

Example of matrix-chain multiplication:

$$c[i,j] = \min \{c[i,k] + c[k+1,j] + d_{i-1} * d_k * d_j\}$$

$$i \leq k < j$$

$A_1 \times A_2 \times A_3 \times A_4$
 $(3 \ 2) \ (2 \ 4) \ (4 \ 2) \ (2 \ 5)$
 $(d_0 \ d_1) \ (d_1 \ d_2) \ (d_2 \ d_3) \ (d_3 \ d_4)$

		j	1	2	3	4
Cost(c)	1		0	24	28	58
	2			0	16	36
	3				0	40
i	4					0

		1	2	3	4
k value	1		1	1	3
	2			2	3
	3				3
	4				

$$c[1,1] = 0$$

$$c[1,2] = \min_{1 \leq k < 2} \{c[1,1] + c[2,2] + d_0 * d_1 * d_2 = 3 * 2 * 4 = 24\}$$

$$c[2,3] = \min_{2 \leq k < 3} \{c[2,2] + c[3,3] + d_1 * d_2 * d_3 = 2 * 4 * 2 = 16\}$$

$$c[3,4] = \min_{3 \leq k < 4} \{c[3,3] + c[4,4] + d_2 * d_3 * d_4 = 4 * 2 * 5 = 40\}$$

$$c[1,3] = \min_{1 \leq k < 3} \left[\begin{array}{l} k=1 : c[1,1] + c[2,3] + d_0 * d_1 * d_3 = 16 + (3 * 2 * 2) = 28 \\ k=2 : c[1,2] + c[3,3] + d_0 * d_2 * d_3 = 24 + (3 * 4 * 2) = 48 \end{array} \right]$$

→ min = 28 for k = 1

$$c[2,4] = \min_{2 \leq k < 4} \left[\begin{array}{l} k=2 : c[2,2] + c[3,4] + d_1 * d_2 * d_4 = 40 + (2 * 4 * 5) = 80 \\ k=3 : c[2,3] + c[4,4] + d_1 * d_3 * d_4 = 16 + (2 * 2 * 5) = 36 \end{array} \right]$$

→ min = 36 for k = 3

$$c[1,4] = \min_{1 \leq k < 4} \left[\begin{array}{l} k=1 : c[1,1] + c[2,4] + d_0 * d_1 * d_4 = 36 + (3 * 2 * 5) = 66 \\ k=2 : c[1,2] + c[3,4] + d_0 * d_2 * d_4 = 64 + (3 * 4 * 5) = 124 \\ k=3 : c[1,3] + c[4,4] + d_0 * d_3 * d_4 = 28 + (3 * 2 * 5) = 58 \end{array} \right]$$

→ min = 58 for k = 3

→ Now find parentheses $((A_1).(A_2 . A_3)) . A_4$

From tables: $c[1,4] \rightarrow k=3$
 $c[1,3] \rightarrow k=1$

