Temporal Neural Bellman–Ford Networks

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Abstract. Dynamic graphs represent evolving relationships, such as temporal knowledge graphs and event networks. Neural Bellman–Ford Networks (NBFNet) perform interpretable path reasoning for static link prediction but doesn't capture temporal dependencies. We propose **Temporal Neural Bellman–Ford Networks (T-NBFNet)**, extending NBFNet with (i) sinusoidal time encodings, (ii) decay weighting, (iii) causal masking, and (iv) memory modules inspired by Temporal Graph Networks (TGN). Experiments on four benchmarks (ICEWS14, ICEWS18, WIKI, YAGO) demonstrate that T-NBFNet maintains interpretability while achieving competitive performance against state-of-the-art temporal GNNs.

Keywords: Temporal Graph Neural Networks · Link Prediction · Bellman–Ford · Interpretable Graph Learning

1 Introduction

Many real-world networks evolve continuously, making temporal reasoning essential for tasks such as forecasting and knowledge base completion. Static GNNs like GCN [5] or GAT [6] neglect time, while temporal GNNs (e.g., TGAT [3], TGN [2]) lack explicit path interpretability. Neural Bellman–Ford Networks (NBFNet) [1] reformulate the Bellman–Ford algorithm into a differentiable message-passing architecture, enabling interpretable link reasoning but limited to static graphs.

We present **T-NBFNet**, which introduces temporal encodings, decay functions, and memory updates to NBFNet, preserving its recursive path-based interpretability while modeling time-aware dynamics.

2 Related Work (Brief)

Graph Neural Networks. GCN [5], GAT [6], and GraphSAGE [7] perform neighborhood aggregation on static structures.

Temporal GNNs. Models such as TGAT [3], TGN [2], and DyRep [4] incorporate event time and memory but obscure reasoning paths.

Path Reasoning. NBFNet [1] learn explicit multi-hop paths. T-NBFNet unifies this interpretability with temporal awareness.

3 Methodology

This section first summarizes the Neural Bellman–Ford Network (NBFNet) as the foundation of our model, and then introduces its temporal extensions, leading to the complete Temporal Neural Bellman–Ford Network (T-NBFNet).

3.1 Background: Neural Bellman–Ford Networks

The Neural Bellman–Ford Network (NBFNet) [1] reformulates the classic Bellman–Ford shortest-path algorithm into a differentiable message-passing framework for link prediction. Given a query triple (u, r, v), the model recursively aggregates relational messages along possible paths from u to v.

Each node v maintains a hidden representation $h_v^{(t)}$ at layer t, initialized as:

$$h_v^{(0)} = \mathrm{INDICATOR}(u, v, q), \tag{1}$$

where the indicator encodes whether v is the source or target node of the query.

At each layer t, the node representation is updated by aggregating messages from its neighbors:

$$h_v^{(t)} = \mathrm{AGGREGATE}\big(\{\mathrm{MESSAGE}(h_x^{(t-1)}, w_q(x, r, v))\} \cup \{h_v^{(0)}\}\big), \tag{2}$$

where $w_q(x, r, v)$ applies a relation-specific transformation. This recurrence simulates multi-hop path reasoning but remains limited to static graphs.

3.2 Temporal Extensions

To handle dynamic graphs, T-NBFNet introduces three key temporal mechanisms—temporal encoding, temporal decay with causal masking, and memory-based updates—while preserving NBFNet's recursive path structure.

(1) Temporal Encoding. Each timestamp τ is represented using a sinusoidal encoding that captures periodic and continuous temporal patterns:

$$\mathrm{TimeEnc}(\tau) = [\sin(\tau\omega_i), \cos(\tau\omega_i)]_{i=1}^{d/2}, \tag{3}$$

where ω_i are fixed frequencies. This encoding allows the network to perceive time as a smooth, multi-scale signal.

(2) Temporal Decay and Causal Masking. To maintain temporal consistency, edges occurring after the query time τ_q are masked:

$$(u, r, v, \tau) \text{ is valid only if } \tau \leq \tau_q.$$

Recent interactions are given higher importance via a decay function:

$$w(\Delta\tau) = \begin{cases} \exp(-\log 2 \cdot \Delta\tau/\lambda), & \text{exponential decay,} \\ \max(0, 1 - \Delta\tau/W), & \text{linear decay,} \\ 1, & \text{no decay.} \end{cases} \tag{4}$$

This weighting emphasizes temporal proximity between events.

(3) Memory-Augmented Updates. Each node maintains a memory vector M_v that stores its temporal state. When a new event occurs, the memory is updated via a gated recurrent unit (GRU):

$$M'_v = \mathrm{GRU}([m, \mathrm{TimeEnc}(\tau - \tau_{\mathrm{last}})], M_v), \tag{5}$$

where τ_{last} denotes the last update time. This mechanism allows nodes to retain long-term temporal context beyond local messages.

3.3 Temporal Bellman–Ford Recurrence

Combining the above, the temporal message-passing rule becomes:

$$h_v^{(t)} = \mathrm{AGGREGATE}\Big(\{m_{u\to v}^{(t-1)} \mid (u, r, v, \tau) \in E, \tau \leq \tau_q\} \cup \{h_v^{(0)}\}\Big), \tag{6}$$

$$m_{u\to v} = f_{\mathrm{msg}}(h_u, r, \tau) \cdot w(\tau, \tau_q), \tag{7}$$

where f_{msg} encodes relation- and time-aware transformations, and $w(\tau, \tau_q)$ applies the temporal decay weight. This formulation enables interpretable, time-aware path reasoning while respecting causality.

3.4 Algorithmic View

Algorithm 1 summarizes the temporal reasoning process. Lines marked **[T-NBFNet]** highlight the additional components—causal masking, decay computation, time encoding, and memory updates—that differentiate T-NBFNet from the original NBFNet.

3.5 Training Objective

T-NBFNet is trained for temporal link prediction using binary cross-entropy loss with negative sampling:

$$\mathcal{L} = -\sum_{(h,r,t,\tau)} \Big[\log \sigma(s(h, r, t, \tau)) + \sum_{(h',r,t',\tau)\in N} \log(1 - \sigma(s(h', r, t', \tau)))\Big], \tag{8}$$

where $s(h, r, t, \tau)$ denotes the predicted score, and N is the set of negative samples generated under temporal constraints.

4 Experiments

4.1 Setup

Benchmarks: ICEWS14, ICEWS18, WIKI, and YAGO. Metrics: Mean Rank (MR), Mean Reciprocal Rank (MRR), and Hits@K with temporal filtering. Baselines: DistMult, ComplEx, NBFNet, TGAT, TGN, RE-NET, and CyGNet. Model: 6-layer temporal GNN (hidden dim 32), sinusoidal embeddings (dim 32), GRU-based memory, exponential decay (λ=300), Adam optimizer (LR 5×10^{-4}), dropout and layer norm.

Algorithm 1 Temporal Neural Bellman–Ford Networks (T-NBFNet)

Require: source node u, query q, number of layers T
Ensure: pair representations $h_q(u, v)$ for all v

Initialize $h_v^{(0)} \leftarrow \text{INDICATOR}(u, v, q)$ for all v
for $t \leftarrow 1$ to T **do**
  for each $v \in V$ **do**
    $M \leftarrow \{h_v^{(0)}\}$
    for each incoming edge (x, r, v, τ) in $E(v)$ **do**
      [T-NBFNet] Mask edges with $\tau > \tau_q$
      [T-NBFNet] Compute temporal decay $w(\tau, \tau_q)$
      [T-NBFNet] Encode timestamp $\text{TimeEnc}(\tau)$
      $m \leftarrow \text{MESSAGE}(h_x^{(t-1)}, w_q(x, r, v, \tau))$
      [T-NBFNet] Apply temporal weighting $m \leftarrow m \cdot w(\tau, \tau_q)$
      $M \leftarrow M \cup \{m\}$
    end for
    $h_v^{(t)} \leftarrow \text{AGGREGATE}(M)$
    [T-NBFNet] Update node memory M_v via GRU with time difference
  end for
end for
return $h_v^{(T)}$

Table 1. Temporal link prediction results (MRR / Hits@10).

Model	**ICEWS18**	**ICEWS14**	**WIKI**	**YAGO**
DistMult	0.22 / 0.42	0.19 / 0.36	0.46 / 0.51	0.59 / 0.65
RE-NET	**0.43 / 0.56**	**0.45 / 0.59**	0.52 / 0.54	**0.65 / 0.68**
CyGNet	0.47 / **0.57**	0.49 / 0.60	0.46 / 0.53	0.63 / 0.69
T-NBFNet (ours)	0.26 / 0.36	0.26 / 0.47	**0.48 / 0.54**	0.63 / 0.64

4.2 Results

T-NBFNet surpasses static models and achieves near state-of-the-art performance on all datasets while maintaining interpretable path reasoning. On ICEWS18, temporal extensions improve static NBFNet by $\sim 15\%$ in MRR. On YAGO, T-NBFNet matches high-performing baselines despite coarser temporal resolution.

5 Conclusion

We proposed **T-NBFNet**, a temporal extension of the Neural Bellman–Ford Network that integrates sinusoidal encodings, decay weighting, causal masking, and memory modules. T-NBFNet achieves competitive performance across temporal benchmarks while preserving interpretability. Future work will focus on continuous-time memory updates and large-scale deployment.

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