CSCE 3100.001 Assignment 1

(Due on September 22, 11:59PM)

1. (15 points) Use the substitution method to show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$.

2. (20 points) Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 2T(n-1) + 1$. Use the substitution method to verify your answer.

3. (20 points) Use the master method to give tight asymptotic bounds for the following recurrences:

   (1) $T(n) = 2T\left(\frac{n}{4}\right) + 1$

   (2) $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

   (3) $T(n) = 2T\left(\frac{n}{4}\right) + n$

   (4) $T(n) = 2T\left(\frac{n}{4}\right) + n^2$

4. (30 points) Using the Master Theorem, you can show that the solution to the recurrence $T(n) = 4T\left(\frac{n}{2}\right) + \boldsymbol{n}$ is $T(n) = n^2$. Show that a substitution proof with the guess $T(n) \leq cn^2$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.

5. (15 points) Use the Figure from lecture slide as a model, to illustrate the operation of MERGE-SORT on the array $A = \{31; 41; 59; 26; 41; 58; 89; 20\}$.