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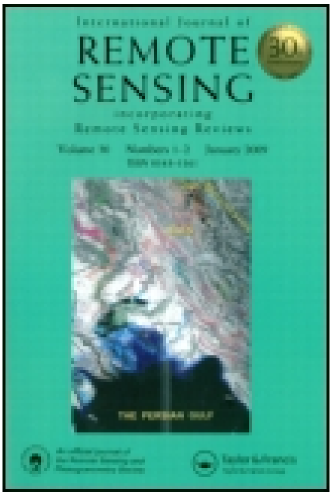

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Quantity, exchange, and shift components of difference in a square contingency table

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Quantity, exchange, and shift components of difference in a square contingency table

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A common task is to measure the difference between two maps that show the same spatial extent for the same categorical variable, such as land-cover type. One popular technique is to express the overall difference as the sum of two components called quantity and allocation. This article shows how to take an additional step to express allocation difference as the sum of two components called exchange and shift. Exchange exists for a pair of pixels when one pixel is classified as category A in the first map and as category B in the second map, while simultaneously the paired pixel is classified as category B in the first map and as category A in the second map. If there are more than two categories, then it is possible to have a component called shift, which is allocation difference that is not exchange. Our article shows how to compute all three components of overall difference: quantity, exchange, and shift. We show also how to compute the three components for each category and to reveal the category pairs that account for the largest exchanges. Our article applies the principles to characterize both temporal changes and classification errors using land-cover maps from suburban Massachusetts, USA.

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1. Introduction

A fundamental operation in our profession is to compare two maps of the same spatial extent, where both maps show the same categorical variable, such as land-cover type. Change analysis is the application when the two maps derive from two time points. Error assessment is the application when the two maps derive from a reference data-set that is considered correct and a comparison data-set that is being tested. For both applications, an obvious first step is to compute the portion of overall difference between the two maps. Pontius and Millones (2011) showed how to partition the overall difference into two mutually exclusive components called quantity difference and allocation difference. Quantity difference is the amount of difference between the two maps that derives from a less than perfect match in the amount of the categories. Allocation difference is the additional difference between the two maps that derives from a less than maximum match in the allocation of the categories, given the quantities of the categories. Many researchers have found it helpful for practical interpretation to separate the overall difference into these two components.

For example, Sarmento et al. (2012) compared reference soil maps to soil maps that were derived using various sampling densities and four machine-learning classification algorithms. Among the various algorithms, the self-organizing map neural network and

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the Gini decision tree gave the smallest quantity differences but the largest allocation differences.

Sloan and Pelletier (2012) obtained an overall difference of 15% when they compared a reference map to their projected map of forest in Panama. They partitioned the difference into 6% quantity and 9% allocation, which is helpful because quantity difference is likely more important than allocation difference when estimating carbon changes for projects intended to reduce carbon dioxide emissions from deforestation and forest degradation.

Thies et al. (2014) created a spatially explicit business-as-usual scenario to project categories of land change from 2001 to 2006 in southern Ecuador. One set of procedures projected the change in the quantity of each category, while a different set of procedures projected the allocation of the categories. They computed quantity and allocation differences for: the observed change during 2001–2006, the predicted change during 2001–2006, and the error between the observed categories at 2006 *versus* the predicted categories at 2006. In nearly all cases, the allocation difference was larger than the quantity difference.

Sohl et al. (2012) used the quantity and allocation measurements to evaluate the differences between pairs of land-change scenarios from 2010 to 2100 for the Great Plains of the USA. Most of scenario pairs showed that quantity difference became larger than allocation difference as the scenarios projected further in time, as the scenario story lines diverged in terms of quantities of the various categories.

Investigators have gained insights by separating difference into two components of quantity and allocation, because the two components can address different aspects of the research goals. For example, if the goal is to compare the quantity of each category, then quantity difference is more important than allocation difference. Investigators might gain additional insight by separating allocation difference further into two interpretable components. For example, some allocation difference in error assessment might be caused by certain pairs of categories being confused with each other; for example, deciduous forest and coniferous forest might be easily confused, and grassland and cropland might be easily confused. These types of pairwise confusions can cause allocation disagreement, while nonpairwise confusions can also cause allocation disagreement.

Our manuscript introduces a method to separate the allocation difference into two components, called exchange and shift. Exchange is the component of allocation difference that pairwise confusions cause. Shift is the component of allocation difference that nonpairwise confusions cause. Our manuscript also gives new methods to compute the components for individual categories and to reveal the category pairs that account for the largest exchanges.

2. Methods

2.1. Components of difference

Figure 1 gives seven cases to demonstrate the difference between a map at time t and a map at time $t + 1$. If the application concerns error assessment, then the map at time t is the comparison map and the map at time $t + 1$ is the reference map. Each map consists of six pixels, where each pixel belongs to exactly one of the three categories: A, B or C. Figure 1 also shows for each case the square contingency table, including marginal totals. Quantity difference is zero if and only if the right column of marginal totals at time t matches the bottom row of marginal totals at time $t + 1$. The title for each case gives the

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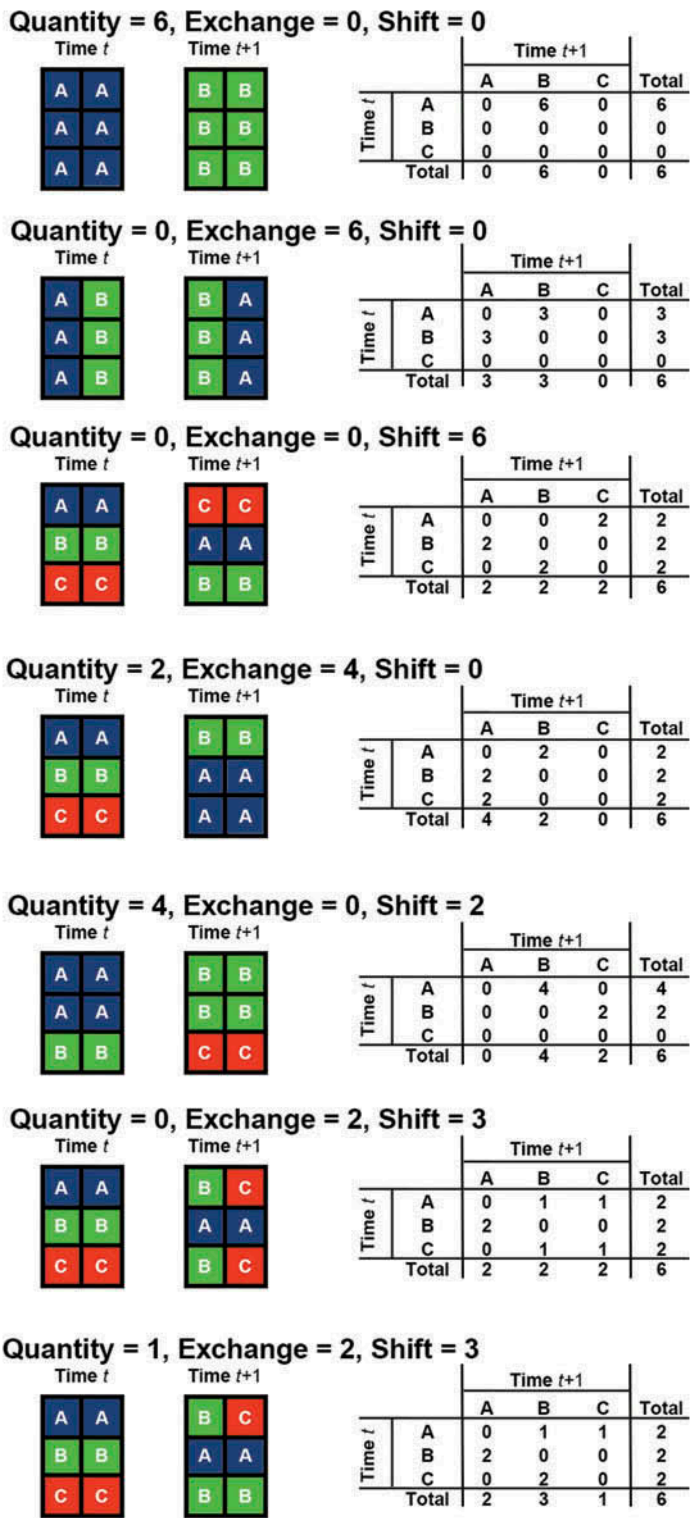

Figure 1. Seven map comparisons to illustrate difference components of quantity, exchange and shift. Letters A, B and C denote categories.

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number of pixels of quantity, exchange and shift. The first case at the top shows an example of complete quantity difference. The second case shows complete exchange difference, as three pixels are A at t and B at $t+1$ while three other pixels are B at t and A at $t+1$. The third case shows complete shift difference and demonstrates that at least three categories are necessary for shift to occur. This third case has no exchange between A and C because two pixels are A at t and C at $t+1$ while no other pixels are C at t and A at $t+1$; similarly, there is no exchange between the other two possible pairs of categories. The remaining cases in Figure 1 show various combinations of components of quantity, exchange and shift.

2.2. Notation and equations

This subsection gives the notation and equations to compute each of the components. Table 1 shows the layout of a square contingency table where i denotes the categories at time t in the rows, j denotes the categories at time $t+1$ in the columns, and C_{tij} equals the number of pixels that are category i at time t and category j at time $t+1$. The sequence of categories in the rows is the same as the sequence of the categories in the columns. Therefore, if $i = j$, then C_{tij} is on the diagonal and indicates agreement between the two maps, while if $i \neq j$, then C_{tij} is off the diagonal and indicates disagreement between the two maps. Table 2 gives the mathematical notation that the equations use. Equations (1)–(4) express annual percentages of the study extent, because each numerator includes a factor of 100% while each denominator is the product of the time interval's duration (Y_{t+1}–Y_t) and the size of the study extent expressed by the double summation. Equation (1) computes the difference for an arbitrary category j, because the numerator's summation of C_{tij} computes the total in the contingency table's column j and the numerator's summation of C_{tji} computes the total in the contingency table's row j. The summation in the numerator counts the diagonal entry C_{tjj} twice; therefore, Equation (1) subtracts C_{tjj} twice. Equation (2) defines the quantity component for an arbitrary category j, because the numerator's summation gives the difference between the contingency table's column total for category j and the contingency table's row total for category j. Equation (3) defines the exchange between categories i and j. Exchange consists of a transition from i to j in some pixels and a transition from j to i in an identical number of other pixels. Therefore, we compute exchange for only $i > j$, and we define exchange as zero for $i \leq j$; thus, $\varepsilon_{ij} = 0$ or $\varepsilon_{ji} = 0$. Equation (3) uses multiplication by two because each exchange derives from a pair of entries in the contingency table. Equation (3) uses the minimum function because each pair is constrained by the smaller of C_{tij} and C_{tji}. Equation (4) gives the exchange component for category j, which is the sum of all exchanges that involve category j. Equation (4) subtracts C_{tjj} because the numerator's summation includes C_{tjj}.

Table 1. Layout of a square contingency table for three categories.

		Time $t+1$		
		$j = 1$	$j = 2$	$j = 3$
Time t	$i = 1$	C_{t11}	C_{t12}	C_{t13}
	$i = 2$	C_{t21}	C_{t22}	C_{t23}
	$i = 3$	C_{t31}	C_{t32}	C_{t33}

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Table 2. Mathematical notation.

Symbol	Description
J	Number of categories
i	Index for a category of a row
j	Index for a category of a column
t	Index for a time interval
C_{tij}	Size of spatial extent that is in row i and column j of contingency table for interval t
Y_t	Year at beginning time point of interval t
Y_{t+1}	Year at ending time point of interval t
d_{tj}	Difference for category j for interval t
q_{tj}	Quantity component for category j for interval t
ε_{tij}	Exchange between categories i and j for interval t
e_{tj}	Exchange component for category j for interval t
s_{tj}	Shift component for category j for interval t
D_t	Overall difference for interval t
Q_t	Overall quantity component for interval t
E_t	Overall exchange component for interval t
S_t	Overall shift component for interval t

Equation (5) gives the shift component for category j and shows how the shift component for category j is the difference d_{tj} minus both the quantity component q_{tj} and the exchange component e_{tj}.

$$d_{tj} = \frac{\left\{\left[\sum_{i=1}^{J}\left(C_{tij} + C_{tji}\right)\right] - 2 \times C_{tjj}\right\} \times 100\ \%}{\left(Y_{t+1} - Y_t\right) \times \sum_{i=1}^{J}\sum_{j=1}^{J} C_{tij}}, \tag{1}$$

$$q_{tj} = \frac{\left|\sum_{i=1}^{J}\left(C_{tij} - C_{tji}\right)\right| \times 100\%}{\left(Y_{t+1} - Y_t\right) \times \sum_{i=1}^{J}\sum_{j=1}^{J} C_{tij}}, \tag{2}$$

$$\begin{aligned} \varepsilon_{tij} &= \frac{2 \times \text{MINIMUM}\left(C_{tij}, C_{tji}\right) \times 100\ \%}{\left(Y_{t+1} - Y_t\right) \times \sum_{i=1}^{J}\sum_{j=1}^{J} C_{tij}} \quad \text{for } i > j \\ &= 0 \quad \text{for } i \leq j' \end{aligned} \tag{3}$$

$$e_{tj} = \sum_{i=1}^{J}\left(\varepsilon_{tij} + \varepsilon_{tji}\right) = \frac{2 \times \left\{\left[\sum_{i=1}^{J} \text{MINIMUM}\left(C_{tij}, C_{tji}\right)\right] - C_{tjj}\right\} \times 100\%}{\left(Y_{t+1} - Y_t\right) \times \sum_{i=1}^{J}\sum_{j=1}^{J} C_{tij}}, \tag{4}$$

$$s_{tj} = d_{tj} - q_{tj} - e_{tj}. \tag{5}$$

Equations (2), (4)–(5) give the three components of difference for an arbitrary category j; then, Equations (6)–(8) sum across all J categories to compute the overall components for the study extent. Equation (6) computes the overall quantity component, Equation (7) computes the overall exchange component, and Equation (8) computes the overall shift component. Equation (9) computes the overall difference and shows how the overall difference is the sum of the three components of quantity, exchange and shift. Equations (6)–(9) use division by two because each numerator's summation double-counts the difference. For example, if a pixel is category i at time t and a different category j at time t + 1, then the pixel is counted as a difference for both category i and category j. Therefore, summation over all categories counts that pixel twice. Division by two in Equations (6)–(9) neutralizes the double-counting.

$$Q_t = \frac{\sum_{j=1}^{J} q_{tj}}{2}, \tag{6}$$

$$E_t = \frac{\sum_{j=1}^{J} e_{tj}}{2}, \tag{7}$$

$$S_t = \frac{\sum_{j=1}^{J} s_{tj}}{2}, \tag{8}$$

$$D_t = \frac{\sum_{j=1}^{J} d_{tj}}{2} = Q_t + E_t + S_t. \tag{9}$$

Division by (Y_{t+1}–Y_t) in Equations (1)–(4) makes the equations express results in units of area per year to facilitate comparison among time intervals, which matches the logic of the flow matrix to compare time intervals that have various durations (Runfola and Pontius 2013). If the application is error assessment, then the map comparison lacks a time dimension, in which case the map at time t is the comparison map and the map at time t + 1 is the reference map. The notation for error assessment does not have a subscript of t, and then, Equations (1)–(4) do not have (Y_{t+1}–Y_t) in the denominator.

2.3. Data

For a first illustration, we compute the components to characterize change among three categories in Plum Island Ecosystems (PIE) study area in suburban Massachusetts, USA. Figure 2 shows the maps at the years 1971, 1985, 1991 and 1999; thus, we analyse three time intervals. Forest, Built, and Other are the three categories, which were aggregated from the 21 categories in the maps from the State of Massachusetts (MassGIS 2013).

For a second illustration, we compute the components to characterize errors in the map that Clark University's team called HOLMES classified (Polsky et al. 2012). HOLMES stands for HERO Object-based Lawn Mapping Exploration of Suburbia, and HERO

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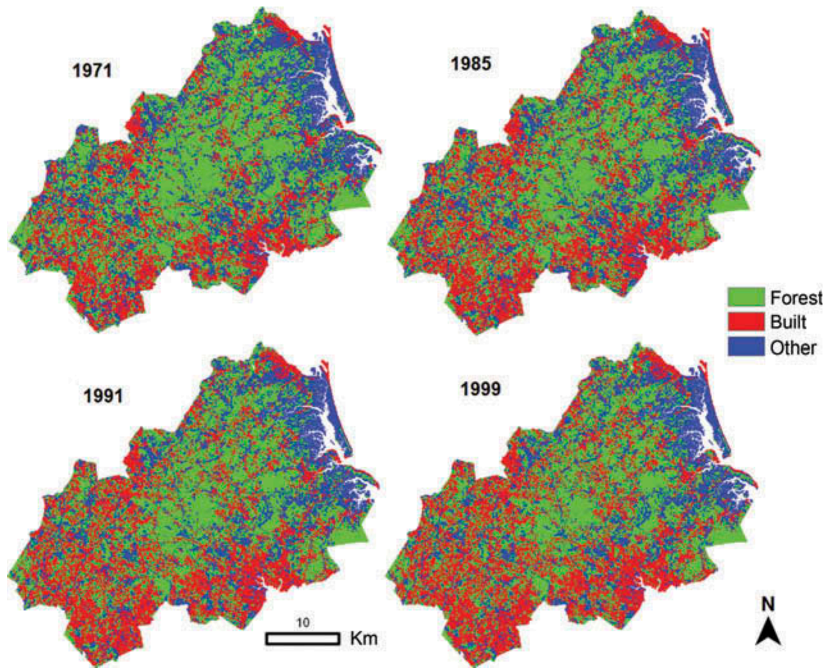

Figure 2. Land categories of the Plum Island Ecosystems at 1971, 1985, 1991 and 1999. The centre of the study area has approximate coordinates 42° 40′ N, 71° 00′ W.

stands for Human-Environment Regional Observatory. The map derives from 0.5 m spatial resolution aerial orthophotographs acquired on April 2005 (MassGIS 2013). HOLMES classified the aerial imagery into seven categories: Bare Soil, Coniferous, Deciduous, Fine Green, Impervious, Water, and Wetlands. The reference data derived from a visual inspection of points for the same orthophotographs. The points were derived from a stratified random sample; therefore, we converted the stratified sample table to the estimated population table by using Equation (1) in Pontius and Millones (2011). We analysed the estimated population table for three towns in the PIE study area.

3. Results

Figure 3 shows that overall annual change is slowest during 1971–1985 and fastest during 1985–1991. The distance between the time points along the horizontal axis is proportional to the duration of the time interval defined by each pair of time points; thus, the size of each rectangle in Figure 3 is proportional to the size of the change during the time interval. Most of the change during each time interval is quantity difference. Exchange is larger than shift during the latter two time intervals. Figure 4 reveals that Forest and Built are mainly responsible for the quantity difference, while Forest is a net losing category and Built is a net gaining category. Table 3 shows that the pairwise exchange between Forest and Other accounts for most of the overall exchange during the latter two time intervals.

Figure 5 characterizes the errors in the HOLMES data for the towns of Ipswich, Newburyport, and Woburn. Ipswich has the most overall error, while both Newburyport and Woburn have nearly the same amount of overall error. Both Ipswich and Newburyport

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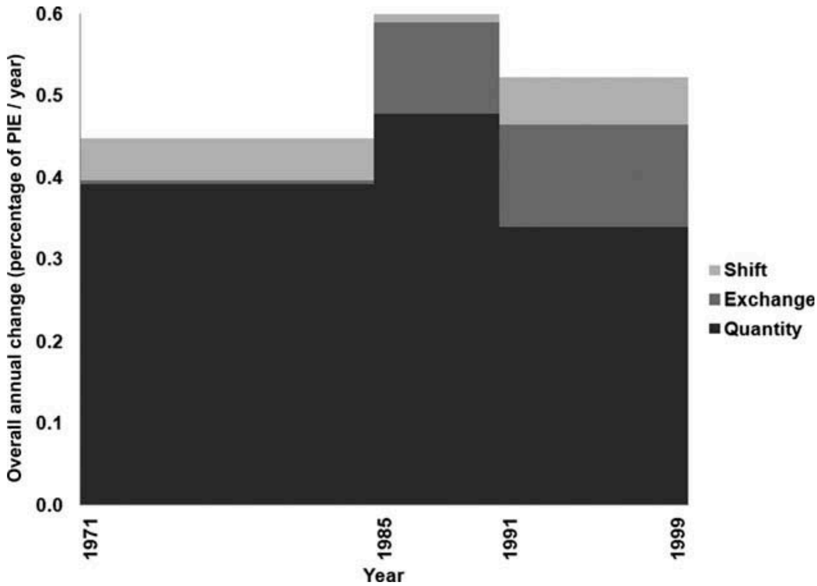

Figure 3. Overall annual changes derived from Equations (6)–(8) during three time intervals for the Plum Island Ecosystems.

have the same quantity error, while Woburn has the largest quantity error. Figure 6 shows the three components of error for each category in Ipswich. The comparison map shows more Fine Green and Wetland than the reference map shows, while the comparison map shows less of all other categories than the reference map shows. Table 4 reveals that the pair consisting of Coniferous and Deciduous accounts for the largest exchange in Ipswich.

4. Discussion

The components in Figure 5 are important for interpretation concerning the errors in the three towns. Ipswich has the largest overall error, but the quantity component for Ipswich is the same as for Newburyport and is smaller than for Woburn. Therefore, if the only goal is to estimate the size of each category, then Ipswich has as much error as Newburyport and less error than Woburn. Figure 5 shows also that exchange is the largest component of error for Ipswich. Table 4 shows that Coniferous and Deciduous form the pair of categories that account for most of the exchange component. If Coniferous and Deciduous were aggregated to form one category called forest, then the exchange error between Coniferous and Deciduous would vanish, which would eliminate error in 6.49% of Ipswich.

For remote sensing, the three components of difference can give insight concerning the causes of the errors. Two types of classifier problems can create quantity error. First, if a classifier uses a statistical characterization of the reflectance that is too narrow for a category, then the narrowness might cause the classifier to classify less of that category than the reference information indicates. Second, if a classifier uses a statistical characterization of the reflectance that is too broad for a category, then the broadness might cause the classifier to classify more of that category than the reference information indicates. If two categories have very similar spectral signatures, then the similarity might cause

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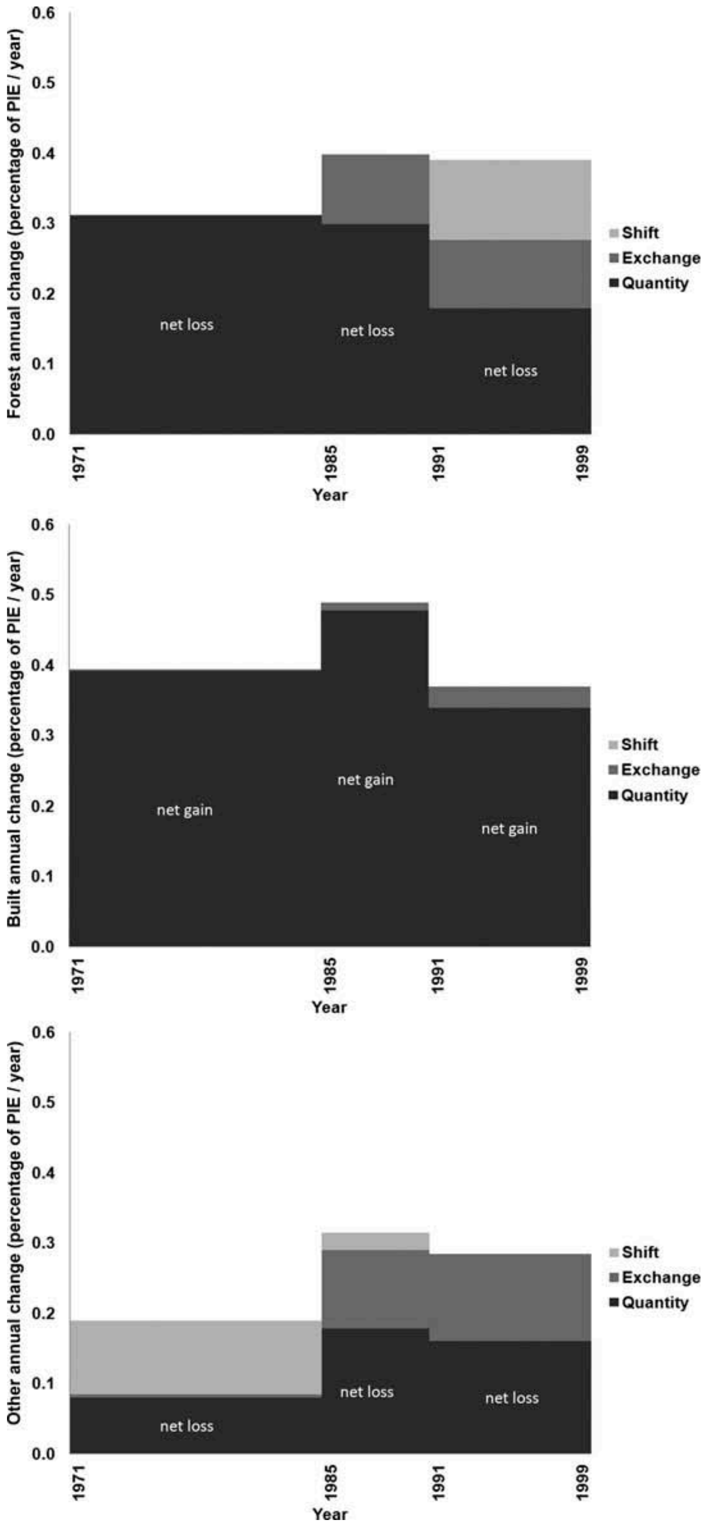

Figure 4. Categorical annual changes derived from Equations (2), (4)–(5) during three time intervals for the Plum Island Ecosystems.

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Table 3. Exchanges derived from Equation (3) expressed as annual percentages of the Plum Island Ecosystems during three time intervals.

		Forest	Built
1971–1985	Built	0.000	
	Other	0.002	0.003
1985–1991	Built	0.000	
	Other	0.100	0.012
1991–1999	Built	0.002	
	Other	0.095	0.028

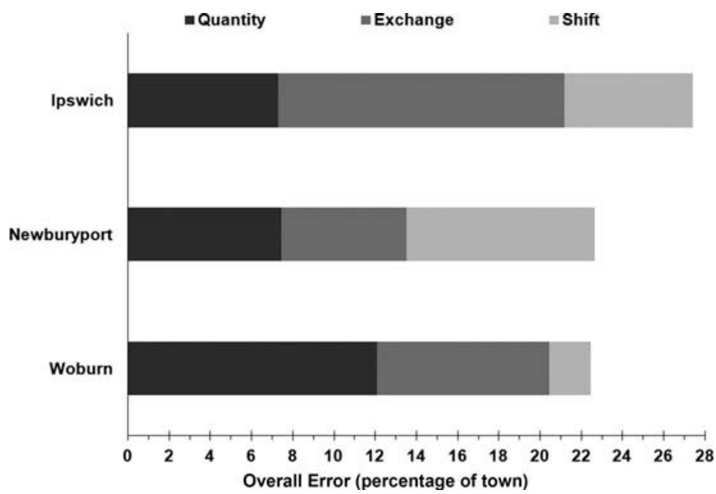

Figure 5. Overall errors derived from Equations (6)–(8) for three towns.

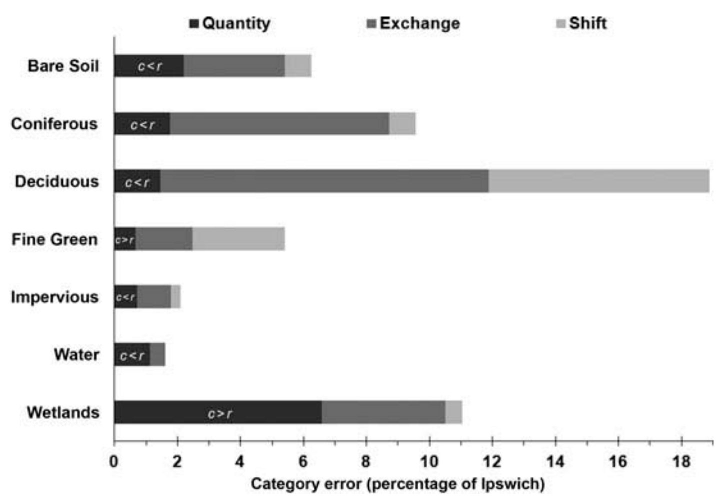

Figure 6. Categorical errors derived from Equations (2), (4)–(5) for each of seven categories for Ipswich. The label $c < r$ means that the area of the category in the comparison map is less than the area of the category in the reference map, and the label $c > r$ means that the area of the category in the comparison map is greater than the area of the category in the reference map.

Table 4. Exchanges derived from Equation (3) expressed as percentages of the town of Ipswich.

		Reference					
		Bare Soil	Coniferous	Deciduous	Fine Green	Impervious	Water
Comparison	Coniferous	0.01					
	Deciduous	0.71	6.49				
	Fine Green	0.54	0.42	0.62			
	Impervious	0.61	0.03	0.21	0.21		
	Water	0.01	0.00	0.13	0.00	0.00	
	Wetlands	1.32	0.00	2.26	0.00	0.00	0.33

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exchange error. Attempts to rectify quantity error are likely to influence both exchange and shift; therefore, investigators should first attempt to rectify quantity error, then re-examine the components to determine the next step for rectification.

For land-change science, the three components of difference can give insight to help to link patterns with processes of land change. Figure 3 shows that the quantity component accounts for most of the difference during all three time intervals in PIE. Figure 4 shows that the quantity component derives from a net loss of Forest and a net gain of Built, as the largest transition is from Forest to Built during all three time intervals. The process is an expansion of residential use on a landscape where most of the available land is Forest. Figure 4 and Table 3 show that exchange between Forest and Other accounts for most of the allocation difference during the latter two time intervals. Some places in PIE experience transition from Forest to Other, while simultaneously different places experience transition from Other to Forest. A closer look at these two transitions reveals that the Other category includes vacant areas of no vegetation; thus, a substantial amount of allocation change involves two transitions: from Forest to vacant areas of no vegetation and from vacant areas of no vegetation to Forest. Exchange is a pattern that indicates that two opposite processes may be working at various places on the landscape.

5. Conclusions

This article gives equations to compare two maps of the same spatial extent for a categorical variable to separate the difference into components of quantity, exchange and shift. The new intellectual contribution of our manuscript is to show how to separate allocation difference into two components of exchange and shift. Our equations compute the components for each individual category and overall. Results reveal the particular pairs of categories contribute to exchange. It is useful to separate overall change into these components, because each component is interpretable based on the particular application. Applications include change characterization, error assessment' and map comparison. The method derives entirely from a square contingency table for which the categories in the rows match the categories in the columns; hence, there are many potential applications that extend beyond the analysis of maps. A spreadsheet to perform the procedure is available for free at www.clarku.edu/~rpontius.

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Disclosure statement

The authors declare no conflict of interest concerning any financial interest or benefit from the direct applications of their research.

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