MSE:

$$L_i = \frac{1}{n} \sum_{i=1}^{n} (y_i - g_i)^2 \rightarrow loss\ function, for\ n = 2:$$

$$L = \frac{1}{2}(y_1 - g_1)^2 + \frac{1}{2}(y_2 - g_2)^2$$

Forward Pass:

$$y_i = a * hi + c$$

$$o_i = W_i x + W_h h_i + b$$

$$h_i = \sigma(o_i)$$

Known Parameters and Calculations for Forward Pass:

$$with\ x_0 = 1, x_1 = 0\ , g_1 = 1, g_2 = 1, h_0 = 0\ and$$

$$W_i = 1, W_h = 1, a = 1, b = 1, c = 1$$

$$o_1 = W_i x_0 + W_h h_0 + b = 1 + 0 + 1 = 2$$

$$h_1 = \sigma(o_1) = \sigma(2) = \frac{1}{1 + e^{-2}} = 0.880797$$

$$y_1 = a * h_1 + c = 1 * 0.880797 + 1 = 1.880797$$

$$o_2 = W_i x_1 + W_h h_1 + b = 0 + 1 * 0.880797 + 1 = 1.880797$$

$$h_2 = \sigma(o_2) = \sigma(1.880797) = \frac{1}{1 + e^{-1.880797}} = 0.867703$$

$$y_2 = a * h_2 + c = 1 * 0.867703 + 1 = 1.867703$$

Derivations:

$$\frac{dL_1}{dy_1} \rightarrow (y_1 - g_1) = 0.880797\ and\ \frac{dL_2}{dy_2} \rightarrow (y_2 - g_2) = 0.867703$$

$$\frac{dy_1}{dh_1} \rightarrow a = 1\ and\ \frac{dy_2}{dh_2} \rightarrow a = 1$$

$$\frac{dh_1}{do_1} \rightarrow h_1(1 - h_1) = 0.880797 * 0.119203 = 0.104994$$

$$\frac{dh_2}{do_2} \rightarrow h_2(1 - h_2) = 0.867703 * 0.132297 = 0.114795$$

$$\frac{do_2}{dh_1} = W_h = 1$$

$$\frac{do_1}{dW_i} \rightarrow x_0 = 1 \text{ and } \frac{do_2}{dW_i} \rightarrow x_1 + \frac{do_2}{dh_1} * \frac{dh_1}{do_1} * \frac{do_1}{dW_i} = 0 + 1 * 0.104994 * 1 = 0.104994$$

$$\frac{do_1}{dW_h} = h_0 = 0 \text{ and } \frac{do_2}{dW_h} = h_1 + \frac{do_2}{dh_1} * \frac{dh_1}{do_1} * \frac{do_1}{dW_h} = 0.880797 + 1 * 0.104994 * 0 = 0.880797$$

$$\frac{dy_1}{da} = h_1 = 0.880797 \text{ and } \frac{dy_2}{da} = h_2 = 0.867703$$

$$\frac{do_1}{db} = 1 \text{ and } \frac{do_2}{db} = 1 + \frac{do_2}{dh_1} * \frac{dh_1}{do_1} * \frac{do_1}{db} = 1 + 1 * 0.104994 * 1 = 1.104994$$

$$\frac{dy_1}{dc} = 1 \text{ and } \frac{dy_2}{dc} = 1$$

dWi:

$$dW_i \rightarrow \frac{dL_1}{dW_i} + \frac{dL_2}{dW_i} = \frac{dL_1}{dy_1} * \frac{dy_1}{dh_1} * \frac{dh_1}{do_1} * \frac{do_1}{dW_i} + \frac{dL_2}{dy_2} * \frac{dy_2}{dh_2} * \frac{dh_2}{do_2} * \frac{do_2}{dW_i} =$$

$$0.880797 * 1 * 0.104994 * 1 + 0.867703 * 1 * 0.114795 * 0.104994 = 0.102937$$

dWh:

$$dW_h \rightarrow \frac{dL_1}{dW_h} + \frac{dL_2}{dW_h} = \frac{dL_1}{dy_1} * \frac{dy_1}{dh_1} * \frac{dh_1}{do_1} * \frac{do_1}{dW_h} + \frac{dL_2}{dy_2} * \frac{dy_2}{dh_2} * \frac{dh_2}{do_2} * \frac{do_2}{dW_h} =$$

$$0.880797 * 1 * 0.104994 * 0 + 0.867703 * 1 * 0.114795 * 0.880797 = 0.087734$$

da:

$$da \rightarrow \frac{dL_1}{da} + \frac{dL_2}{da} = \frac{dL_1}{dy_1} * \frac{dy_1}{da} + \frac{dL_2}{dy_2} * \frac{dy_2}{da} =$$

$$0.880797 * 0.880797 + 0.867703 * 0.867703 = 1.528712$$

db:

$$db \rightarrow \frac{dL_1}{db} + \frac{dL_2}{db} = \frac{dL_1}{dy_1} * \frac{dy_1}{dh_1} * \frac{dh_1}{do_1} * \frac{do_1}{db} + \frac{dL_2}{dy_2} * \frac{dy_2}{dh_2} * \frac{dh_2}{do_2} * \frac{do_2}{db} =$$

$$0.880797 * 1 * 0.104994 * 1 + 0.867703 * 1 * 0.114795 * 1.104994 = 0.202545$$

dc:

$$dc \rightarrow \frac{dL_1}{dc} + \frac{dL_2}{dc} = \frac{dL_1}{dy_1} * \frac{dy_1}{dc} + \frac{dL_2}{dy_2} * \frac{dy_2}{dc} =$$

$$0.880797 * 1 + 0.867703 * 1 = 1.7485$$