(2) (b) $x = \frac{t}{1-t} \Rightarrow x - xt = t \Rightarrow t = \frac{x}{1+x}$

$$\frac{dx}{dt} = \frac{(1-t)+t}{(1-t)^2} = \frac{1}{(1-t)^2}$$

$x = 0 \Rightarrow t = 0$

$x \to \infty \Rightarrow t \to 1$

$$\Rightarrow \int_0^\infty e^{-x} \cos^2(x)\, dx$$

$$= \int_0^1 e^{\frac{t}{t-1}} \cos^2\left(\frac{t}{1-t}\right) \frac{1}{(1-t)^2}\, dt$$

$x^{1/2}$

(3) (a) $\int_0^1 \sin(\sqrt{x})\, dx = \int_0^1 \sin\sqrt{x} - \sqrt{x} + \sqrt{x}\, dx$

$$= \int_0^1 \sin\sqrt{x} - \sqrt{x}\, dx + \int_0^1 \sqrt{x}\, dx$$

$$= \int_0^1 \sin\sqrt{x} - \sqrt{x}\, dx + \Big|_0^1 \frac{2}{3} x^{3/2}$$

$$= \int_0^1 \sin\sqrt{x} - \sqrt{x}\, dx + \frac{2}{3} \quad \square$$

(4) $R(1,1) = R(1,0) + \frac{R(1,0) - R(0,0)}{4-1}$

$$= \frac{1}{2}R(0,0) + hf(a + (2-1)h) + \frac{\frac{1}{2}R(0,0) + hf(a+(2-1)h) - R(0,0)}{3}$$

$$= \frac{1}{2}R(0,0) + hf(a+h) - \frac{\frac{1}{2}R(0,0)}{3} + \frac{hf(a+h)}{3}$$

$$= \frac{1}{3}R(0,0) + \frac{4hf(a+h)}{3}$$

$$= \frac{h(f(a)+f(b))}{3} + \frac{4hf(a+h)}{3}$$

$$= \frac{2hf(a) + 2hf(b) + 8hf(a+h)}{6}$$

$$= \frac{h}{6} \cdot \left(2f(a) + 8f(a+h) + 2f(b)\right) = \frac{a+b}{6}\left(f(a) + 4f(a+b) + f(b)\right) \quad \square$$