# Probability on Trees and Networks

Winter 2023

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| **Instructor:** | Luc Devroye | **Time:** | M 11-12, W 10-11 |
| **Email:** | lucdevroye@gmail.com | **Place:** | McConnell Eng. 300N |

**Course Page:** http://arthurayestas.xyz/comp480.html

**Textbook:** This course will follow selected sections of the following textbook:

- Lyons, R., & Peres, Y., *Probability on Trees and Networks*, Cambridge University Press, 2017.

**Prerequisites:** Comp 690 - Probabilistic Analysis of Algorithms

**Tentative Course Outline:**

- Random Walks and Electric Networks
  1. Circuit Basics and Harmonic Functions
  2. More Probabilistic Interpretations
  3. Network Reduction
  4. Energy
  5. Transience and Recurrence
  6. Rough Isometries and Hyperbolic Graphs
  7. Hitting, Commute, and Cover Times
  8. The Canonical Gaussian Field
- Special Networks
  1. Flows, Cutsets, and Random Paths
  2. Trees
  3. Growth of Trees
  4. Cayley Graphs
- Uniform Spanning Trees
  1. Generating Uniform Spanning Trees
  2. Electrical Interpretations
  3. The Square Lattice $\mathbf{Z}^2$

- Branching Processes, Second Moments, and Percolation
  1. Galton-Watson Branching Processes
  2. The First-Moment Method
  3. The Weighted Second-Moment Method
  4. Quasi-independent Percolation
  5. Transience of Percolation Clusters in $\mathbf{Z}^d$
  6. Reversing the Second-Moment Inequality
  7. Surviving Galton-Watson Trees
  8. Harris's Inequality
  9. Galton-Watson Networks
- Hausdorff Dimension
  1. Basics
  2. Coding by Trees
  3. Galton-Watson Fractals
  4. Hölder Exponent
  5. Derived Trees
- Isoperimetric Inequalities
  1. Flows and Submodularity
  2. Spectral Radius
  3. Nonbacktracking Paths and Cogrowth
  4. Relative Mixing Rate, Spectral Gap, and Expansion in Finite Networks
  5. Planar Graphs
  6. Euclidean Lattices and Entropy
  7. Expansion Profiles and Decay of Transition Probabilities
  8. Anchored Isoperimetric Profiles and Transience
  9. Anchored Expansion and Percolation

**Evaluation:** Five Oral Presentations (duration of 1 hr each) (100% = 5 * 20%)

**Course Schedule:** We will meet two times a week for the entirety of the semester.

**Class Policy:** Complete attendance is essential and expected.