(3) $f(x) = \frac{1}{1+x^2}$

$x_1 = 0$ $\quad y_1 = \frac{1}{1+0} = 1$

$x_2 = 1$ $\quad y_2 = \frac{1}{1+1} = \frac{1}{2}$

$x_3 = 3$ $\quad y_3 = \frac{1}{1+9} = \frac{1}{10}$

(4) $l_i(x) = \prod_{\substack{j=1 \\ j\neq i}}^{3} \frac{x - x_j}{x_i - x_j}$

$l_1(x) = \frac{x-1}{0-1} \cdot \frac{x-3}{0-3} = \frac{(x-1)(x-3)}{3}$

$l_2(x) = \frac{x-0}{1-0} \cdot \frac{x-3}{1-3} = \frac{x^2-3x}{-2}$

$l_3(x) = \frac{x-0}{3-0} \cdot \frac{x-1}{3-1} = \frac{x^2-x}{6}$

$P(x) = \sum_{i=1}^{3} f(x_i)\, l_i(x)$

$= \frac{1}{1} \cdot \frac{x^2-3x-x+3}{3} + \frac{1}{2} \cdot \frac{x^2-3x}{-2} + \frac{1}{10} \cdot \frac{x^2-x}{6}$

$= \frac{1}{3}x^2 - \frac{1}{4}x^2 + \frac{1}{60}x^2 - \frac{4}{3}x + \frac{3}{4}x - \frac{1}{60}x + 1$

$= \frac{1}{10}x^2 - \frac{3}{5}x + 1$

(5) $P_0(x) = y_0 = \frac{1}{1} = 1$

$P_1(x) = 1 + \frac{\frac{1}{2}-1}{1-0}(x-0) = 1 - \frac{1}{2}x$

$P_2(x) = 1 - \frac{1}{2}x + c_2(x-0)(x-1) = 1 - \frac{1}{2}x + c_2(x^2-x)$

Let's set $P_2(3) = \frac{1}{10}$ and solve c_2

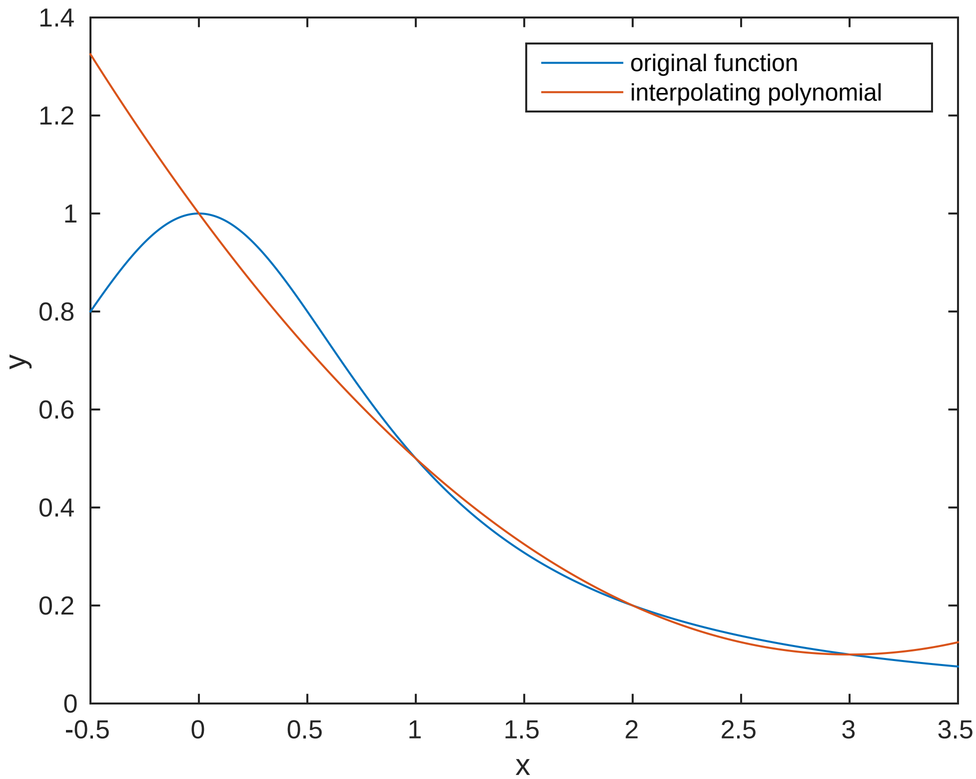
$\frac{1}{10} = 1 - \frac{3}{2} + c_2(9-3)$

$c_2 = \frac{3}{5 \cdot 6} = \frac{1}{10}$

$P_2(x) = 1 - \frac{1}{2}x + \frac{1}{10}x^2 - \frac{1}{10}x$

$= \frac{1}{10}x^2 - \frac{3}{5}x + 1$

We can easily see both polynomials are equal


original function
interpolating polynomial
1.4
1.2
1
0.8
0.6
0.4
0.2
0
y
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
x