

Probabilities: $k_1 < k_2 < k_3 < k_4$
 $\frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{8} \quad \frac{1}{8}$

M(Cost Table)

i \ j	0	1	2	3	4
1	0	$\frac{3}{8}$	$\frac{9}{8}$	$\frac{11}{8}$	$\frac{14}{8}$
2		0	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{8}{8}$
3			0	$\frac{1}{8}$	$\frac{3}{8}$
4				0	$\frac{1}{8}$
5					0

R

i \ j	0	1	2	3	4
1	0	1	1 or 2	2	2
2		0	2	2	2
3			0	3	3 or 4
4				0	4
5					0

Keep
Optimal
root

$$M[i][j] = \min_{i \leq k \leq j} (M[i][k-1] + M[k+1][j]) + \sum_{m=i}^j P_m$$

$$M[1][2] = \min_{1 \leq k \leq 2} \left\{ \begin{array}{l} \overbrace{M[1][0] + M[2][2]}^{K=1: \quad 0} \\ \underbrace{M[1][1] + M[3][2]}_{K=2: \quad \frac{3}{8}} \end{array} \right\} + \underbrace{\sum_{m=1}^2 P_m}_{\frac{6}{8}} = \frac{9}{8} \rightarrow k=1 \text{ or } k=2$$

$$M[2][3] = \min_{2 \leq k \leq 3} \left\{ \begin{array}{l} \overbrace{M[2][1] + M[3][3]}^{K=2: \quad 0} \\ \underbrace{M[2][2] + M[4][3]}_{K=3: \quad \frac{3}{8}} \end{array} \right\} + \underbrace{\sum_{m=2}^3 P_m}_{\frac{4}{8}} = \frac{5}{8} \rightarrow k=2$$

$$M[3][4] = \min_{3 \leq k \leq 4} \left\{ \begin{array}{l} \overbrace{M[3][2] + M[4][4]}^{K=3: \quad 0} \\ \underbrace{M[3][3] + M[5][4]}_{K=4: \quad \frac{1}{8}} \end{array} \right\} + \underbrace{\sum_{m=3}^4 P_m}_{\frac{2}{8}} = \frac{3}{8} \rightarrow k=3 \text{ or } k=4$$

$$M[1][3] = \min_{1 \leq k \leq 3} \begin{cases} K=1: & \cancel{M[1][0]} + \overbrace{M[2][3]}^{\frac{5}{8}} \\ K=2: & M[1][1] + M[3][3] + \underbrace{\sum_{m=1}^3 P_m}_{\frac{7}{8}} = \frac{11}{8} \rightarrow k=2 \\ K=3: & \underbrace{M[1][2]}_{\frac{9}{8}} + \cancel{M[4][3]}_0 \end{cases}$$

$$M[2][4] = \min_{2 \leq k \leq 4} \begin{cases} K=2: & \cancel{M[2][1]} + \overbrace{M[3][4]}^{\frac{3}{8}} \\ K=3: & M[2][2] + M[4][4] + \underbrace{\sum_{m=2}^4 P_m}_{\frac{5}{8}} = \frac{8}{8} \rightarrow k=2 \\ K=4: & \underbrace{M[2][3]}_{\frac{5}{8}} + \cancel{M[5][4]}_0 \end{cases}$$

$$M[1][4] = \min_{1 \leq k \leq 4} \begin{cases} K=1: & \cancel{M[1][0]} + \overbrace{M[2][4]}^{\frac{8}{8}} \\ K=2: & M[1][1] + M[3][4] + \underbrace{\sum_{m=1}^4 P_m}_{\frac{8}{8}} = \frac{14}{8} \rightarrow k=2 \\ K=3: & M[1][2] + M[4][4] \\ K=4: & \underbrace{M[1][3]}_{\frac{11}{8}} + \cancel{M[5][4]}_0 \end{cases}$$

From R Table:

