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Assignment 1

1.) $T(n) = T(n-1) + n$ is $O(n^2)$. Prove by substitution

$T(n) [T(n-2) + n-1] + n$

$T(n) = T(n-2) + (n-1) + n$

$[T(n-3) + n-2] + (n-1) + n$

$T(n) = T(n-3) + (n-2) + (n-1) + n$

$T(n) = T(n-k) + (n-k+1) + (n-k+2) + \dots + n$

$T(n) = T(1) + \frac{n(n+1)}{2}$

$T(n) = \underline{O(n^2)}$

2.) $T(n) \quad -1 \qquad 2T(n-1)+1$

$T(n-1) \quad T(n-1) \qquad 1 \times 2 = 2$

$T(n-2) \; T(n-2) \; T(n-2) \; T(n-2) \qquad 1 \times 4 = 4$

cont. 2^i

$\vdots$

$T(1) = 2^{n-1} \rightarrow 2^n - 1 = \underline{O(2^n)}$

$T(1) = 2T(0) + 1 = 1$

$T(k) \leq c \cdot 2^k$

$T(n) \leq 2(c \cdot 2^{n-1}) + 1 = c \cdot 2^n \cancel{+1}$

$c \cdot 2^n$

$T(n) = \underline{O(2^n)}$

$$T(n)=aT\left(\frac{n}{b}\right)+f(n)$$

3a.) $T(n)=2T\left(\frac{n}{4}\right)+1 \quad a=2 \quad b=4 \quad f(n)=1=O(n^0)$

$$\log_4 2=\frac{\log 2}{\log 4}=\frac{\log 2}{2\log 2}=\frac{1}{2}\to \log_b a=\frac{1}{2}$$

$$n^{\log_b a}=n^{1/2}$$

$$T(n)=O(n^{\log_b a})=O(n^{1/2})=O(\sqrt{n}) \quad 0<\tfrac{1}{2} \Rightarrow \#1$$

$$T(n)=2T\left(\frac{n}{4}\right)+1=\underline{O(\sqrt{n})}$$

3b.) $T(n)=2T\left(\frac{n}{4}\right)+\sqrt{n} \quad a=2 \quad b=4 \quad f(n)=\sqrt{n}=n^{1/2}$

$$\log_4 2=\frac{\log 2}{\log 4}=\frac{1}{2}\to f(n)=n^{1/2} \text{ and } n^{\log_b a}=n^{1/2}$$

$$f(n)=O(n^{\log_b a})\Rightarrow \#2$$

$$T(n)=O(n^{\log_b a}\log n)=O(n^{1/2}\log n)$$

$$T(n)=\underline{O(\sqrt{n}\log n)}$$

3c.) $T(n)=2T\left(\frac{n}{4}\right)+n \quad a=2 \quad b=4 \quad f(n)=n$

$$\log_4 2=\frac{\log 2}{\log 4}=\frac{1}{2} \quad n^{\log_b a}=n^{1/2}$$

$$f(n)=n>n^{1/2}\Rightarrow \#3$$

$$af\left(\frac{n}{b}\right)=2f\left(\frac{n}{4}\right)=2\cdot\frac{n}{4}=\frac{n}{2}\leq cn$$

$$T(n)=O(f(n))=\underline{O(n)}$$

3d) $T(n)=2T\left(\frac{n}{4}\right)+n^2 \quad a=2 \quad b=4 \quad f(n)=n^2$

$$\log_4 2=\frac{1}{2}\Rightarrow n^{1/2}<n^2\Rightarrow \#3$$

$$af\left(\frac{n}{b}\right)=2f\left(\frac{n}{4}\right)2\left(\frac{n}{4}\right)^2=2\cdot\frac{n^2}{16}=\frac{n^2}{8}\leq cn^2$$

$$T(n)=O(f(n))=\underline{O(n^2)}$$

4.) $T(n)=4T(\frac{n}{2})+n$ G $\Rightarrow T(n)\le cn^2$

$$T(n)\le 4(c(\tfrac{n}{2})^2)+n$$
$$T(n)\le 4(c\cdot\tfrac{n^2}{4})+n$$
$$T(n)\le cn^2+n$$ fails bc "+n"

G $\Rightarrow T(n)\le cn^2-dn$

$$T(n)\le 4(c(\tfrac{n}{2})^2-d(\tfrac{n}{2}))+n$$
$$T(n)\le 4(c\cdot\tfrac{n^2}{4}-d\cdot\tfrac{n}{2})+n$$
$$T(n)\le cn^2-2dn+n$$
$$cn^2-2dn+n\le cn^2-dn$$
$$\underline{T(n)\le cn^2-dn \quad d\ge 1}$$

5.) {31, 41, 59, 26, 41, 58, 89, 20}

{31, 41, 59, 26}   {41, 58, 89, 20}

{31, 41} {59, 26}   {41, 58} {89, 20}

31 41 59 26   41 58 89 20

31, 41   26, 59   41, 58   20, 89

26, 31, 41, 59   20, 41, 58, 89

{20, 26, 31, 41, 41, 58, 59, 89}