An Effective Field Equation for Emergent Gravity from Quantum Entanglement

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HC-005-DOC

April 14, 2025

Abstract

We propose an effective gravitational field equation derived from the structure of quantum entanglement entropy in quantum many-body systems. By positing that curvature emerges from second derivatives of local entanglement entropy, we establish a novel connection between entropic quantum information structures and classical gravitational behavior. Our equation generalizes Einstein's field equations, replacing energy-momentum sources with entanglement curvature tensors. We also show how this framework connects with Verlinde's entropic gravity, and we demonstrate how informational wells may give rise to effective gravitational potentials.

1 Introduction

Emergent gravity theories suggest spacetime and gravity may not be fundamental but instead arise from underlying quantum informational processes, such as entanglement. Recent research has indicated that quantum entanglement entropy gradients might effectively mimic gravitational curvature, offering novel insights into gravitational phenomena without invoking dark matter or modifications to classical relativity.

2 Entanglement Curvature Tensor

Consider a quantum many-body system represented by a reduced density matrix $\rho_{\Omega(x)}$, defined within a spatial region around point x. The local von Neumann entropy field $S(x)$ is:

$$S(x) = -\text{Tr}[\rho_{\Omega(x)} \log \rho_{\Omega(x)}]. \tag{1}$$

We define the entanglement curvature tensor $E_{\mu\nu}$ as the covariant second derivative of this scalar entropy field:

$$E_{\mu\nu}(x) = \nabla_\mu \nabla_\nu S(x). \tag{2}$$

3   Effective Field Equation

Motivated by Einstein's equations of general relativity, we propose the following effective field equation for emergent gravity:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa_E \left(\nabla_\mu \nabla_\nu S(x) - g_{\mu\nu} \Box S(x) \right) + \Lambda_E g_{\mu\nu}, \tag{3}$$

where κ_E is the entanglement-gravitational coupling constant, and Λ_E is a cosmological-like constant arising naturally from the entropy background.

This field equation suggests that gravitational dynamics are sourced by entanglement entropy gradients, directly linking quantum information structure to spacetime curvature.

4   Connection to Verlinde's Entropic Gravity

Verlinde proposed that gravity emerges as an entropic force, arising from changes in entropy associated with the position of a test mass near an information-theoretic screen. In his framework, the entropic force is given by:

$$F = T \frac{\Delta S}{\Delta x}, \tag{4}$$

where T is the temperature associated with the screen, and ΔS is the entropy change due to displacement.

This idea aligns conceptually with our framework: gravity is not fundamental but a manifestation of entropy gradients. While Verlinde used a thermodynamic argument, our approach formalizes these gradients in terms of differential geometry:

$$F(r) \propto -\frac{d}{dr} S(r), \tag{5}$$

which, when combined with a mutual information decay $S(r) \sim \log I(A:B) \sim -1/r$, yields an inverse-square law:

$$F(r) \sim \frac{1}{r^2}. \tag{6}$$

Thus, our entanglement field equation may be viewed as a geometric generalization of Verlinde's entropic force concept.

5   Informational Gravity Wells

In the weak-field, static limit, let us consider a spherically symmetric entropy field $S(r)$. The Laplacian becomes:

$$\nabla^2 S(r) = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dS}{dr} \right). \tag{7}$$

Let the gravitational potential $\Phi(r)$ obey:

$$\nabla^2 \Phi(r) = \kappa_E \nabla^2 S(r) \quad \Rightarrow \quad \Phi(r) = \kappa_E S(r) + C, \tag{8}$$

where C is an integration constant. Thus, local minima of the entropy field $S(r)$ act as attractive wells for test particles, consistent with gravitational attraction. These informational wells generate potential gradients in the same way mass distributions would in Newtonian gravity.

6 Physical Interpretations

6.1 Galactic Rotation Curves

If gravitational attraction arises from long-range entanglement correlations, galaxy rotation curves could naturally flatten without additional dark matter, as entanglement entropy gradients may decay more slowly than classical mass density.

6.2 Gravitational Lensing and Microlensing

Regions of high entanglement curvature can deflect geodesics analogously to classical gravitational lenses, reproducing observed microlensing events without introducing massive compact halo objects (MACHOs).

6.3 Galaxy Cluster Collisions and Dark Matter

The discrepancy between visible mass and gravitational lensing, as in the Bullet Cluster, may reflect the persistence and non-locality of entanglement networks rather than invisible particle-based dark matter.

7 Experimental and Numerical Probes

The validity of this equation can be examined by:

- Analyzing spatial entanglement entropy fields in quantum simulations,

- Comparing theoretical entanglement curvature fields to observed gravitational fields,

- Searching for entanglement-induced deviations from classical gravitational predictions in astrophysical observations.

8 Conclusions

We introduced a novel effective gravitational field equation grounded in quantum entanglement entropy. Our framework provides a conceptual shift from energy-based gravitational sources to informational ones. By connecting with Verlinde's entropic gravity and deriving informational gravity wells in the weak-field limit, we establish a pathway for reconciling geometric gravity with quantum informational foundations. Future numerical and observational work will clarify whether gravitational phenomena, traditionally attributed to dark matter or modifications of gravity, might instead be emergent effects of entanglement.

Acknowledgments

References

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