

$$③ f(x) = \frac{1}{1+x^2}$$

$$x_1 = 0$$

$$x_2 = 1$$

$$x_3 = 3$$

$$y_1 = \frac{1}{1+0} = 1$$

$$y_2 = \frac{1}{1+1} = \frac{1}{2}$$

$$y_3 = \frac{1}{1+9} = \frac{1}{10}$$

$$④ l_i(x) = \prod_{\substack{j=1 \\ j \neq i}}^3 \frac{x - x_j}{x_i - x_j}$$

$$l_1(x) = \frac{x-1}{0-1} \cdot \frac{x-3}{0-3} = \frac{(x-1)(x-3)}{3}$$

$$l_2(x) = \frac{x-0}{1-0} \cdot \frac{x-3}{1-3} = \frac{x^2-3x}{-2}$$

$$l_3(x) = \frac{x-0}{3-0} \cdot \frac{x-1}{3-1} = \frac{x^2-x}{6}$$

$$P(x) = \sum_{i=1}^3 f(x_i) l_i(x)$$

$$= \frac{1}{1} \cdot \frac{x^2-3x-x+3}{3} + \frac{1}{2} \cdot \frac{x^2-3x}{-2} + \frac{1}{10} \cdot \frac{x^2-x}{6}$$

$$= \frac{1}{3}x^2 - \frac{1}{4}x^2 + \frac{1}{60}x^2 - \frac{4}{3}x + \frac{3}{4}x - \frac{1}{60}x + 1$$

$$= \frac{1}{10}x^2 - \frac{3}{5}x + 1$$

$$⑤ P_0(x) = y_0 = \frac{1}{1} = 1$$

$$P_1(x) = 1 + \frac{\frac{1}{2}-1}{1-0}(x-0) = 1 - \frac{1}{2}x$$

$$P_2(x) = 1 - \frac{1}{2}x + c_2(x-0)(x-1) = 1 - \frac{1}{2}x + c_2(x^2-x)$$

Let's set $P_2(3) = \frac{1}{10}$ and solve c_2

$$\frac{1}{10} = 1 - \frac{3}{2} + c_2(9-3)$$

$$c_2 = \frac{\frac{3}{2}-1}{6} = \frac{1}{10}$$

$$P_2(x) = 1 - \frac{1}{2}x + \frac{1}{10}x^2 - \frac{1}{10}x$$

$$= \frac{1}{10}x^2 - \frac{3}{5}x + 1$$

We can easily see both polynomials are equal