Foundations & Data Structures With C++

Assignment 17 –Dynamic Programming

1. On a positive integer, you can perform any one of the following 3 steps.
    a. Subtract 1 from it (n = n - 1)
    b. If it is divisible by 2, divide by 2 (if n % 2 == 0, then n = n / 2)
    c. If it is divisible by 3, divide by 3 (if n % 3 == 0, then n = n / 3)

    Now the question is, given a positive integer n, find the minimum number of steps that makes n to 1. For e.g. –

    For n = 1, output: 0
    For n = 4, output: 2 (4 /2 = 2 /2 = 1)
    For n = 7, output: 3 (7 -1 = 6 /3 = 2 /2 = 1)

2. Given a list of N coins, their values (V1, V2, … , VN), and the total sum S. Find the minimum number of coins, sum of which is S (we can use as many coins of one type as we want), or report that it's not possible to select coins in such a way that they sum up to S.

3. Given a sequence of N numbers – A[1] , A[2] , …, A[N] . Find the length of the longest non-decreasing sequence.

4. A table composed of N x M cells each having a certain quantity of apples, is given. You start from the upper-left corner. At each step you can go down or right one cell. Find the maximum number of apples you can collect.

5. Given a rod of length n inches and an array of prices that contains prices of all pieces of size smaller than n. Determine the maximum value obtainable by cutting up the rod and selling the pieces. For example, if length of the rod is 8 and the values of different pieces are given as following, then the maximum obtainable value is 22 (by cutting in two pieces of lengths 2 and 6)
    a. Length - 1 2 3 4 5 6 7 8
    b. Price - 1 5 8 9 10 17 17 20

6. Given weights and values of n items, put these items in a knapsack of capacity W to get the maximum total value in the knapsack. In other words, given two integer arrays val[0..n-1] and wt[0..n-1] which represent values and weights associated with n items respectively. Also given an integer W which represents knapsack capacity, find out the maximum value subset of val[] such that sum of the weights of this subset is smaller than or equal to W. You cannot break an item, either pick the complete item, or don't pick it (0-1 property).

7. Given a binary matrix, find out the maximum size square sub-matrix with all 1s.

For e.g:

```
0 1 1 0 1
1 1 0 1 0
0 1 1 1 0
1 1 1 1 0
1 1 1 1 1
0 0 0 0 0
```

Output:

```
1 1 1
1 1 1
1 1 1
```