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(1) Multiplication:

$$(x \pm e_x)(y \pm e_y) = xy \pm xe_y \pm e_x y \mp \underbrace{e_x e_y}_{\approx 0} = xy \pm xy r_y \pm xy r_x$$

$$\Rightarrow r_{xy} = \frac{xy - (x \pm e_x)(y \pm e_y)}{xy} = \frac{\cancel{xy} - \cancel{xy} \mp xyr_y \mp xyr_x}{\cancel{xy}} = \mp r_x \mp r_y$$ (sum of errors)

Division:

$$r_{x/y} = \frac{\frac{x}{y} - \frac{x \pm e_x}{y \pm e_y}}{\frac{x}{y}} = \frac{\frac{\cancel{xy} \pm xe_y - \cancel{xy} \mp ye_x}{y(y \pm e_y)}}{\frac{x}{\cancel{y}}} = \frac{\pm xe_y \mp ye_x}{xy \underbrace{\pm xe_y}_{\approx 0}} = \pm \frac{e_y}{y} \mp \frac{e_x}{x} = \pm r_y \mp r_x$$

(small e_y compared to y) (difference of errors)

(3) Let's consider an original binary number B and it's complement C. What we want to prove is that $C + 1 + B = 0$. Because C has been obtained by complementing B, it is clear* that in all positions where B has 0 C has 1 and vice versa. Therefore sum $B + C$ contains only ones. Because summation is commutative $C + 1 + B = (B + C) + 1$ and therefore we can just add 1 to the previously obtained sum $B + C = \underbrace{111\ldots111}_{n \text{ ones}}$ thus getting $B + C + 1 = 1\underbrace{000\ldots000}_{n \text{ zeros}}$

Due to the nature of computers, leading 1 overflows and thus it won't affect the result and we are left with $\underbrace{000\ldots000}_{n \text{ zeros}}$. Thus $C + 1 + B = 0$ i.e. complementing all bits of a number, adding one and adding the original number produces zero. □

*Think about columnar addition: in each column there is exactly one "1" so the result contains only ones

(2) B) Calculating value of harmonic series normally gives a value of approximately 15,40 whereas using harmonic_bunch gives following values:

N	value
50	18,6
100	19,3
500	20,9

This makes sense because even if a single term 1/n is too small compared to the already calculated part of the sum to affect the value of the float, it is very much possible that summing N "too small" floats together results in a float that is big enough to affect the sum.

(4) (B) $e^x \approx 1 + \sum_{i=1}^{n-1} \frac{x^i}{i!} \cdot 1 + R(n)$

$$\approx 1 + \sum_{i=1}^{n-1} \frac{x^i}{i!} + \frac{x^n}{n!} e^{\xi}$$

Near value $x=0$ there's a spike (top row plots)

(C) Using the approximation above we can make f_1 and f_2 easier to compute near zero:

$$f_1 = \frac{e^x - 1}{x}$$

$$\approx \frac{\cancel{1} + \sum_{i=1}^{n-1} \frac{x^i}{i!} - \cancel{1}}{x}$$

$$\approx \sum_{i=1}^{n-1} \frac{x^{i-1}}{i!}$$

$$f_2 = \frac{e^x - e^{-x}}{2x}$$

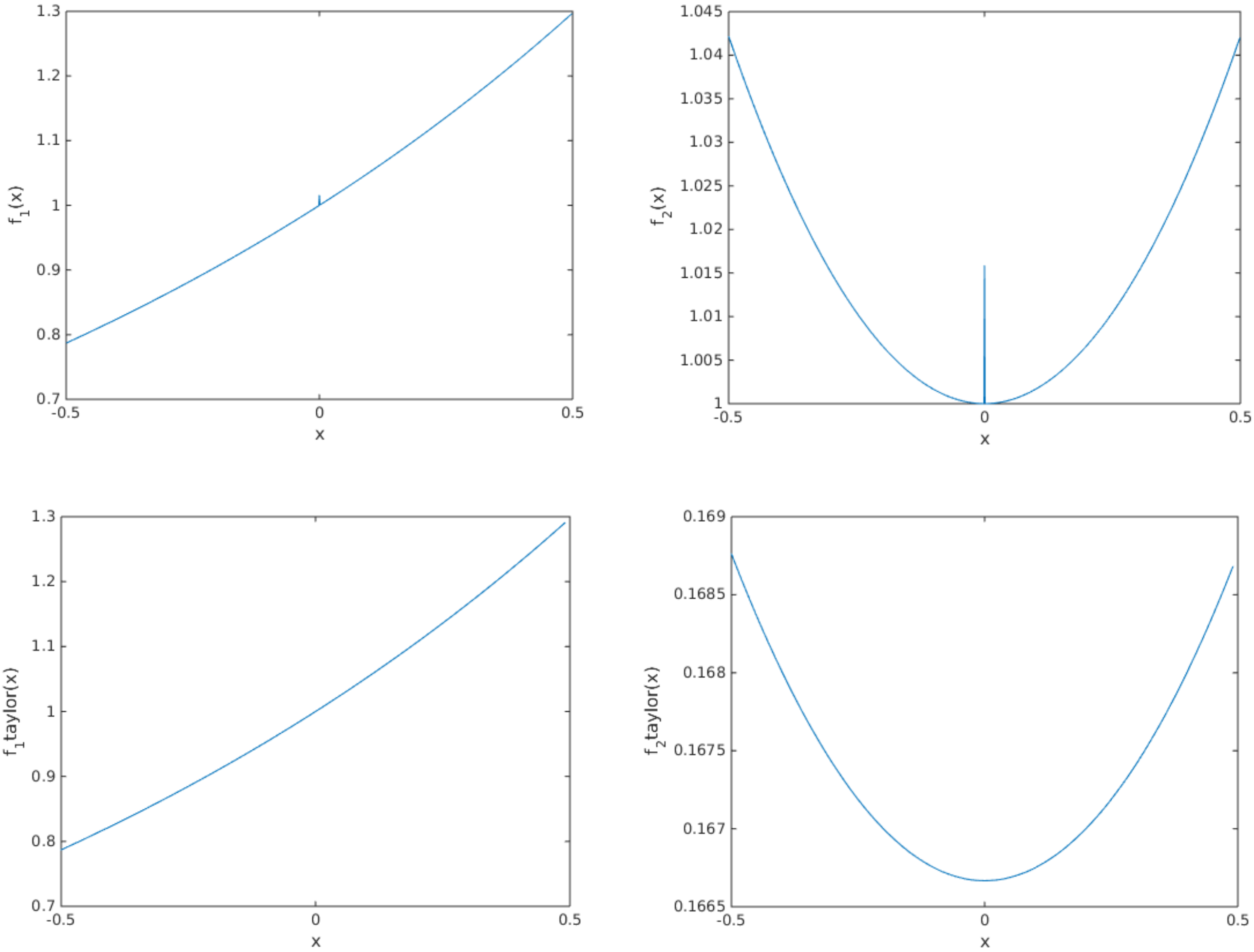
$$\approx \frac{\cancel{1} + \sum_{i=1}^{n-1} \frac{x^i}{i!} - \cancel{1} - \sum_{i=1}^{n-1} \frac{(-x)^i}{i!}}{2x}$$

Even terms cancel each other out and we are left with

$$f_2 \approx \frac{2 \sum_{i=0}^{n-1} \frac{x^{2i+1}}{(2i+1)!}}{2x}$$

$$\approx \sum_{i=0}^{n-1} \frac{x^{2i}}{(2i+1)!}$$

Using these approximations we can see that spikes present in first plots have disappeared.


f1(x)
x
f2(x)
x
f1taylor(x)
x
f2taylor(x)
x