1| Fundamental Equations

In derivatives,

$$dU = TdS - PdV + \mu dN \quad (1a)$$
$$dH = TdS + VdP + \mu dN \quad (1b)$$
$$dG = -SdT + VdP + \mu dN \quad (1c)$$
$$dA = -SdT - PdV + \mu dN \quad (1d)$$

In integrated form,

$$H = U + PV \quad (2a)$$
$$G = U + PV - TS = H - TS \quad (2b)$$
$$A = G - PV = U - TS \quad (2c)$$

2| Maxwell Relations

$$\left.\frac{\partial T}{\partial V}\right|_S = -\left.\frac{\partial P}{\partial S}\right|_V \quad (3a)$$
$$\left.\frac{\partial T}{\partial P}\right|_S = \left.\frac{\partial V}{\partial S}\right|_P \quad (3b)$$
$$\left.\frac{\partial S}{\partial P}\right|_T = -\left.\frac{\partial V}{\partial T}\right|_P \quad (3c)$$
$$\left.\frac{\partial S}{\partial V}\right|_T = \left.\frac{\partial P}{\partial T}\right|_V \quad (3d)$$

3| Measurable Properties

$$C_P = \left.\frac{\partial U}{\partial T}\right|_V = T\left.\frac{\partial S}{\partial T}\right|_V = -T\left.\frac{\partial^2 A}{\partial T^2}\right|_V \quad (4)$$
$$C_V = \left.\frac{\partial H}{\partial T}\right|_P = T\left.\frac{\partial S}{\partial T}\right|_P = -T\left.\frac{\partial^2 G}{\partial T^2}\right|_P \quad (5)$$
$$\kappa_T = \frac{-1}{V}\left.\frac{\partial V}{\partial P}\right|_T = \frac{1}{\rho}\left.\frac{\partial \rho}{\partial P}\right|_T \quad (6)$$
$$\kappa_S = \frac{-1}{V}\left.\frac{\partial V}{\partial P}\right|_S = \frac{1}{\rho}\left.\frac{\partial \rho}{\partial P}\right|_S \quad (7)$$
$$\alpha = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P = \frac{-1}{\rho}\left.\frac{\partial \rho}{\partial T}\right|_P \quad (8)$$
$$C_P - C_V = \frac{Tv\alpha^2}{\kappa_T} \quad (9)$$
$$\mu_{JT} = \frac{V}{C_P}(\alpha T - 1) \quad (10)$$
$$\frac{\kappa_T}{\kappa_S} = \frac{C_P}{C_V} \quad (11)$$

4| Ideal gas and water properties

$C_P = \frac{5}{2}R$ and $C_V = \frac{3}{2}R$. $u_{ig} = \frac{3}{2}RT$.

$$\begin{aligned}\gamma &= 1.4 \quad \text{Air}, N_2, O_2, H_2\\ &= 1.67 \quad Ar, He\\ &= 1.30 \quad CO_2\end{aligned}$$

Water: $V = 18$ cm^3/mol, $T_c = 647$ K, $P_c = 220$ bar, $C_P = 4.18$ J/(g.K).

5| Calculus Tricks

Inversion:

$$\left.\frac{\partial X}{\partial Y}\right|_Z = 1/\left.\frac{\partial Y}{\partial X}\right|_X \quad (13)$$

Triple product:

$$\left.\frac{\partial X}{\partial Y}\right|_Z \left.\frac{\partial Y}{\partial Z}\right|_X \left.\frac{\partial Z}{\partial X}\right|_Y = -1 \quad (14)$$

Non-natural deriavtive:

$$\left.\frac{\partial X}{\partial Y}\right|_Z = \left.\frac{\partial X}{\partial Y}\right|_W + \left.\frac{\partial X}{\partial W}\right|_Y \left.\frac{\partial W}{\partial Y}\right|_Z \quad (15)$$

6| Gibbs-Duhem Equation

$$\sum_i n_i \left.\frac{\partial \mu_i}{\partial n_k}\right|_{T,P,n'} = 0 \iff \sum_i x_i \left.\frac{\partial \mu_i}{\partial x_k}\right|_{T,P,n'} = 0 \quad (16)$$

7| Bakius-Roozeboom Equation

$$\left.\frac{\partial g}{\partial x_j}\right|_{T,P,x'} = \mu_j - \mu_c \quad j = 1\ldots c-1 \quad (17)$$

8| Statistical Mechanics

Stirling's approximation:

$$n! \approx n^n e^{-n}\sqrt{2\pi n} \quad (18)$$

N-site binary system at level s:

$$p(s) = \left(\frac{2}{\pi N}\right)^{\frac{1}{2}} e^{-\frac{2s^2}{N}} \quad (19)$$

N harmonic oscillators at $E = n$:

$$\Omega(N,n) = \frac{(N+n-1)!}{n!(N-1)!} \quad (20)$$