

1| Fundamental Equations

In derivatives,

$$dU = TdS - PdV + \mu dN \quad (1a)$$

$$dH = TdS + VdP + \mu dN \quad (1b)$$

$$dG = -SdT + VdP + \mu dN \quad (1c)$$

$$dA = -SdT - PdV + \mu dN \quad (1d)$$

In integrated form,

$$H = U + PV \quad (2a)$$

$$G = U + PV - TS = H - TS \quad (2b)$$

$$A = G - PV = U - TS \quad (2c)$$

2| Maxwell Relations

$$\left. \frac{\partial T}{\partial V} \right|_S = - \left. \frac{\partial P}{\partial S} \right|_V \quad (3a)$$

$$\left. \frac{\partial T}{\partial P} \right|_S = \left. \frac{\partial V}{\partial S} \right|_P \quad (3b)$$

$$\left. \frac{\partial S}{\partial P} \right|_T = - \left. \frac{\partial V}{\partial T} \right|_P \quad (3c)$$

$$\left. \frac{\partial S}{\partial V} \right|_T = \left. \frac{\partial P}{\partial T} \right|_V \quad (3d)$$

3| Measurable Properties

$$C_P = \left. \frac{\partial U}{\partial T} \right|_V = T \left. \frac{\partial S}{\partial T} \right|_V = -T \left. \frac{\partial^2 A}{\partial T^2} \right|_V \quad (4)$$

$$C_V = \left. \frac{\partial H}{\partial T} \right|_P = T \left. \frac{\partial S}{\partial T} \right|_P = -T \left. \frac{\partial^2 G}{\partial T^2} \right|_P \quad (5)$$

$$\kappa_T = \frac{-1}{V} \left. \frac{\partial V}{\partial P} \right|_T = \frac{1}{\rho} \left. \frac{\partial \rho}{\partial P} \right|_T \quad (6)$$

$$\kappa_S = \frac{-1}{V} \left. \frac{\partial V}{\partial P} \right|_S = \frac{1}{\rho} \left. \frac{\partial \rho}{\partial P} \right|_S \quad (7)$$

$$\alpha = \frac{1}{V} \left. \frac{\partial V}{\partial T} \right|_P = \frac{-1}{\rho} \left. \frac{\partial \rho}{\partial T} \right|_P \quad (8)$$

$$C_P - C_V = \frac{T v \alpha^2}{\kappa_T} \quad (9)$$

$$\mu_{JT} = \frac{V}{C_P} (\alpha T - 1) \quad (10)$$

$$\frac{\kappa_T}{\kappa_S} = \frac{C_P}{C_V} \quad (11)$$

4| Ideal gas and water properties

$$C_P = \frac{5}{2}R \text{ and } C_V = \frac{3}{2}R. \quad u_{ig} = \frac{3}{2}RT.$$

$$\gamma = 1.4 \quad \text{Air, } N_2, O_2, H_2$$

$$= 1.67 \quad \text{Ar, He}$$

$$= 1.30 \quad CO_2$$

Water: $V = 18 \text{ cm}^3/\text{mol}$, $T_c = 647 \text{ K}$, $P_c = 220 \text{ bar}$, $C_P = 4.18 \text{ J/(g.K)}$.

5| Calculus Tricks

Inversion:

$$\left. \frac{\partial X}{\partial Y} \right|_Z = 1 / \left. \frac{\partial Y}{\partial X} \right|_X \quad (13)$$

Triple product:

$$\left. \frac{\partial X}{\partial Y} \right|_Z \left. \frac{\partial Y}{\partial Z} \right|_X \left. \frac{\partial Z}{\partial X} \right|_Y = -1 \quad (14)$$

Non-natural deriavtive:

$$\left. \frac{\partial X}{\partial Y} \right|_Z = \left. \frac{\partial X}{\partial Y} \right|_W + \left. \frac{\partial X}{\partial W} \right|_Y \left. \frac{\partial W}{\partial Y} \right|_Z \quad (15)$$

6| Gibbs-Duhem Equation

$$\sum_i n_i \left. \frac{\partial \mu_i}{\partial n_k} \right|_{T,P,n'} = 0 \iff \sum_i x_i \left. \frac{\partial \mu_i}{\partial x_k} \right|_{T,P,n'} = 0 \quad (16)$$

7| Bakius-Roozeboom Equation

$$\left. \frac{\partial g}{\partial x_j} \right|_{T,P,x'} = \mu_j - \mu_c \quad j = 1 \dots c-1 \quad (17)$$

8| Statistical Mechanics

Stirling's approximation:

$$n! \approx n^n e^{-n} \sqrt{2\pi n} \quad (18)$$

N -site binary system at level s :

$$p(s) = \left(\frac{2}{\pi N} \right)^{\frac{1}{2}} e^{-\frac{2s^2}{N}} \quad (19)$$

N harmonic oscillators at $E = n$:

$$\Sigma(N, n) = \frac{(N+n-1)!}{n!(N-1)!} \quad (20)$$