04588>.

Harsh Trivedi, Niranjan Balasubramanian, Tushar Khot, and Ashish Sabharwal. MuSiQue: Multihop questions via single-hop question composition. *Transactions of the Association for Computational Linguistics*, 10:539–554, 2022. doi: 10.1162/tacl\_a\_00475. URL <https://aclanthology.org/2022.tacl-1.31/>.

Jingjin Wang. Proprag: Guiding retrieval with beam search over proposition paths, 2025. URL <https://arxiv.org/abs/2504.18070>.

Jinyu Wang, Jingjing Fu, Rui Wang, Lei Song, and Jiang Bian. Pike-rag: specialized knowledge and rationale augmented generation, 2025a. URL <https://arxiv.org/abs/2501.11551>.

- Nengbo Wang, Xiaotian Han, Jagdip Singh, Jing Ma, and Vipin Chaudhary. Causalrag: Integrating causal graphs into retrieval-augmented generation, 2025b. URL <https://arxiv.org/abs/2503.19878>.
- Junde Wu, Jiayuan Zhu, Yunli Qi, Jingkun Chen, Min Xu, Filippo Menolascina, and Vicente Grau. Medical graph rag: Towards safe medical large language model via graph retrieval-augmented generation, 2024. URL <https://arxiv.org/abs/2408.04187>.
- Xiaobao Wu. Sailing ai by the stars: A survey of learning from rewards in post-training and test-time scaling of large language models, 2025. URL <https://arxiv.org/abs/2505.02686>.
- Yilin Xiao, Junnan Dong, Chuang Zhou, Su Dong, Qianwen Zhang, Di Yin, Xing Sun, and Xiao Huang. Graphrag-bench: Challenging domain-specific reasoning for evaluating graph retrieval-augmented generation, 2025. URL <https://arxiv.org/abs/2506.02404>.
- Tian Xie, Zitian Gao, Qingnan Ren, Haoming Luo, Yuqian Hong, Bryan Dai, Joey Zhou, Kai Qiu, Zhirong Wu, and Chong Luo. Logic-rl: Unleashing llm reasoning with rule-based reinforcement learning, 2025. URL <https://arxiv.org/abs/2502.14768>.
- Derong Xu, Wei Chen, Wenjun Peng, Chao Zhang, Tong Xu, Xiangyu Zhao, Xian Wu, Yefeng Zheng, Yang Wang, and Enhong Chen. Large language models for generative information extraction: a survey. *Front. Comput. Sci.*, 18(6), November 2024. ISSN 2095-2228. doi: 10.1007/s11704-024-40555-y. URL <https://doi.org/10.1007/s11704-024-40555-y>.
- Tianyang Xu, Haojie Zheng, Chengze Li, Haoxiang Chen, Yixin Liu, Ruoxi Chen, and Lichao Sun. Noderag: Structuring graph-based rag with heterogeneous nodes, 2025. URL <https://arxiv.org/abs/2504.11544>.
- An Yang, Anfeng Li, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chang Gao, Chengan Huang, Chenxu Lv, Chujie Zheng, Dayiheng Liu, Fan Zhou, Fei Huang, Feng Hu, Hao Ge, Haoran Wei, Huan Lin, Jialong Tang, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Yang, Jiaxi Yang, Jing Zhou, Jingren Zhou, Junyang Lin, Kai Dang, Keqin Bao, Kexin Yang, Le Yu, Lianghao Deng, Mei Li, Mingfeng Xue, Mingze Li, Pei Zhang, Peng Wang, Qin Zhu, Rui Men, Ruize Gao, Shixuan Liu, et al. Qwen3 technical report, 2025. URL <https://arxiv.org/abs/2505.09388>.
- Zhilin Yang, Peng Qi, Saizheng Zhang, Yoshua Bengio, William Cohen, Ruslan Salakhutdinov, and Christopher D. Manning. HotpotQA: A dataset for diverse, explainable multi-hop question answering. In Ellen Riloff, David Chiang, Julia Hockenmaier, and Jun’ichi Tsujii (eds.), *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing*, pp. 2369–2380, Brussels, Belgium, October–November 2018. Association for Computational Linguistics. doi: 10.18653/v1/D18-1259. URL <https://aclanthology.org/D18-1259/>.
- Yue Zhang, Yafu Li, Leyang Cui, Deng Cai, Lemao Liu, Tingchen Fu, Xinting Huang, Enbo Zhao, Yu Zhang, Yulong Chen, Longyue Wang, Anh Tuan Luu, Wei Bi, Freda Shi, and Shuming Shi. Siren’s song in the ai ocean: A survey on hallucination in large language models, 2023. URL <https://arxiv.org/abs/2309.01219>.
- Wayne Xin Zhao, Kun Zhou, Junyi Li, Tianyi Tang, Xiaolei Wang, Yupeng Hou, Yingqian Min, Beichen Zhang, Junjie Zhang, Zican Dong, Yifan Du, Chen Yang, Yushuo Chen, Zhipeng Chen, Jinhao Jiang, Ruiyang Ren, Yifan Li, Xinyu Tang, Zikang Liu, Peiyu Liu, Jian-Yun Nie, and Ji-Rong Wen. A survey of large language models, 2025. URL <https://arxiv.org/abs/2303.18223>.
- Yaowei Zheng, Richong Zhang, Junhao Zhang, Yanhan Ye, and Zheyan Luo. LlamaFactory: Unified efficient fine-tuning of 100+ language models. In Yixin Cao, Yang Feng, and Deyi Xiong (eds.), *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 3: System Demonstrations)*, pp. 400–410, Bangkok, Thailand, August 2024. Association for Computational Linguistics. doi: 10.18653/v1/2024.acl-demos.38. URL <https://aclanthology.org/2024.acl-demos.38/>.
- Yuxiang Zheng, Dayuan Fu, Xiangkun Hu, Xiaojie Cai, Lyumanshan Ye, Pengrui Lu, and Pengfei Liu. Deepresearcher: Scaling deep research via reinforcement learning in real-world environments, 2025. URL <https://arxiv.org/abs/2504.03160>.

## APPENDIX

## A PROMPTS USED IN GRAPH-R1

## A.1 KNOWLEDGE HYPERGRAPH CONSTRUCTION PROMPT

As shown in Figure 9, we use the same n-ary relation-extraction prompt as HyperGraphRAG (Luo et al., 2025a). Moreover, we streamline knowledge-hypergraph construction by skipping confidence-score calculations and adopting a simpler semantic-retrieval method, reducing construction costs while maintaining equivalent knowledge-representation.

**-Goal-**  
Given a text document that is potentially relevant to this activity and a list of entity types, identify all entities of those types from the text and all relationships among the identified entities Use [language] as output language.

**-Steps-**

1. Divide the text into several complete knowledge segments. For each knowledge segment, extract the following information:
  - knowledge\_segment: A sentence that describes the context of the knowledge segment.Format each knowledge segment as [Hyper-relation]<tuple\_delimiter><knowledge\_segment>
2. Identify all entities in each knowledge segment. For each identified entity, extract the following information:
  - entity\_name: Name of the entity, use same language as input text. If English, capitalize the name.
  - entity\_type: Type of the entity.
  - entity\_description: Comprehensive description of the entity's attributes and activities.Format each entity as <entity><tuple\_delimiter><entity\_name><tuple\_delimiter><entity\_type><tuple\_delimiter><entity\_description>
3. Return output in [language] as a single list of all the entities and relationships identified in steps 1 and 2. Use \*\*[record\_delimiter]\*\* as the list delimiter.
4. When finished, output [completion\_delimiter]

#####
-Examples-  
#####
(examples)

#####
+ Test Data  
#####
Text: [input\_text]  
#####

Figure 9: Prompt for n-ary relation extraction  $\pi_{\text{ext}}$  in Equation 4.

## A.2 AGENTIC KNOWLEDGE REASONING PROMPT

The initial knowledge-reasoning prompt has been shown in Table 1. Through several rounds of interaction with the environment, the Graph-R1 agent keeps adding information, readying the prompt for the next exchange. As shown in Figure 10, we present a case of the final prompt produced by the full agentic knowledge-reasoning process.

Figure 10: Prompt for agentic knowledge reasoning  $\pi_\theta$  in Equation 6.

## B THEORETICAL PROOF

### B.1 PROOF OF PROPOSITION 1

**Proposition 1.** *Graph-structured knowledge boosts agent accuracy by richer representation.*

*Proof.* Let the knowledge base  $\mathcal{K}$  be encoded into two forms: a graph  $R_G$  and a linear chunk set  $R_C$ , where  $R_C = g(R_G)$  is a deterministic transformation that discards edge information. For a query  $Q$  and ground-truth answer  $A^*$ , the agent's internal belief at step  $t$  is  $h_t$ . Each step performs retrieval  $\mathcal{E}_t(R)$  and Bayesian update, forming the recurrence:

$$h_{t+1} = f(h_t, R). \quad (16)$$

Define the Lyapunov function as  $V_R(h_t) = -\log P(A^* | h_t)$ , measuring how far the agent is from certainty. Its update is:

$$\Delta V_R(h_t) = -\log \frac{P(\mathcal{E}_t(R) | A^*)}{\sum_a P(a | h_t)P(\mathcal{E}_t(R) | a)}. \quad (17)$$

Graphs can capture more relevant facts in shorter contexts due to explicit edges, leading to higher information density  $\delta_R$  and more negative  $\Delta V_R(h_t)$  in expectation. Thus,  $V_R(h_t)$  decreases faster with graphs, indicating faster convergence. From an information-theoretic view, the mutual information evolves as:

$$I(A^*; h_{t+1} | Q) = I(A^*; h_t | Q) + I(A^*; \mathcal{E}_t(R) | h_t, Q), \quad (18)$$

and since graphs provide denser evidence, we have  $I_{R_G}(A^*; h_T | Q) \geq I_{R_C}(A^*; h_T | Q)$ . Then by Fano's inequality,

$$P_e(R) \leq \frac{H(A^* | Q) - I_R(A^*; h_T | Q) + 1}{\log |\mathcal{A}|}, \quad (19)$$

which implies  $P_e(R_G) \leq P_e(R_C)$ , i.e.,  $\text{Acc}(R_G) \geq \text{Acc}(R_C)$ , with strict inequality when the graph contains structural relations not recoverable from text.

In summary, the graph-structured representation offers higher information density per retrieval, accelerates belief convergence via Lyapunov descent, and accumulates more mutual information, leading to provably higher answer accuracy.  $\square$

### B.2 PROOF OF PROPOSITION 2

**Proposition 2.** *Multi-turn interaction with the graph environment improves retrieval efficiency.*

*Proof.* Let the graph-structured knowledge base be denoted by  $R_G$ , and let  $A^*$  be the ground-truth answer. Suppose the retrieval cost is measured by the number of tokens retrieved, and we fix a total budget of  $B$  tokens. A single-turn retrieval strategy selects a fixed token set  $\mathcal{E}_{\text{static}}$  of size  $B$ , independent of any intermediate reasoning. This leads to a posterior belief  $P(A^* | Q, \mathcal{E}_{\text{static}})$  and yields total information gain

$$I_{\text{static}} = I(A^*; \mathcal{E}_{\text{static}} | Q) = H(A^* | Q) - H(A^* | Q, \mathcal{E}_{\text{static}}), \quad (20)$$

where  $Q$  is the query and  $H(\cdot)$  denotes entropy. In contrast, an adaptive multi-turn strategy  $\pi$  divides the budget across  $T$  rounds as  $B = \sum_{t=1}^T B_t$ . At each round  $t$ , the agent uses prior evidence  $\mathcal{H}_{t-1} = \{\mathcal{E}_1, \dots, \mathcal{E}_{t-1}\}$  to update its internal belief  $h_{t-1}$  and selects new evidence  $\mathcal{E}_t$  of size  $B_t$  by actively exploring the graph based on current uncertainty. The updated belief  $h_t$  is obtained via Bayesian inference, and the entire process forms a dynamic system:

$$h_t = f(h_{t-1}, \mathcal{E}_t, R_G). \quad (21)$$

To evaluate retrieval progress, we define a Lyapunov-style potential function  $V_t = H(A^* | Q, \mathcal{H}_t)$ , which quantifies the remaining uncertainty after round  $t$ . Each retrieval step reduces entropy by:

$$V_{t-1} - V_t = I(A^*; \mathcal{E}_t | Q, \mathcal{H}_{t-1}), \quad (22)$$

which is precisely the mutual information contributed by  $\mathcal{E}_t$  given past evidence. Let  $\rho_t$  denote the information gain per token at round  $t$ :

$$\rho_t = \frac{I(A^*; \mathcal{E}_t | Q, \mathcal{H}_{t-1})}{B_t}, \quad (23)$$

and define the average information density of the static strategy as:

$$\rho_{\text{static}} = \frac{I_{\text{static}}}{B}. \quad (24)$$

Since the adaptive agent can tailor each  $\mathcal{E}_t$  to the current belief state and dynamically select high-impact regions in the graph, it is expected that  $\rho_t \geq \rho_{\text{static}}$  in each round. When the graph structure allows the agent to prune irrelevant branches based on early evidence, this inequality becomes strict with non-zero probability. Summing over all rounds, the total information gain of the adaptive strategy satisfies:

$$\mathbb{E}_\pi \left[ \sum_{t=1}^T I(A^*; \mathcal{E}_t | Q, \mathcal{H}_{t-1}) \right] = \mathbb{E}_\pi [I(A^*; \mathcal{H}_T | Q)] \geq I_{\text{static}}, \quad (25)$$

with strict inequality under the above conditions. From a Bayesian viewpoint, retrieval efficiency can be seen as how much uncertainty is reduced per token. Because the adaptive policy achieves a greater entropy reduction under the same budget, or requires fewer tokens to reach the same posterior certainty, it is strictly more efficient. Moreover, by Fano's inequality,

$$P_e \leq \frac{H(A^* | Q) - I(A^*; \mathcal{H}_T | Q) + 1}{\log |\mathcal{A}|}, \quad (26)$$

we conclude that the lower the conditional entropy, the lower the expected error. Therefore, greater mutual information directly translates into improved answer accuracy.

In conclusion, multi-turn interaction enables the agent to reason over what has already been retrieved, selectively expanding into the most informative parts of the graph, leading to more efficient and accurate question answering.  $\square$

### B.3 PROOF OF PROPOSITION 3

**Proposition 3.** *End-to-end RL bridges the gap between graph-based knowledge and language.*

*Proof.* Let  $G_H$  denote the graph-structured knowledge base, and  $q$  a given query. The agent (parameterized by  $\theta$ ) interacts over multiple steps, forming a trajectory  $\tau = (s_1, a_1, \dots, s_T, a_T)$  where each  $a_t$  is either a graph query or a natural-language output. The policy induces an answer distribution:

$$P_\theta(y | q, G_H) = \sum_{\tau: \text{answer}(\tau)=y} \pi_\theta(\tau | q, G_H). \quad (27)$$

To align graph usage with answer generation, we define a trajectory-level reward:

$$R(\tau) = r_{\text{fmt}}(\tau) + \mathbb{I}\{r_{\text{fmt}}(\tau) = 1\} \cdot r_{\text{ans}}(y_T, y_q^*) - 1, \quad (28)$$

where  $r_{\text{fmt}}$  ensures proper structure (e.g., retrieve before answer), and  $r_{\text{ans}}$  measures answer quality. Only valid, grounded answers receive positive reward. The expected reward is maximized via policy gradient:

$$\nabla_\theta J(\theta) \propto \mathbb{E}_{\tau \sim \pi_\theta} \left[ \sum_{t=1}^T \nabla_\theta \log \pi_\theta(a_t | s_t; G_H) \cdot \hat{A}(\tau) \right], \quad (29)$$

where  $\hat{A}(\tau)$  derives from  $R(\tau)$ . Trajectories that retrieve the right subgraph and generate correct answers are reinforced, linking graph retrieval to linguistic accuracy. As training progresses, the expected log-likelihood of the gold answer increases:

$$\mathcal{L}_\theta = -\log \sum_{\tau} \pi_\theta(\tau | q, G_H) P(y_q^* | \tau), \quad (30)$$

which lower-bounds the ideal  $\log P^*(y_q^* | q, G_H)$ . In the limit, we approach:

$$P_\theta(\cdot | q, G_H) \rightarrow P^*(\cdot | q, G_H). \quad (31)$$

This also manifests as a reduction in conditional entropy:

$$H_\theta(Y | Q, G_H) < H(Y | Q), \quad (32)$$

since ungrounded answers are discouraged and graph-consistent ones are promoted. By Fano's inequality, lower entropy implies lower error.

Thus, end-to-end RL not only learns to query the graph but also binds retrieved knowledge to answer generation, effectively bridging the gap between structure and language.  $\square$

## C GRAPH-R1 ALGORITHM DETAILS

To illustrate the mechanism of Graph-R1, we present its full workflow in Algorithm 1, comprising three key phases: (1) **Hypergraph Construction**. An LLM-based extractor  $\pi_{\text{ext}}$  extracts  $n$ -ary relational facts from the corpus  $K$  to build a semantic hypergraph  $\mathcal{G}_H = (V, E_H, \phi)$ , where all elements are encoded via  $\text{Enc}(\cdot)$ . (2) **Multi-turn Agentic Reasoning**. Given query  $q$ , the agent performs reflection, intent selection, and action generation over  $T$  steps under policy  $\pi_\theta$ . Queries trigger dual-path hypergraph retrieval; answers terminate reasoning. (3) **End-to-end RL Optimization**. The policy is optimized using GRPO, guided by format and answer rewards. The objective  $\mathcal{J}_{\text{GRPO}}$  is computed from sampled trajectories  $\{\tau_i\}$  with clipped advantage-weighted updates.

---

**Algorithm 1** Graph-R1: Agentic GraphRAG via End-to-end RL

---

**Require:** Query  $q$ , knowledge corpus  $K = \{d_1, \dots, d_N\}$ , policy  $\pi_\theta$ , reward function  $R(\tau)$   
**Ensure:** Final answer  $y_q$

```

1: // 1: Knowledge Hypergraph Construction
2: Initialize hypergraph  $\mathcal{G}_H = (V, E_H, \phi)$ 
3: for each document  $d \in K$  do
4:   Extract relational facts:  $\{(h_i, \mathcal{V}_{h_i})\} \sim \pi_{\text{ext}}(d)$ 
5:   for each  $(h_i, \mathcal{V}_{h_i})$  do
6:      $E_H \leftarrow E_H \cup \{h_i\}$ ,  $V \leftarrow V \cup \mathcal{V}_{h_i}$ 
7:      $\phi(h_i) \leftarrow \text{Enc}(h_i)$ ,  $\phi(v) \leftarrow \text{Enc}(v)$  for  $v \in \mathcal{V}_{h_i}$ 
8:   end for
9: end for

10: // 2: Multi-turn Graph Reasoning
11: Initialize state  $s_1 \leftarrow q$ , trajectory  $\tau \leftarrow \emptyset$ 
12: for  $t = 1$  to  $T$  do
13:   Generate reasoning plan:  $\mathbf{a}_t^{\text{think}} \sim \pi_\theta(\cdot | s_t)$ 
14:   Choose intent:  $\alpha_t \sim \pi_\theta(\cdot | \mathbf{a}_t^{\text{think}}, s_t)$ 
15:   if  $\alpha_t = (\text{answer})$  then
16:     Output answer:  $\mathbf{a}_t^{\text{ans}} \sim \pi_\theta(\cdot | \mathbf{a}_t^{\text{think}}, s_t)$ 
17:      $\tau \leftarrow \tau \cup \{(s_t, \mathbf{a}_t^{\text{query}}, \mathbf{a}_t^{\text{ans}})\}$ ; return  $y_q = \mathbf{a}_t^{\text{ans}}$ 
18:   else if  $\alpha_t = (\text{query}, \text{retrieve})$  then
19:     Generate query:  $\mathbf{a}_t^{\text{query}} \sim \pi_\theta(\cdot | \mathbf{a}_t^{\text{think}}, s_t)$ 
20:     Entity retrieval:  $\mathcal{R}_V = \text{argmax}_{v \in V}^k \text{sim}(\phi(v), \phi(V_{\mathbf{a}_t^{\text{query}}}))$ 
21:     Hyperedge retrieval:  $\mathcal{R}_H = \text{argmax}_{h \in E_H}^k \text{sim}(\phi(h), \phi(\mathbf{a}_t^{\text{query}}))$ 
22:     Rank fusion:  $\mathbf{a}_t^{\text{ret}} = \text{Top-}k \left( \mathcal{F}_V^* \cup \mathcal{F}_H^*, \text{Score}(f) = \frac{1}{r_V(f)} + \frac{1}{r_H(f)} \right)$ 
23:     Update state  $s_{t+1} \leftarrow s_t \cup \{(s_t, \mathbf{a}_t^{\text{think}}, \mathbf{a}_t^{\text{query}}, \mathbf{a}_t^{\text{ret}})\}$ 
24:      $\tau \leftarrow \tau \cup \{(s_t, \mathbf{a}_t^{\text{think}}, \mathbf{a}_t^{\text{query}}, \mathbf{a}_t^{\text{ret}})\}$ 
25:   end if
26: end for

27: // 3: End-to-end Policy Optimization (GRPO)
28: Sample  $N$  trajectories  $\{\tau_i\} \sim \pi_{\theta_{\text{old}}}$ 
29: for each  $\tau_i$  do
30:   Compute reward:  $R(\tau_i) = -1 + R_{\text{format}}(\tau_i) + \mathbb{I}\{R_{\text{format}} = 1\} \cdot R_{\text{answer}}(y_T, y_q^*)$ 
31:   Compute advantage:  $\hat{A}(\tau_i) = \frac{R(\tau_i) - \text{mean}(\{R(\tau_j)\})}{\text{std}(\{R(\tau_j)\})}$ 
32: end for
33: Update policy via GRPO:  $\mathcal{J}_{\text{GRPO}} \sim \sum_{i=1}^N \sum_{t=1}^{|\tau_i|} \min \left( \rho_\theta(a_t^{(i)}) \hat{A}(\tau_i), \text{clip}(\rho_\theta(a_t^{(i)}), 1 \pm \epsilon) \hat{A}(\tau_i) \right)$ 
34: where  $\rho_\theta(a_t^{(i)}) = \frac{\pi_\theta(a_t^{(i)} | s_{t-1}^{(i)})}{\pi_{\theta_{\text{old}}}(a_t^{(i)} | s_{t-1}^{(i)})}$ 

```

---

*Complexity Analysis.* Graph-R1 involves three computational components corresponding to phases. First, hypergraph construction scales with the total token count  $T_K$  of the knowledge corpus and the number of extracted relational facts  $F$ , yielding complexity  $O(T_K) + O(F)$ . Second, during multi-turn reasoning, the agent performs  $T$  steps of action sampling and dual-path retrieval. At each step, similarity computations over  $|V|$  nodes and  $|E_H|$  hyperedges with embedding dimension  $d$  yield  $O((|V| + |E_H|)d)$  per step. Third, for policy optimization, GRPO processes  $N$  sampled trajectories of max length  $T$ , with gradient updates costing  $O(NTd)$ . Each component is computationally tractable and benefits from parallelization and localized retrieval over compact hypergraph subsets.

## D DATASET DETAILS

We conduct experiments on six widely-used RAG benchmarks selected from the FlashRAG toolkit (Jin et al., 2025b), covering both single-hop and multi-hop question answering tasks:

- **2WikiMultiHopQA (2Wiki.)** (Ho et al., 2020): A multi-hop dataset requiring reasoning across two Wikipedia documents.
- **HotpotQA** (Yang et al., 2018): A challenging multi-hop QA dataset with sentence-level supporting facts and diverse question types.
- **Musique** (Trivedi et al., 2022): Multi-hop questions needing chains of inference, often involving three or more reasoning steps.
- **Natural Questions (NQ)** (Kwiatkowski et al., 2019): A large-scale single-hop QA dataset grounded in real Google search questions with Wikipedia passages.
- **PopQA** (Mallen et al., 2023): An open-domain QA dataset focused on popular culture questions sourced from Wikipedia.
- **TriviaQA** (Joshi et al., 2017): A large-scale dataset containing trivia-style questions with distantly supervised evidence documents.

To ensure consistency across datasets and maintain manageable training and evaluation workloads, we uniformly sample 5,120 instances per dataset for training and 128 instances for testing.

## E BASELINE DETAILS

Our experiments compare Graph-R1 with two groups of baselines using different backbone LLMs:

### E.1 BASELINES WITH GPT-4O-MINI

- **NaiveGeneration (GPT-4o-mini)**: Zero-shot generation using GPT-4o-mini without retrieval, evaluating base model capacity.
- **StandardRAG (GPT-4o-mini)** (Lewis et al., 2020): Chunk-based RAG using GPT-4o-mini as the generator with retrieval over text chunks.
- **GraphRAG** (Edge et al., 2025): Graph-structured retrieval baseline that constructs entity graphs and performs one-shot retrieval with GPT-4o-mini for answer generation.
- **LightRAG** (Guo et al., 2025): A lightweight GraphRAG variant that builds compact graphs for more efficient retrieval and GPT-4o-mini generation.
- **PathRAG** (Chen et al., 2025): Retrieval via path-based pruning on entity graphs, followed by GPT-4o-mini answer synthesis.
- **HippoRAG2** (Gutiérrez et al., 2025): Hierarchical path planner over knowledge graphs to improve retrieval efficiency, with GPT-4o-mini used for generation.
- **HyperGraphRAG** (Luo et al., 2025a): Constructs n-ary relational hypergraphs to support a single retrieval step, and uses GPT-4o-mini for answer writing.

### E.2 BASELINES WITH QWEN2.5 (1.5B, 3B, 7B)

- **NaiveGeneration**: Direct generation by Qwen2.5 given the question prompt, without any retrieval, serving as a lower bound baseline.
- **StandardRAG** (Lewis et al., 2020): Classic chunk-based retrieval-augmented generation pipeline with semantic retriever and Qwen2.5 decoder.
- **SFT** (Zheng et al., 2024): Supervised fine-tuning of Qwen2.5 on QA pairs, without multi-turn reasoning or reinforcement optimization.
- **R1** (Shao et al., 2024): A GRPO-trained policy that generates final answers directly from question prompts without retrieval, optimized only on answer quality.
- **Search-R1** (Jin et al., 2025a): A multi-turn chunk-based retrieval method trained with GRPO, capable of iterative query refinement and retrieval under a unified policy.
- **R1-Searcher** (Song et al., 2025): A two-stage GRPO-based method with chunk-based retrieval: first using only format-level rewards to produce structured traces, then adding answer rewards.

## F EVALUATION DETAILS

Inspired by Luo et al. (2025a); Jin et al. (2025a), we evaluate model performance using four metrics:

**(i) Exact Match (EM).** EM measures whether the predicted answer exactly matches the ground truth. Let  $\text{norm}(\cdot)$  denote the normalization function:

$$\text{EM} = \frac{1}{N} \sum_{i=1}^N \mathbb{I}\{\text{norm}(y_i) = \text{norm}(y_i^*)\}. \quad (33)$$

**(ii) F1 Score.** The F1 score measures the token-level overlap between the predicted answer  $y_i$  and the ground-truth answer  $y^*$  using the harmonic mean of precision and recall:

$$\text{F1} = \frac{1}{N} \sum_{i=1}^N \frac{2 \cdot |\text{tokens}(y_i) \cap \text{tokens}(y_i^*)|}{|\text{tokens}(y_i)| + |\text{tokens}(y_i^*)|}. \quad (34)$$

"comprehensiveness": (	"knowledgeability": (	"correctness": (	"relevance": (	"diversity": (	"logical coherence": (	"factuality": (
"comprehensiveness", "whether the thinking considers all important aspects and is thoughtful" "Scoring Guide" (0-10): - 10: Extremely thorough, covering all relevant angles and considerations with depth. - 8-9: Covers most key aspects clearly and thoughtfully; only minor omissions. - 6-7: Covers some important aspects, but lacks depth or overlooks notable areas. - 4-5: Tries to cover all relevant points, but overall lacks substance or completeness. - 1-3: Sparse or shallow treatment of one topic; misses most key aspects. - 0: No comprehensiveness at all; completely superficial or irrelevant.***" )	"knowledgeability": ( "whether the thinking is rich in insightful domain-relevant knowledge" "Scoring Guide" (0-10): - 10: Demonstrates exceptional depth and insight with strong domain-specific knowledge. - 8-9: Shows clear domain knowledge with good insight; has notable gaps. - 6-7: Displays some understanding, but lacks depth or has notable gaps. - 4-5: Shows some knowledge, but understanding is basic or somewhat flawed. - 1-3: Poor grasp of relevant knowledge; superficial or mostly inaccurate. - 0: No evidence of meaningful knowledge.***" )	"correctness": ( "correctness", "whether the reasoning and answer are logically and factually correct" "Scoring Guide" (0-10): - 10: Fully accurate and logically sound; no flaws in reasoning or facts. - 8-9: Shows clear domain knowledge with good insight; has notable gaps. - 6-7: Partially accurate; some key flaws or inconsistencies present. - 4-5: Noticeable incorrect reasoning or factual errors throughout. - 1-3: Largely incorrect, misleading, or illogical. - 0: Entirely wrong or nonsensical.***" )	"relevance": ( "relevance", "whether the reasoning and answer are highly relevant and helpful to the question" "Scoring Guide" (0-10): - 10: Highly focused on the question; highly relevant and helpful. - 8-9: Mostly correct with minor inaccuracies or small logical flaws. - 6-7: Partially relevant, but includes distractions or less helpful parts. - 4-5: Limited relevance; much of the reasoning is off-topic or unrelated. - 1-3: Barely related to the question or largely unhelpful. - 0: Entirely irrelevant.***" )	"diversity": ( "diversity", "whether the reasoning is thought-provoking, covering varied or multiple perspectives" "Scoring Guide" (0-10): - 10: Exceptionally rich and original, demonstrates multiple fresh and thought-provoking ideas. - 8-9: Contains a diverse range of ideas, or interesting perspectives, but a few are redundant or weakly connected parts. - 6-7: Some variety, but generally safe or conventional. - 4-5: Mostly standard thinking; limited diversity. - 1-3: Very predictable or monotonous. - 0: No diversity or originality at all.***" )	"logical coherence": ( "logical coherence", "whether the reasoning is internally consistent, clear, and well-structured" "Scoring Guide" (0-10): - 10: All facts are accurate and verifiable. - 8-9: Mostly accurate; only minor factual issues. - 6-7: Contains some factual inaccuracy or unverified claims. - 4-5: Several significant factual errors. - 1-3: Mostly false or misleading. - 0: Completely fabricated or factually wrong throughout.***" )	"factuality": ( "factuality", "whether the reasoning and answer are based on accurate and verifiable facts" "Scoring Guide" (0-10): - 10: All facts are accurate and verifiable. - 8-9: Mostly accurate; only minor factual issues. - 6-7: Contains some factual inaccuracy or unverified claims. - 4-5: Several significant factual errors. - 1-3: Mostly false or misleading. - 0: Completely fabricated or factually wrong throughout.***" )

Figure 11: Seven Dimensions for Generation Evaluation.

## G IMPLEMENTATION DETAILS

As shown in Table 5, we summarize the detailed hyperparameter configurations used throughout our experiments, including model backbone, input limits, training configuration, and retrieval setup.

Method	Backbone	Batch Size	Max Length	Top-K	Algo	Epochs
NaiveGeneration	Qwen2.5 / GPT-4o-mini	—	$\infty$	N/A	—	—
StandardRAG	Qwen2.5 / GPT-4o-mini	—	$\infty$	5 Chunks	—	—
GraphRAG	GPT-4o-mini	—	$\infty$	60	—	—
LightRAG	GPT-4o-mini	—	$\infty$	60	—	—
PathRAG	GPT-4o-mini	—	$\infty$	60	—	—
HippoRAG2	GPT-4o-mini	—	$\infty$	60	—	—
HyperGraphRAG	GPT-4o-mini	—	$\infty$	60	—	—
SFT	Qwen2.5 (1.5B, 3B, 7B)	16	4096	N/A	LoRA	3
R1	Qwen2.5 (1.5B, 3B, 7B)	128	4096	N/A	GRPO	3
Search-R1	Qwen2.5 (1.5B, 3B, 7B)	128	4096	5 Chunks / Turn	GRPO	3
R1-Searcher	Qwen2.5 (1.5B, 3B, 7B)	128	4096	5 Chunks / Turn	GRPO	3
Graph-R1 (ours)	Qwen2.5 (1.5B, 3B, 7B)	128	4096	5 / Turn	GRPO	3

Table 5: Hyperparameter settings for baselines and Graph-R1.

## H LIMITATIONS AND FUTURE WORK

While Graph-R1 achieves strong performance, several limitations remain. **First**, the cost of hypergraph construction, especially relation extraction and encoding, remains non-trivial. Future work may explore more efficient methods for zero-cost extraction. **Second**, current retrieval lacks structural reasoning. Integrating GNNs or trainable message-passing could improve both accuracy and scalability. **Third**, Graph-R1 currently supports only textual knowledge; extending it to multi-modal inputs is a promising direction. **Finally**, we aim to further apply Graph-R1 in knowledge-intensive domains such as healthcare, law, and finance, where robust and interpretable reasoning is essential.