Supply Chain Optimization using Capacitated Plant Location Problem

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Abstract

This report presents a mathematical modeling approach to solving the Capacitated Plant Location Problem (CPLP) in supply chain optimization. The goal is to minimize the total costs associated with operating plants and shipping goods between locations. This includes both fixed costs of opening plants and variable costs of transportation. The optimization model is implemented in Python using a Jupyter Notebook.

1 Introduction

In today's competitive market, efficient supply chain management is crucial for minimizing costs and maximizing profitability. The Capacitated Plant Location Problem (CPLP) is a classical optimization problem in which the objective is to determine the optimal locations for plants and the distribution of goods between these plants and demand locations. This report explores the mathematical formulation of the CPLP and provides a Python-based implementation for solving it.

2 Mathematical Model

The objective function in the Capacitated Plant Location Problem aims to minimize the total costs associated with operating the plants and shipping goods between locations. The total cost comprises two main components: fixed costs and variable costs.

2.1 Objective Function

$$\text{Minimize } Z = \sum_{i \in \text{loc}} \sum_{s \in \text{size}} (\text{fixed\_costs}_{is} \times y_{is} \times 1000) + \sum_{i \in \text{loc}} \sum_{j \in \text{loc}} (\text{var\_cost}_{ij} \times x_{ij}) \tag{1}$$

2.1.1 Fixed Costs

$$\sum_{i \in \text{loc}} \sum_{s \in \text{size}} (\text{fixed\_costs}_{is} \times y_{is} \times 1000) \tag{2}$$

- **Fixed Costs ($\textbf{fixed\_costs}_{is}$)**: These are the costs associated with opening and operating a plant of size s at location i. This includes expenses like construction, machinery, labor, and maintenance.
- **Decision Variable (y_{is})**: This is a binary variable indicating whether a plant of size s is opened at location i ($y_{is} = 1$ if a plant is opened, otherwise $y_{is} = 0$).
- **Multiplier (1000)**: This multiplier is used to scale the fixed costs appropriately, assuming that the fixed costs are given on a per thousand basis.

2.1.2 Variable Costs

$$\sum_{i \in \text{loc}} \sum_{j \in \text{loc}} (\text{var\_cost}_{ij} \times x_{ij}) \tag{3}$$

- **Variable Costs ($\textbf{var\_cost}_{ij}$)**: These are the costs associated with transporting goods from location i to location j. These costs can depend on factors like distance, fuel prices, and shipping methods.
- **Decision Variable (x_{ij})**: This represents the amount of goods transported from location i to location j.

2.2 Interpretation

The objective function balances two types of costs:

- **Fixed Costs**: Costs incurred by opening and operating plants at different locations. Higher fixed costs are typically associated with larger plants or plants in more expensive locations.
- **Variable Costs**: Costs incurred by transporting goods from one location to another. Higher variable costs can result from longer distances, higher transportation fees, or greater quantities of goods being moved.

By minimizing the objective function, the model seeks the optimal balance between opening enough plants to meet demand efficiently while also minimizing the transportation costs between these plants and the locations where demand exists.

3 Methodology

The Python code in the Jupyter notebook implements the above mathematical model using a linear programming solver. The code is structured as follows:

- **Data Input**: Locations, plant sizes, fixed costs, and variable costs are provided as inputs.

- **Decision Variables**: Binary and continuous decision variables are defined to represent plant openings and goods transportation.
- **Objective Function**: The objective function is coded to minimize the total cost as described in the mathematical model.
- **Constraints**: Capacity constraints for plants and demand satisfaction constraints are added to ensure feasible solutions.
- **Solver Execution**: The optimization problem is solved using a suitable linear programming solver.
- **Results Output**: The optimal plant locations, sizes, and transportation routes are output, along with the minimized cost.

4 Results

The results of the optimization are as follows:

- **Optimal Plant Locations**:

Location	**Low**	**High**
USA	0	1
Germany	0	0
Japan	0	1
Brazil	1	0
India	0	1

Table 1: Plant Size Distribution across Locations

- **Total Cost**: Total Costs = 92,981,000 (\$/Month)

These results provide actionable insights into the most cost-effective plant locations and distribution strategies for the given data.

5 Conclusion

In this project, we developed and solved a mathematical model for the Capacitated Plant Location Problem to optimize supply chain costs. The model successfully identifies the optimal plant locations and transportation routes to minimize the total cost. Future work could involve extending the model to consider additional real-world complexities such as varying demand over time or multi-modal transportation.