$$T(n) = 8\,T(n/4) + n^{3/2}$$

$a = 8, b = 4, f(n) = n^{3/2}$

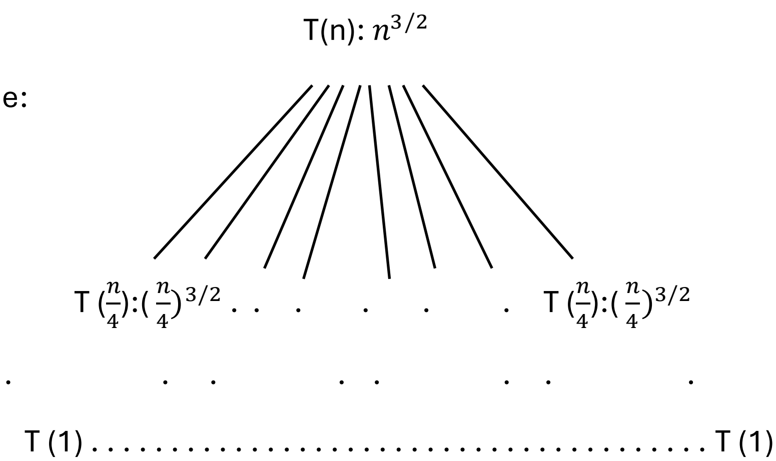
$n^{(\log_b a)} = n^{(\log_4 8)} = n^{(3/2)}$

$n^{(3/2)} = f(n) \rightarrow \textit{Case 2}$

$T(n) = \Theta\,(n^{(3/2)} \log n)$

$\boxed{\begin{array}{l} \log_4 8 = x \rightarrow 4^x = 8 \\ \rightarrow (2^2)^{x} = 2^3 \\ \rightarrow 2x = 3 \\ \rightarrow x = 3/2 \end{array}}$

Recursion Tree:



In level L, we have 8^L nodes. Work in each node = $(n/\,4^L)^{3/2}$

levels (tree height) => 1 = n/4^L => n = 4^L =>L = $\log_4 n$

Merging cost at level 1: 8 $(\frac{n}{4})^{3/2}$= $n^{3/2}$

Merging cost at level 2: 8^2 $(n/4^2)^{3/2}$= $n^{3/2}$

…

Total Amount of work = L* $n^{3/2}$= O $((n)^{3/2}$log n) → O $((n)^{3/2}$log n)

Or:

Total Amount of work = C $\sum_{L=0}^{\log_4 n} (n/4^L)^{3/2} * 8^L$ = C$(n)^{3/2}$ $\sum_{L=0}^{\log_4 n} 4^{-3L/2} * 8^L$

C$(n)^{3/2}$ $\sum_{L=0}^{\log_4 n} 2^{-3L} * 2^{3L}$ = C$(n)^{3/2} \sum_{L=0}^{\log_4 n} 1$

= C$(n)^{3/2}$ (log n + 1) ⇒ O $((n)^{3/2}$log n)

$\boxed{\sum_{i=1}^{n} C = Cn}$