A Toy Model in Three Dimensions Indicating Inverse-Square-Law Emergence from Quantum Entanglement

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Abstract

We present a simplified extension of a one-dimensional toy model to three dimensions, demonstrating that an effective force with an inverse-square radial dependence may, in principle, emerge from the decay of mutual information between spatially separated quantum subsystems. This work serves as a sanity check to complement previous 1D investigations, highlighting key challenges and potential avenues toward a more realistic emergent-gravity framework.

1 Introduction

Recent theoretical work has explored the idea that spacetime and gravitational dynamics may emerge from the entanglement properties of an underlying quantum state. In one spatial dimension, a toy model has shown that a power-law decay of mutual information between two subsystems can be associated with an effective force obeying an inverse-square law. This note extends that idea into three dimensions, providing a higher-dimensional generalization and exploring the conditions under which similar inverse-square-law behavior emerges.

2 Three-Dimensional Setup

Consider a three-dimensional cubic lattice composed of qubits. Each lattice site is labeled by a coordinate vector

$$\mathbf{x} = (x_1, x_2, x_3),$$

with $x_i \in \{1, 2, \ldots, L\}$ for a lattice of side length L. We assume either open or periodic boundary conditions.

Define two disjoint subsystems, A and B, each consisting of a contiguous block (or cluster) of qubits. Let $\mathbf{r}_A$ and $\mathbf{r}_B$ denote the centers of mass of the subsystems, and define the separation

$$r = \|\mathbf{r}_A - \mathbf{r}_B\|.$$

The quantum correlations between A and B are quantified by the mutual information,

$$I(A:B) = S(A) + S(B) - S(A \cup B),$$

where $S(X)$ is the von Neumann entropy of subsystem X.

3 Power-Law Decay of Mutual Information

We postulate that the mutual information decays with the distance r according to a power law:

$$I(A:B) \sim \frac{c}{r^{\alpha}},$$

where $c > 0$ is a constant and $\alpha > 0$ characterizes the decay. In many three-dimensional systems—especially those at criticality or in gapless phases—power-law decays of correlation functions (and thus entanglement measures) are commonplace.

4 Entanglement Potential and Effective Force

We define an *entanglement potential* as

$$V_{\text{ent}}(r) = -kI(A:B),$$

with $k > 0$ setting the overall coupling strength. Following classical mechanics, we define an effective force as the negative radial derivative of the potential:

$$F(r) = -\frac{dV_{\text{ent}}}{dr} = k\frac{d}{dr}I(A:B).$$

Substituting our power-law ansatz,

$$I(A:B) \sim \frac{c}{r^{\alpha}},$$

we compute

$$F(r) = k\frac{d}{dr}\left(\frac{c}{r^{\alpha}}\right) = -k\,\alpha\,c\,\frac{1}{r^{\alpha+1}}.$$

5 Recovering the Inverse-Square Law

To mimic the Newtonian gravitational force, which scales as $1/r^2$, we require that

$$\frac{1}{r^{\alpha+1}} \propto \frac{1}{r^2}.$$

This condition implies:

$$\alpha + 1 = 2 \quad \Longrightarrow \quad \alpha = 1.$$

Thus, if the mutual information decays as $I(A:B) \sim 1/r$, the effective force becomes

$$F(r) \sim \frac{1}{r^2},$$

which is the familiar inverse-square law observed in Newtonian gravity.

6 Discussion and Outlook

This three-dimensional extension represents a straightforward, yet important, step toward more realistic models of emergent gravity:

- **Exponent Tuning:** In a true 3D quantum many-body system, engineering a precise $1/r$ decay may require fine-tuning or may naturally occur only under special critical conditions.
- **Static versus Dynamic Models:** Our analysis is entirely static. For a complete gravitational theory, one must consider time-dependent entanglement correlations and their impact on emergent spacetime dynamics.
- **Towards Continuum Theories:** Ultimately, it is desirable to derive a continuum description linking entanglement dynamics with an effective field theory that includes gravitational degrees of freedom.

While this model does not provide a complete theory of gravity, it offers a proof-of-principle that quantum entanglement can encode an effective inverse-square interaction—thereby complementing earlier 1D studies and guiding future research in emergent gravity scenarios.

7 Conclusion

We have extended a toy model based on the power-law decay of mutual information from one to three dimensions. By positing that $I(A:B) \sim 1/r$ for well-chosen subsystems in a 3D lattice, the resulting entanglement potential yields an effective force that scales as $1/r^2$. Although highly idealized, this extension reinforces the possibility that gravity-like interactions may emerge from the underlying quantum entanglement structure, and it serves as a stepping stone for more comprehensive explorations connecting quantum information theory with the geometry of spacetime.

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