# Scientific computing III 2017

*Exercise 6*
*Return by Wednesday 25.2.2017 23:00 to Moodle*
*Exercise session: Friday 28.3.2017*

**Problem 1.** *(pen and paper)* (6 points)
The recursion relation of the Richardson extrapolation is

$$D(n,m+1)=\frac{4^m}{4^m-1}D(n,m)-\frac{1}{4^m-1}D(n-1,m)\,,$$

where

$$D(n,m)=L+\sum_{k=m}^{\infty}A(k,m)\left(\frac{h}{2^n}\right)^{2k}$$

and neither $L$ nor $A(k,m)$ depend on $h$. Prove that $D(n,m+1)$ is a better approximation to $L$ than $D(n,m)$ by $O(h^2)$, when $h\ll 1$.

**Problem 2.** *(computer)* (6 points)

A) Write a program function named `re_der(x)` which calculates the derivative of the mathematical function $f(x)=\sin(e^x)$ at a given point x using Richardson extrapolation (RE). For RE parameters use $N=5$, $h=0.1$. Put your function in a file named "re_der".
B) Calculate the error of the value of the derivative at $x$=1 and compare it with the error when the derivative is calculated using the central difference form $f'(x)\approx\frac{f(x+h)-f(x-h)}{2h}$, using various step sizes $h=10^{-n}$, with $n=4,6,8,10,12,14$.

**Problem 3.** *(computer)* (6 points)
Plot the cubic spline interpolants of random data[1] with 5 and 20 points. For each set of nodes use three different 'boundary conditions':

$$S''(x_0)=S''(x_N)=\begin{cases}0\\1\\10\end{cases}.$$

In your plots the original random data points must be clearly visible in markers and the interpolating spline in three different style lines for the three different boundary conditions. Give a brief explanation of the method and tools you used (no need to return any code).

1 E.g. `gawk 'BEGIN {srand(); N=10; for (i=0;i<N;i++) print i,rand()}'`

**Problem 4.** *(pen and paper - computer)* (6 points)
B-splines of degree $k$ (and order $k+1$ ) for nodes $t_i, i=0,\ldots,n$ are defined recursively as

$$N_i^0(t)=\begin{cases}1, & t\in[t_i, t_{i+1})\\ 0, & \text{otherwise}\end{cases}, \quad i=0,\ldots,n-1$$

$$N_i^k(t)=\left(\frac{t-t_i}{t_{i+k}-t_i}\right)N_i^{k-1}(t)+\left(\frac{t_{i+k+1}-t}{t_{i+k+1}-t_{i+1}}\right)N_{i+1}^{k-1}(t), \quad i=0,\ldots,n-k-1$$

$N_i^k(t)\neq 0$ , only when $t\in[t_i, t_{i+k+1})$ .

Write down the B-splines for $k=1,2,3$ and for the minimum possible number of nodes *n*. You can use a symbolic math programs (Maxima or the like) to assist your work if you want to. Plot the spline functions in the same diagram. You may assume that the distance between the nodes is one and starting from zero; i.e. $t_i=i$ .