1. To find the first root I ran bisect with limits 0 and 0.5 which converged to 0.3639226269 after 32 iterations and newton with initial guess 0.25 which converged to same value after 4 iterations. For second root I used limits 0.5 and 0.7 with bisect to find value 0.5718029815 after 33 iterations. Newton's method converged to same value after 4 iterations when given initial guess 0.6.

In general Newton's method is faster but it's efficiency is also somewhat more difficult to predict. Bisection method's use of binary search ensures logarithmic time complexity and it will always find a root in given range. Newton's method on the other hand requires a good initial guess and for some functions the found root might be quite far from the closest root.

2. Results with different values of B using reasonable initial guesses are shown below.

x0	B	iterations	result
-0.7	0.1	3	-0.7164258891
-0.5	1	3	-0.5884017765
-0.3	10	3	-0.3264020101
-0.1	100	5	-0.1398945646

When x_0=1 first two values of B produce results after 4 and 5 iterations but with greater values of B iteration gets stuck in a loop where we never get an answer. This is easy to understand when looking at plots of g(10) and g(100). Next value of x we get from x=0 is quite far from zero but there g(x) is almost linear which in turn results in next x value being very close to zero.

4. My function gives results that are compatible with ones from WolframAlpha. For example when given array [1, -6, 10, 3] corresponding to polynomial $3x^3+10x^2-6x+1$ my function outputs these three solutions:

-3.872084 + 0.000000*i
0.269375 + 0.116290*i
0.269375 + -0.116290*i

Makefile can be found in tar archive.

(3) $\begin{cases} x_{i+1} = \mu x_i(1-x_i) \\ x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)} \end{cases}$

$$\mu x_i - \mu x_i^2 = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\frac{f(x_i)}{\frac{df(x_i)}{dx_i}} = \mu x_i^2 + x_i(1-\mu)$$

$$\frac{f(x_i)}{df(x_i)} = (\mu x_i^2 + x_i(1-\mu))\frac{1}{dx}$$

$$\frac{1}{f(x_i)}\,df(x_i) = \frac{1}{\mu x_i^2 + x_i(1-\mu)}\,dx \quad \Big| \int$$

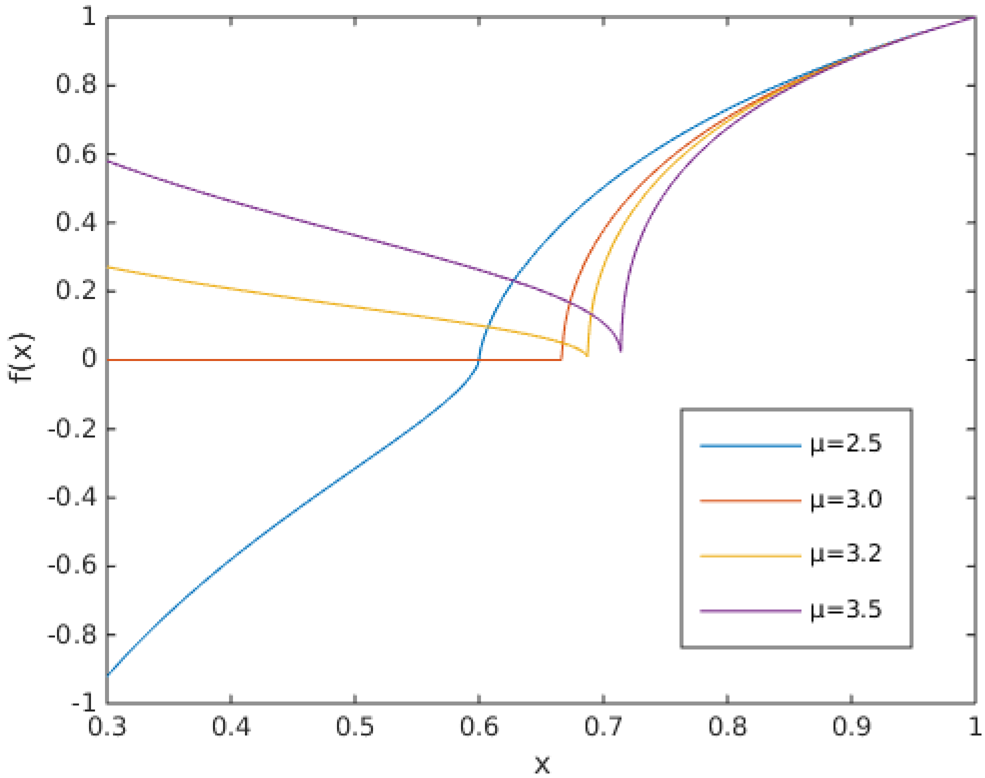
$$\ln|f(x_i)| = \frac{\ln(\mu x_i - \mu + 1) - \ln(x_i)}{\mu - 1} + c_1 \quad \Big| e^x$$

$$|f(x_i)| = c_2 e^{\frac{\ln(\mu x_i - \mu + 1) - \ln(x_i)}{\mu - 1}}$$

When applying Newton's method we are interested in zeros and x-axis intercepts of slopes, neither of which change when taking the absolute value, so we can say

$$\underline{\underline{f(x_i) = e^{\frac{\ln(\mu x - \mu + 1) - \ln(x_i)}{\mu - 1}}}}$$

3.



Bifurcation in the original plot can be explained by Newton's method not finding a single value but instead jumping between two or more values. Blue curve here is nice and smooth giving good convergence but for yellow curve, landing too far right from f(x)=0 causes the next value of x be too far left which bring value after that too far right and so on. On violet curve (μ=3.5) this happens between four points. This means there is not a single value x_N to which the approximation would converge but instead it jumps between two or more values which can be seen as bifurcations in the given plot.