(4) Let's consider finding a polynomial $P_N(x) = \sum_{i=0}^{N} c_i e^{ix} = \sum_{i=0} c_i (e^x)^i$ which goes through points $(x_1, y_1), (x_2, y_2), \ldots$ and (x_N, y_N). It is clear that if we performing change of variables $t = e^x \Rightarrow x = \ln t$. Now

$P_N(t) = \sum_{i=0}^{N} c_i t^i$ and points the polynomial goes through are $(\ln t_1, y_1), (\ln t_2, y_2), (\ln t_3, y_3), \ldots$ and $(\ln t_N, y_N)$. We can recognize $P_N(t) = \sum c_i t^i$ as interpolating polynomial for given points $(\ln t_i, y_i)$. It is proven in lecture notes (interpolation, page 5→) that there is a solution for these c_i and that solution is unique. □