A Toy Model Indicating Inverse-Square-Law Emergence from Quantum Entanglement

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Abstract

We present a simplified spin-chain toy model demonstrating that a force with an inverse-square radial dependence can, in principle, emerge from the decay of mutual information between separated quantum subsystems. Although this caricature does not constitute a full theory of gravity, it illustrates why entanglement-based approaches to emergent gravity are not immediately ruled out.

1 Introduction

Proposals that spacetime and gravity could arise from the underlying entanglement structure of quantum states have attracted significant interest in recent years. In particular, certain holographic frameworks have shown promising hints that geometry can be encoded in patterns of quantum correlations. However, the leap from these ideas to reproducing the familiar $1/r^2$ gravitational potential is far from trivial.

In this note, we outline a minimal "toy model" scenario in which the mutual information between two subsystems decays in such a way that one can define an "effective potential" scaling as $1/r^\alpha$. Under the particular condition $\alpha = 1$ (in a one-dimensional setup), the associated "force" can be made to look like $1/r^2$. Although heavily idealized, this suggests that entanglement-driven interactions can *mimic* inverse-square forces, at least in certain carefully tuned quantum states.

2 Mutual Information as an Effective Potential

2.1 Setup

Consider a one-dimensional chain of N qubits, labeled by sites $x = 1, 2, \ldots, N$. Let A and B denote two disjoint blocks or "subsystems" of qubits, with the distance between the centers of A and B denoted by r. Define the *mutual information* between A and B as

$$I(A : B) \;=\; S(A) + S(B) - S(A \cup B), \tag{1}$$

where $S(\cdot)$ is the von Neumann entropy.

We posit a simple ansatz for the distance dependence of $I(A:B)$, namely

$$I(A:B) \;\sim\; \frac{c}{r^{\alpha}}, \tag{2}$$

for some constant $c > 0$ and exponent $\alpha > 0$. This power-law behavior might arise, for example, in a gapless or critical spin system.

2.2 Defining an Effective Force

We define an "entanglement potential" $V_{\text{ent}}(r)$ as

$$V_{\text{ent}}(r) \;=\; -\,k\,I(A:B), \tag{3}$$

where $k > 0$ is a constant that sets the overall strength. The resulting effective force $F(r)$ is then given by

$$F(r) \;=\; -\,\frac{dV_{\text{ent}}}{dr} \;=\; k\,\frac{d}{dr}\Big[I(A:B)\Big]. \tag{4}$$

Inserting the ansatz of Eq. (2) leads to

$$I(A:B) \;=\; \frac{c}{r^{\alpha}} \quad\Longrightarrow\quad F(r) \;=\; k\,c\,\alpha\,\frac{1}{r^{\alpha+1}}. \tag{5}$$

Thus, if $\alpha + 1 = 2$, i.e. $\alpha = 1$, then

$$F(r) \;\sim\; \frac{1}{r^{2}}, \tag{6}$$

which reproduces the familiar inverse-square dependence of Newtonian gravity in this highly stylized scenario.

3 Interpretation and Caveats

Although this toy calculation shows how an entanglement-based "force" can exhibit an inverse-square law, one should note several important caveats:

1. **Dimensionality and Tuning.** Achieving $\alpha = 1$ in a genuine spin system typically requires special critical conditions. Moreover, this derivation is in one spatial dimension; real gravitational interactions in 3+1 dimensions require more elaborate constructions.

2. **Emergent Gravity vs. Force Analogy.** Demonstrating an inverse-square force *alone* does not suffice to show that spacetime curvature, geodesic motion, or other hallmarks of gravity truly emerge.

3. **Microscopic Realizations.** Detailed Hamiltonians that yield the exact $I(A:B) \sim 1/r$ fall outside the scope of this short note, but one might look toward certain gapless systems or carefully designed tensor network states to realize (or approximate) such scaling.

4. **Deeper Dynamical Constraints.** A fully fleshed-out model must ensure unitarity, consistency with quantum field theory, and a mechanism for incorporating matter fields and local Lorentz invariance.

4 Discussion and Outlook

Despite these restrictions, the simple analysis above indicates that the notion of an "entanglement-induced" force replicating the $1/r^2$ scaling is not obviously inconsistent. Indeed, it illustrates why researchers exploring emergent gravity from quantum information continue to develop more sophisticated models (e.g. using holography or tensor networks) in hopes of recovering richer gravitational phenomena.

In a more ambitious framework, one would systematically show how Einstein-like equations or their generalizations emerge when one treats "entanglement geometry" in a large-N, continuum, or holographic limit. The toy example here serves mainly as a sanity check that an *inverse-square law from entanglement* is conceptually plausible at a basic level.

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