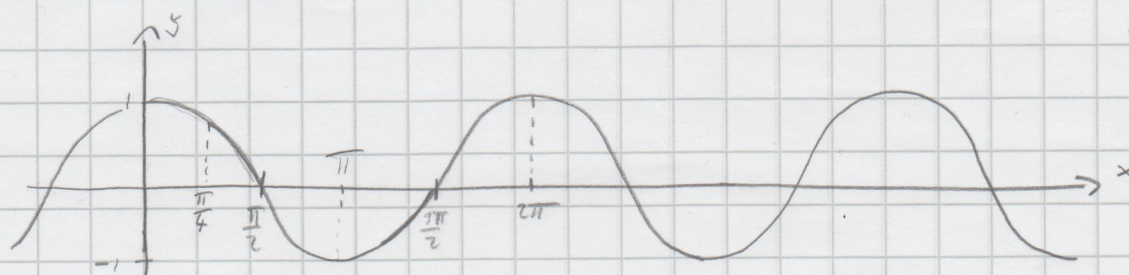
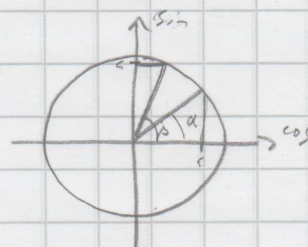


2 6



First we can "move" all x values to range $[0, 2\pi]$ by setting $x' = |x - \lfloor \frac{x}{2\pi} \rfloor \cdot 2\pi|$



Now it is fairly easy to "mirror" the function to be defined in four parts:

$$\alpha = \frac{\pi}{2} - \beta$$

$$\sin \alpha = \cos \beta$$

$$\cos(x') = \begin{cases} \cos(x') & , 0 \leq x' \leq \frac{\pi}{2} \\ -\cos(-(x' - \pi)) & , \frac{\pi}{2} \leq x' < \pi \\ -\cos(x' - \pi) & , \pi \leq x' < \frac{3\pi}{2} \\ \cos(-(x' - 2\pi)) & , \frac{3\pi}{2} \leq x' \leq 2\pi \end{cases}$$

$$= \begin{cases} \cos(x') \\ -\cos(\pi - x') \\ -\cos(x' - \pi) \\ \cos(2\pi - x') \end{cases}$$



Now we have reduced $\cos x$ to $[0, \frac{\pi}{2}]$. Now we have to reduce it further to $[0, \frac{\pi}{4}]$. Let's use the knowledge $\sin(\frac{\pi}{2} - x) = \cos x$ when $x \in [\frac{\pi}{4}, \frac{\pi}{2}]$ and $\sin x = \sqrt{1 - \cos^2 x}$. Therefore when $x' \in [0, \frac{\pi}{4}]$ $\cos(x') = \cos(x')$ when $x' \in [\frac{\pi}{4}, \frac{\pi}{2}]$ $\cos(x') = \sqrt{1 - \cos^2(\frac{\pi}{2} - x')}$

Now combining this with earlier form:

$$\cos(x') = \begin{cases} \cos(x') & , 0 \leq x' < \frac{\pi}{4} \\ \sqrt{1 - \cos^2(\frac{\pi}{2} - x')} & , \frac{\pi}{4} \leq x' < \frac{\pi}{2} \\ -\sqrt{1 - \cos^2(x' - \frac{\pi}{2})} & , \frac{\pi}{2} \leq x' < \frac{3\pi}{4} \\ -\cos(\pi - x') & , \frac{3\pi}{4} \leq x' < \pi \\ -\cos(x' - \pi) & , \pi \leq x' < \frac{5\pi}{4} \\ -\sqrt{1 - \cos^2(\frac{3\pi}{2} - x')} & , \frac{5\pi}{4} \leq x' < \frac{3\pi}{2} \\ \sqrt{1 - \cos^2(x' - \frac{3\pi}{2})} & , \frac{3\pi}{2} \leq x' < \frac{7\pi}{4} \\ \cos(2\pi - x') & , \frac{7\pi}{4} \leq x' < 2\pi \end{cases} \quad \text{where } x' = |x - \lfloor \frac{x}{2\pi} \rfloor \cdot 2\pi|$$