A Model C Framework for Membrane-Actin Systems

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1| Theory

1.1 Model formulation

The grand free energy of the membrane-actin system is given by

$$F(\phi, \mathbf{Q}) = \int d\mathbf{r} \Big\{ f_0(\phi) + f_1(\phi, \mathbf{Q}) + \alpha |\mathbf{Q}\cdot\nabla\phi|^2 + \frac{1}{2}\left[\kappa|\nabla\phi|^2 + L_1(\partial_i Q_{jk})(\partial_i Q_{jk}) + L_2(\partial_i Q_{ik})(\partial_j Q_{jk})\right] \Big\} , \tag{1}$$

where the bulk free energy ϕ is the usual double-well potential,

$$f_0(\phi) = \frac{\phi^4}{4} - \frac{\phi^2}{2} , \tag{2}$$

and the bulk free energy of $\mathbf{Q}$ comes from the Landau-de Gennes free energy,

$$f_1(\phi, \mathbf{Q}) = \frac{\beta}{4}(\mathrm{Tr}(\mathbf{Q}^2))^2 - \left(\frac{1+\phi}{2}\right)\mathrm{Tr}(\mathbf{Q}^2) . \tag{3}$$

The ϕ field follows conserved dynamics:

$$\frac{\partial \phi}{\partial t} = \gamma \nabla^2 \frac{\delta F}{\delta \phi} \ , \tag{4}$$

where

$$\frac{\delta F}{\delta \phi} = \phi^3 - \phi - \frac{1}{2}\mathrm{Tr}(\mathbf{Q}^2) - \nabla \cdot [1 + \chi \mathrm{Tr}(\mathbf{Q}^2)]\nabla \phi \ . \tag{5}$$

The $\mathbf{Q}$ field follows non-conserved dynamics:

$$\frac{\partial \mathbf{Q}}{\partial t} = -\frac{\delta F}{\delta \mathbf{Q}} \ , \tag{6}$$

where

$$-\frac{\delta F}{\delta \mathbf{Q}} = [1 + \phi - 2\alpha|\nabla\phi|^2 - \beta \mathrm{Tr}(\mathbf{Q})^2]\mathbf{Q} + E_1 \nabla^2 \mathbf{Q} + E_2 \nabla(\nabla \cdot \mathbf{Q}) \ . \tag{7}$$

Putting everything together, the final model is:

$$\frac{\partial \phi}{\partial t} = \gamma \nabla^2 [f'_{eff} - \nabla \cdot (\kappa_{eff} \nabla \phi)] \ , \tag{8}$$

$$\frac{\partial \mathbf{Q}}{\partial t} = [1 + \phi - 2\chi|\nabla\phi|^2 - \beta \mathrm{Tr}(\mathbf{Q}^2)]\mathbf{Q} + E_1 \nabla^2 \mathbf{Q} + E_2 \nabla(\nabla \cdot \mathbf{Q}) \ , \tag{9}$$

where the effective bulk free energy is

$$f'_{eff} = \phi^3 - \phi - \frac{1}{2}\mathrm{Tr}(\mathbf{Q}^2) \ , \tag{10}$$

and the effective surface tension is

$$\kappa_{eff} = 1 + \chi \mathrm{Tr}(\mathbf{Q}^2) = \begin{cases} 1, & Q_{ij} = 0 \\ 1 + 4\chi S^2, & Q_{ij} \neq 0 \end{cases} \ . \tag{11}$$

The length- and time-scales normalized by a characteristic scale, defined as $(l_c, t_c) = (\sqrt{\kappa}, \Gamma^{-1})$ in our problem. The non-dimensional parameters in the model are

$$\gamma = \frac{1/\Gamma}{\kappa/M} = \frac{\text{nematic timescale}}{\text{molecular timescale}} \ ,$$
$$\chi = \frac{\alpha}{\kappa} = \frac{\text{anchoring strength}}{\text{surface tension}} \ ,$$
$$E_i = \frac{L_i}{\kappa} = \frac{\text{elastic constants}}{\text{surface tension}} \ .$$

1.2 Linear stability

1.3 Theory of domain growth for inhomogenous surface tension

2| Numerics

3| Computation

```
import numpy as np
import matplotlib.pyplot as plt
import cupy as cp
```

4| Results

4.1 Finite size effects

4.2 Understanding the parameters

4.3 Varying actin density

4.3.1 Domain growth

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4.4 Varying actin concentration

4.4.1 Domain growth

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4.5 Analyzing domain growth

4.6 Analyzing structure

4.7 Analyzing relxation