Implementing a Dynamic Delta-Hedging Strategy

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# (a) Problem Addressed and Models Used

**Problem**:

Replicate a short European call via daily delta hedging from to evaluated on:

* simulated GBM paths
* real GOOG option data to assess Black–Scholes validity.

Models

We model the underlying under the assignment’s discrete-time GBM Euler step, where one business-day increment evolves as

Option values and Greeks are computed with the Black–Scholes (BS) model for a European call with spot , strike , continuously-compounded risk-free rate , volatility , and time to expiry . The BS price and delta are

with

where is the standard normal CDF. The daily delta-hedging P&L is tracked via the cumulative hedging error (HE) and the cash (bank) account . Defining , for trading day (one business day after ) we use

where is the market mid quote for the option on day and year.

For market data (Task 2), the implied volatility on each day is obtained by solving the BS pricing equation for via a robust bisection search over and that implied is used to compute for re-hedging.

# (b) Structure of the Model Implementation

Languages/tools: C++17 for core logic and CSV I/O; Python (pandas/matplotlib) for plotting.

Key components (Check Appendix for class design):

|  |  |
| --- | --- |
| **File(s)** | **Purpose** |
| bs.h | Black–Scholes helpers: price, delta, vega, implied\_vol\_call (bisection). |
| option.h | EuropeanCall class encapsulating pricing/greeks for given (K, T). |
| date.h | Business-day counting (weekends only), lightweight parsing/formatting. |
| hedger.h / hedger.cpp | Engines for Task 1 (simulation) and Task 2 (market hedging). |
| io.h / io.cpp | CSV loaders for interest, stock, and option mid quotes. |
| main.cpp | Main executable point (mode=sim or mode=market). |
| plot\_\*.py | Plots for paths, prices, IV, delta, PNL vs hedged PNL, HE histogram. |

# (c) Unit Tests

Two unit-tests are implemented and are reported with outcomes (pass/fail):

**Implied-volatility recovery.** We validate the IV solver by a consistency check against Black–Scholes. First, we pick a known parameter set (e.g., , , yr, ) and compute the call price . We then run the IV routine on and assert that the recovered volatility matches within a tight tolerance (e.g., ). It would fail if prices were miscomputed, if the solver didn’t properly bracket the root, or if tolerances were too loose/tight.

**Delta correctness (edge cases + numeric cross-check).** We confirm the analytic delta behaves correctly in limiting regimes: for a deep-ITM call (e.g., ) should be near 1, and for a deep-OTM call (e.g., ) should be near 0; we assert thresholds like and . This catches sign/scale errors in , misuse of time units (days vs years), or an incorrect normal CDF. As a stronger numerical check, we compare the analytic delta to a symmetric finite-difference estimate using the same BS pricer and require close agreement (e.g., error ). Passing these tests gives confidence that the daily pipeline in Task 2 (IV → → rebalance) rests on correct building blocks; remaining discrepancies in hedged PnL are then attributable to model limits (discrete hedging, IV vs realized vol) rather than implementation bugs.

# (d) Results, Analysis, Plots, and Conclusions

**Task 1** – All simulated stock paths and option prices and Hedging Error

A graph of colored lines

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A graph of a distribution of a normal fit

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Across the simulation, the stock paths spread out over time with average upward drift ( and growing dispersion proportional to ; the corresponding call‐price series co-move with S and show visible convexity (gamma), rising more on rallies than they fall on similar-sized declines.

The terminal hedging errors () are approximately bell-shaped and centered near zero: the fitted normal has mean ≈ **0.023** and variance ≈ **0.256** (sd ≈ **0.51**). This near-zero mean is consistent with delta hedging in a BS-consistent world, while the non-zero variance reflects **discrete (daily) rebalancing** and the Euler time-step used for simulation, in other words, unhedged intraday moves accumulate small errors.

In practice, increasing hedge frequency (smaller ) or using the exact GBM scheme would reduce this dispersion, whereas larger σ or fewer rebalances would widen it.

**Task 2** – Market Data hedging

Time series for S, V, implied volatility, delta.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **date** | **S** | **V** | **imp\_vol** | **delta** | **HE** | **PNL** | **PNL\_with\_hedge** |
| **2011-07-05** | 532.440000 | 44.200000 | 0.259686 | 0.722654 | 0.000000 | 0.000000 | 0.000000 |
| **2011-07-06** | 535.360000 | 46.900000 | 0.269337 | 0.733318 | -0.594987 | -2.700000 | -0.594987 |
| **2011-07-07** | 546.600000 | 55.300000 | 0.269861 | 0.787576 | -0.755109 | -11.100000 | -0.755109 |
| **2011-07-08** | 531.990000 | 43.950000 | 0.268563 | 0.719396 | -0.914573 | 0.250000 | -0.914573 |
| **2011-07-11** | 527.280000 | 41.000000 | 0.275950 | 0.691509 | -1.355218 | 3.200000 | -1.355218 |
| **2011-07-12** | 534.010000 | 46.400000 | 0.286788 | 0.722767 | -2.103552 | -2.200000 | -2.103552 |
| **2011-07-13** | 538.260000 | 49.300000 | 0.287137 | 0.745056 | -1.934231 | -5.100000 | -1.934231 |
| **2011-07-14** | 528.940000 | 41.150000 | 0.272178 | 0.706903 | -0.730400 | 3.050000 | -0.730400 |
| **2011-07-15** | 597.620000 | 99.650000 | 0.281490 | 0.940745 | -10.682291 | -55.450000 | -10.682291 |
| **2011-07-18** | 594.940000 | 97.650000 | 0.300413 | 0.926444 | -11.206303 | -53.450000 | -11.206303 |
| **2011-07-19** | 602.550000 | 103.800000 | 0.266228 | 0.960290 | -10.308833 | -59.600000 | -10.308833 |
| **2011-07-20** | 595.350000 | 97.800000 | 0.299707 | 0.931925 | -11.226193 | -53.600000 | -11.226193 |
| **2011-07-21** | 606.990000 | 108.150000 | 0.275666 | 0.964257 | -10.732118 | -63.950000 | -10.732118 |
| **2011-07-22** | 618.230000 | 118.700000 | 0.248420 | 0.985999 | -10.447736 | -74.500000 | -10.447736 |
| **2011-07-25** | 618.980000 | 119.950000 | 0.294243 | 0.971588 | -10.962216 | -75.750000 | -10.962216 |
| **2011-07-26** | 622.520000 | 123.250000 | 0.286970 | 0.978582 | -10.826701 | -79.050000 | -10.826701 |
| **2011-07-27** | 607.220000 | 108.650000 | 0.304805 | 0.957691 | -11.203146 | -64.450000 | -11.203146 |
| **2011-07-28** | 610.940000 | 112.100000 | 0.302652 | 0.964977 | -11.094570 | -67.900000 | -11.094570 |
| **2011-07-29** | 603.690000 | 106.800000 | 0.370045 | 0.924709 | -12.794724 | -62.600000 | -12.794724 |

A graph of a graph with numbers and lines

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On the GOOG window (2011-07-05→07-29), the daily table includes all required fields (S, V, implied vol, delta, HE, PNL, and PNL with hedge). The time-series plot shows that the unhedged PNL experiences a sharp drawdown around mid-July and bottoms near −$60 to −$80, while the hedged PNL stays much closer to flat, drifting only modestly negative (roughly −$10 to −$20). This gap indicates that delta hedging materially reduced directional risk and PNL volatility over the period. The remaining negative drift in the hedged PNL is typical and can be explained by a combination of discrete (daily) rebalancing, implied volatility differing from realized volatility (likely realized > implied during the drawdown), and time varying IV that causes day-to-day changes in delta. In short, the hedge did its principal job which is to dampen exposure to the underlying’s move. The residual losses reflect model and timing frictions rather than implementation errors.

# Screenshots of Implementation

Console outputs from runs (mode=sim and mode=market).:

A screenshot of a computer

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Folder Structure:

Interim\_Project/

README. md

REPORT.docx

hedger.cpp

i0. cpp

main

main.cpp

plot\_delta\_vs\_price.py

plot\_market

results.py

plot sim paths.py

tests.cpp

tests bin

data/

interest.csv

op G00G. csv

sec G00G. Csv

include/

bs.h

date.h

hedger.h

i0.h

option.h

utils.h

out/

fig HE hist.png

fig delta vs price.png

fig market IV.png

fig\_market\_S.png

fig\_market\_V.png

fig\_market\_delta.png

fig market\_pnl\_vs\_hedged.png

fig\_option\_prices\_all.png

fig\_paths\_all.png

result.csv

sim hedge errors.csv

sim\_option\_prices.csv

sim\_paths.csv