T(n) = 2T(n/2) + O($\sqrt{n}$) → a = 2, b=2, f(n) = $O(\sqrt{n})$

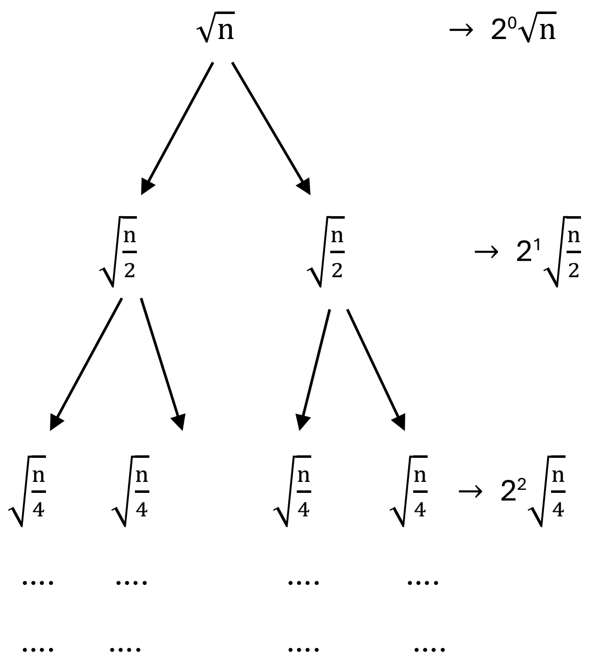
<u>Master's theorem</u>

$n^{\log_b a} = n^{\log_2 2} = n$

f(n) = O($n^{1/2}$) = O($n^{1-1/2}$) = $O\left(n^{\log_2 2 - \frac{1}{2}}\right)$ = $O\left(n^{\log_b a - \varepsilon}\right)$ where $\varepsilon = \frac{1}{2} > 0$

→ case 1 of master's theorem =>T(n) = $\theta(n)$

<u>Tree method:</u>



total work = $2^l.\sqrt{\frac{n}{2^l}}$

↙ ↘

Number of nodes work on each node

T(1) = T($\sqrt{\frac{n}{2^l}}$) => $\sqrt{\frac{n}{2^l}} = 1 => \sqrt{n} = \sqrt{2^l} => n = 2^l \rightarrow l = \log_2 n$: tree height

Total level:

$\sum_{l=0}^{log_2 n}(2^l.\sqrt{n.2^{-l}})$=$\sqrt{n}\sum_{l=0}^{log_2 n}(2^l.2^{-l/2})$=$\sqrt{n}\sum_{l=0}^{log_2 n} 2^{l/2}$

$=\sqrt{n}\sum_{l=0}^{log_2 n}(\sqrt{2})^l$ ---------→ forms geometric series

$= \sqrt{n}\ .\ \frac{(\sqrt{2})^{\log_2 n+1}-1}{\sqrt{2}-1} = \sqrt{n}\ .\ \frac{1}{\sqrt{2}-1}\ .\ ((\sqrt{2})^{\log_2 n+1} - 1) \quad \rightarrow \left\{\frac{1}{\sqrt{2}-1}\ is\ constant\right\}$

= c . $\sqrt{n}$. ($2^{\log_2 \sqrt{n}+(\frac{1}{2})}$ - 1) = c. $\sqrt{n}$. ($\sqrt{n}.\sqrt{2}$ - 1) → $O(n)$

Geometric series:

$$a + ar + ar^2 + ar^3 + \ldots\ldots\ldots. + ar^n = \sum_{k=0}^{n} ar^k = \frac{a(r^{n+1}-1)}{(r-1)}$$

$(\sqrt{2})^{\log_2 n} = 2^{\frac{1}{2}\log_2 n} = 2^{\log_2 n^{\frac{1}{2}}} = 2^{\log_2 \sqrt{n}} = \sqrt{n}$