ME609
“Optimization”

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Summary

This is a presentation for the Programming Assignments given by our instructor for the ME609 course Prof. Deepak Sharma. We will present our optimal results for several objective functions. We used Bounding Phase method to bracket the optima and Secant method to find the exact optimal solution.

PHASE_1

Programming Project Question Set (Phase-1)

1. Minimize

   $$f(x) = x^3 + 5x^2 - 3 \quad \text{in interval (-1, 1)}$$

2. Maximize

   $$f(x) = (2x-5)^4 - (x^2-1)^3 \quad \text{in interval (-10, 0)}$$

3. Maximize

   $$f(x) = 8 + x^3 - 2x - 2e^x \quad \text{in interval (-2, 1)}$$

4. Minimize

   $$f(x) = 3x^2 + \frac{12}{x^3} - 5 \quad \text{in interval (0.5, 5)}$$

5. Maximize

   $$f(x) = 4x(\sin x) \quad \text{in interval } (0.5, \pi)$$

6. Minimize

   $$f(x) = 2(x-3)^2 + e^{0.5x^2} \quad \text{in interval (-2, 3)}$$

7. Minimize

   $$f(x) = 2e^x - x^3 - 10x \quad \text{in interval (0, 4)}$$

8. Minimize

   $$f(x) = x^2 - 10e^{(0.1x)} \quad \text{in interval (-6, 6)}$$

9. Maximize

   $$f(x) = 20\sin x - 15x^2 \quad \text{in interval (-4, 4)}$$

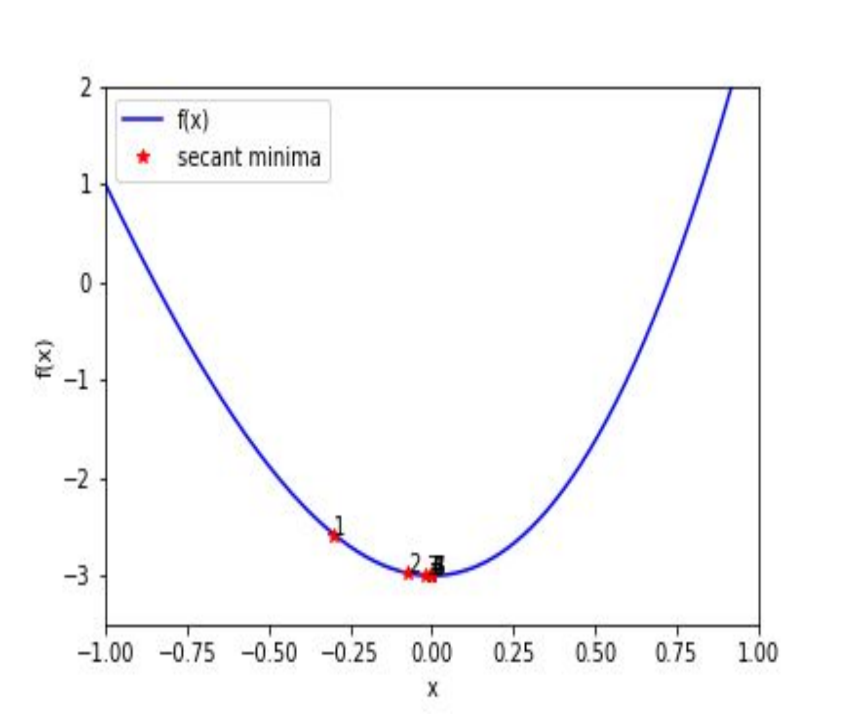
10. Find at least one root of the following function

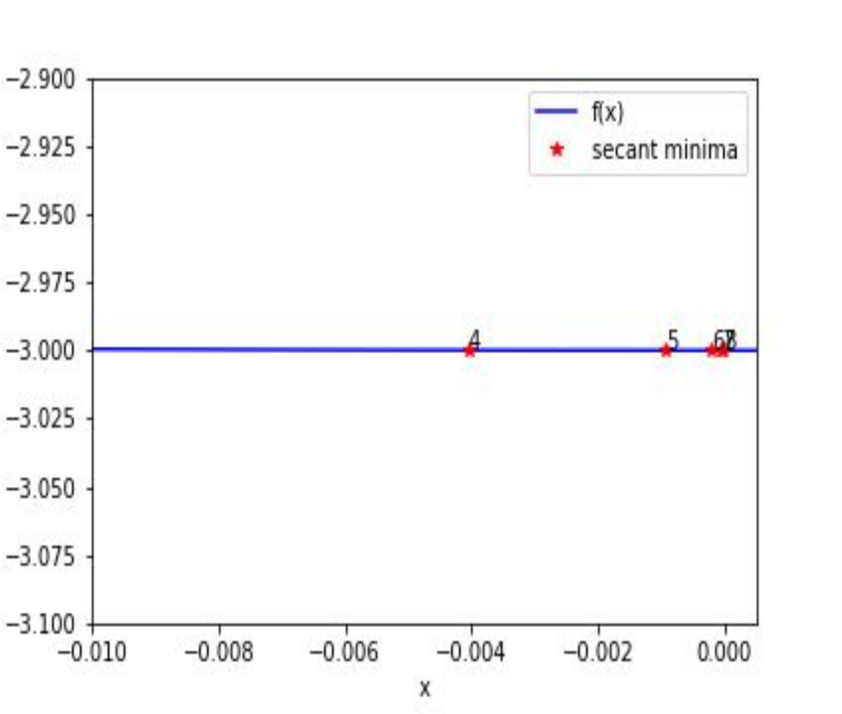
    $$f(x) = e^x - x^3$$

Minimize

$f(x) = x^3 + 5x^2 - 3$ in interval (-1, 1)

Epsilon = 0.0001
Minima = -2.75157e-06
Total 9 iterations.

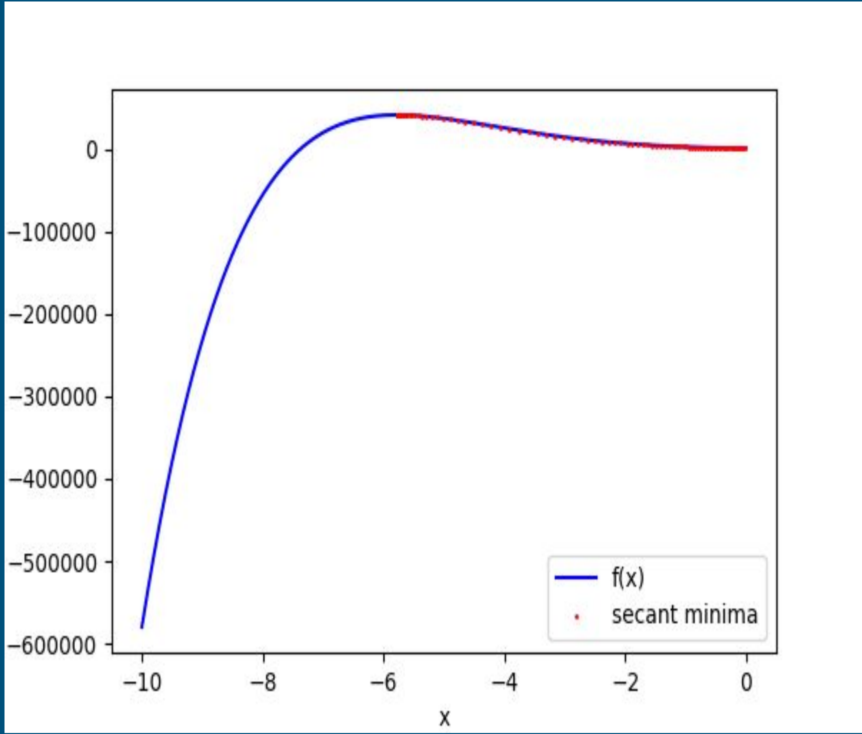


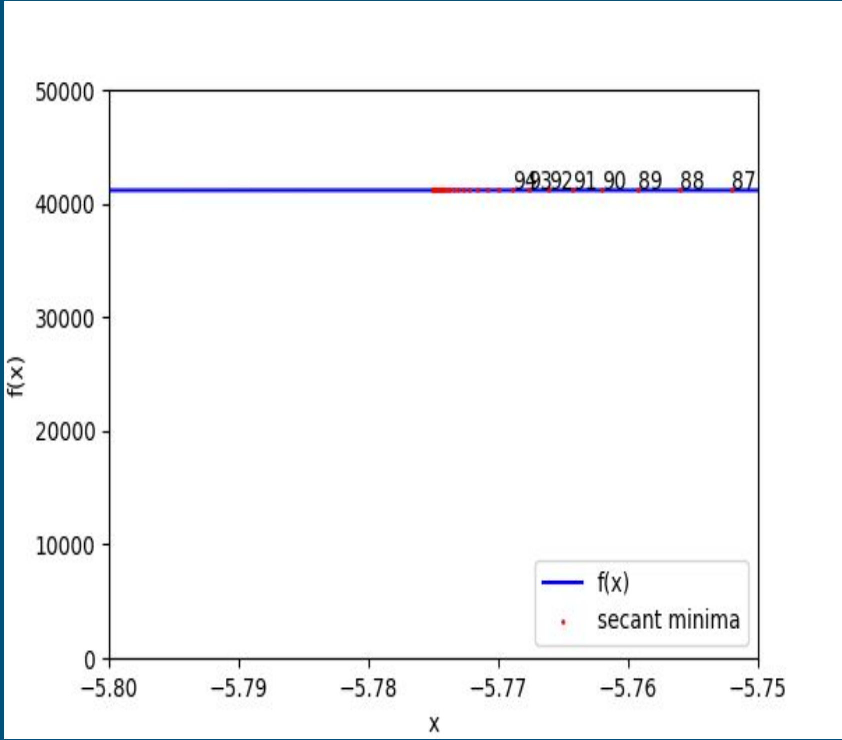


Maximize

$f(x) = (2x-5)^4 - (x^2-1)^3$ in interval (-10, 0)

Epsilon = 0.0001
Maxima = -5.77507
Total 168 iterations.

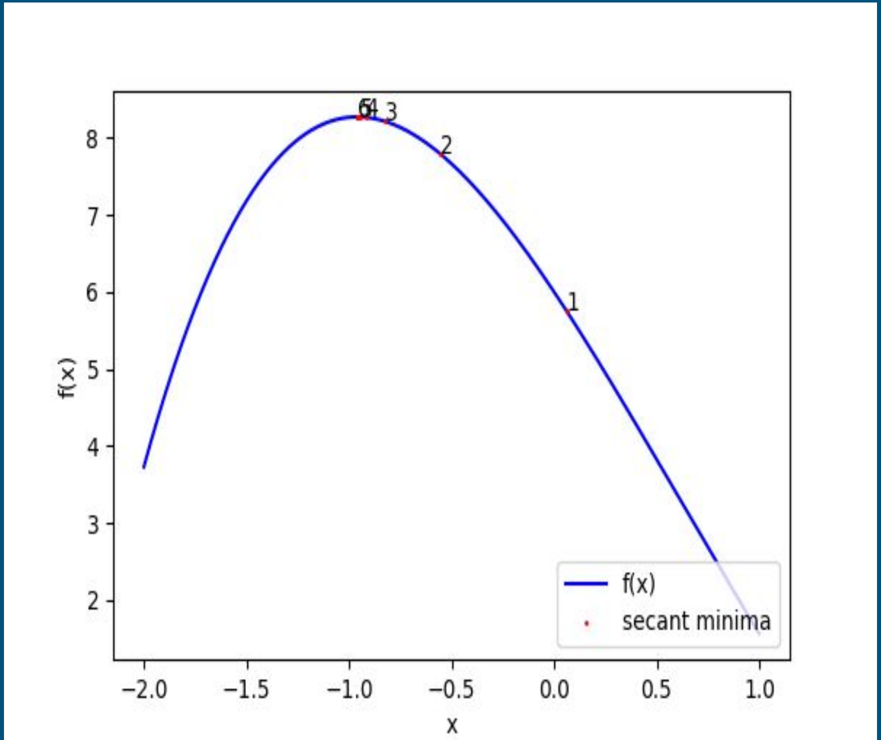


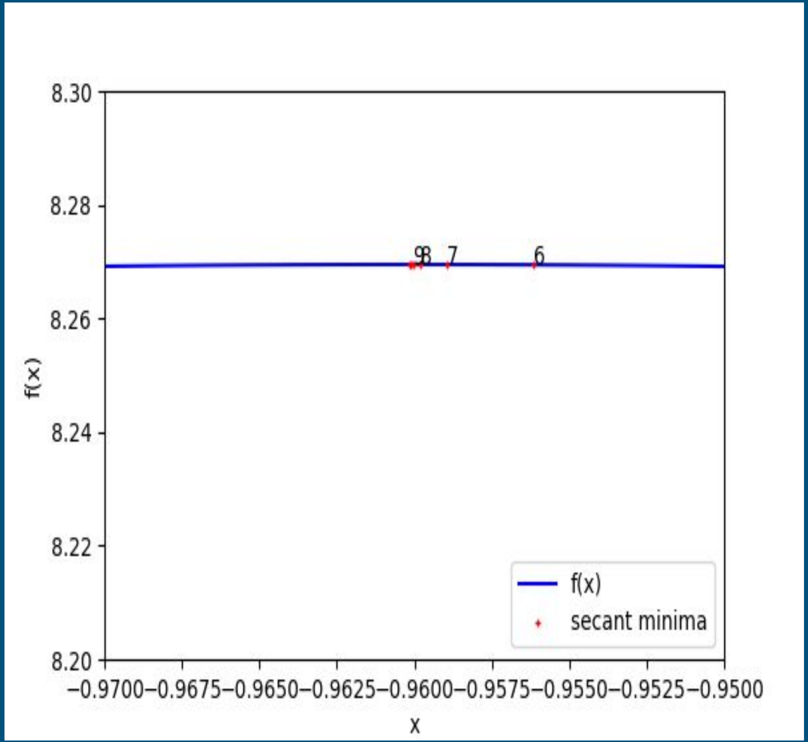


Maximize

$f(x) = 8 + x^3 - 2x - 2e^x$ in interval (-2, 1)

Epsilon = 0.0001
Maxima = -0.960141
Total 11 iterations.

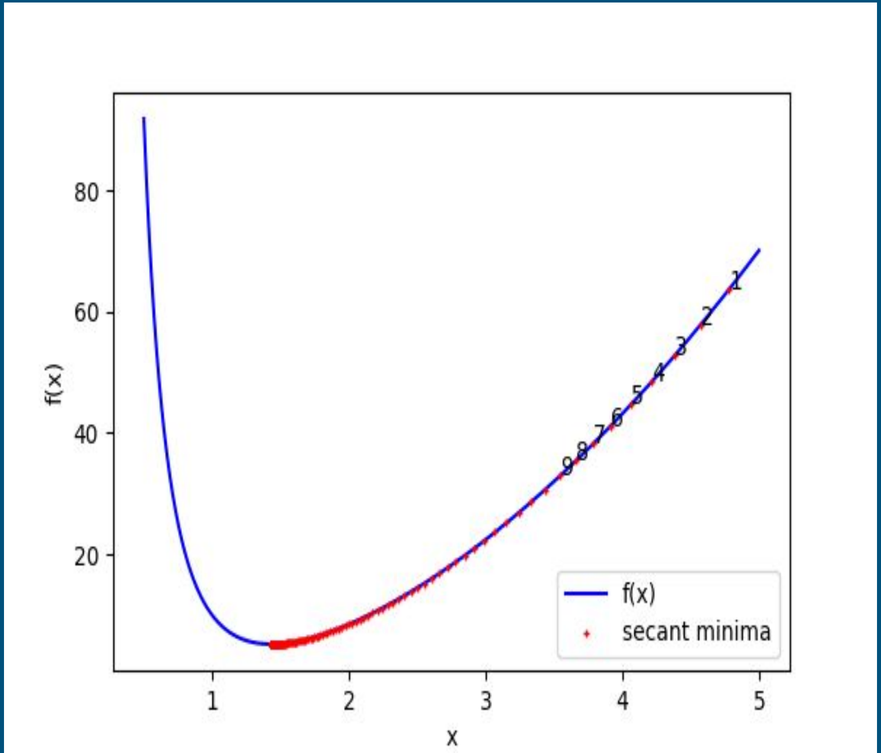


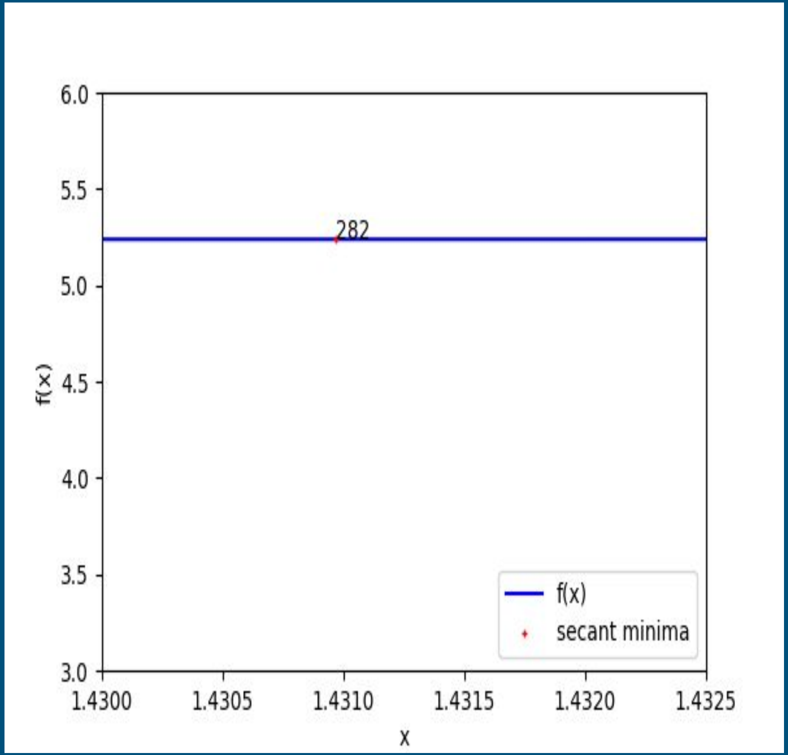


Minimize

$$f(x) = 3x^2 + \frac{12}{x^3} - 5 \quad \text{in interval } (0.5, 5)$$

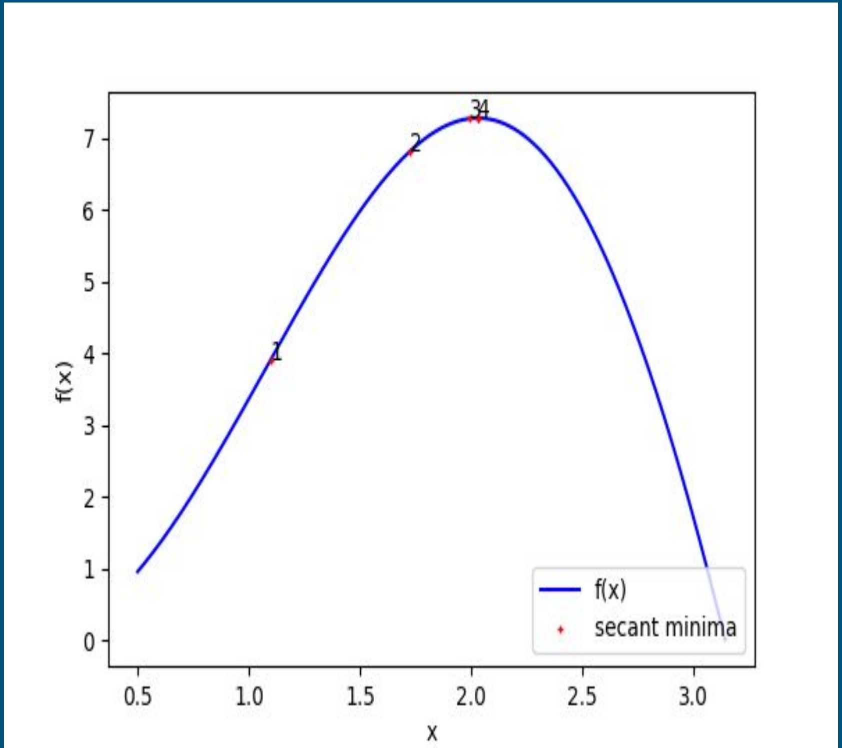
Epsilon = 0.0001
Minima = 1.43097
Total 282 iterations.

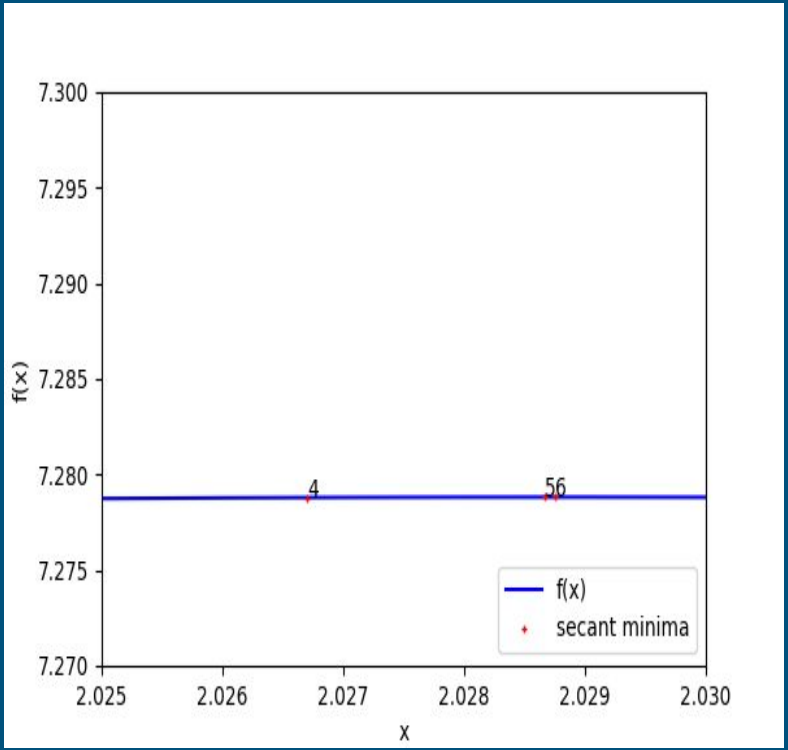




Maximize
$f(x) = 4x(\sin x)$ in interval $(0.5, \pi)$

Epsilon = 0.0001
Maxima = 2.02875
Total 6 iterations.

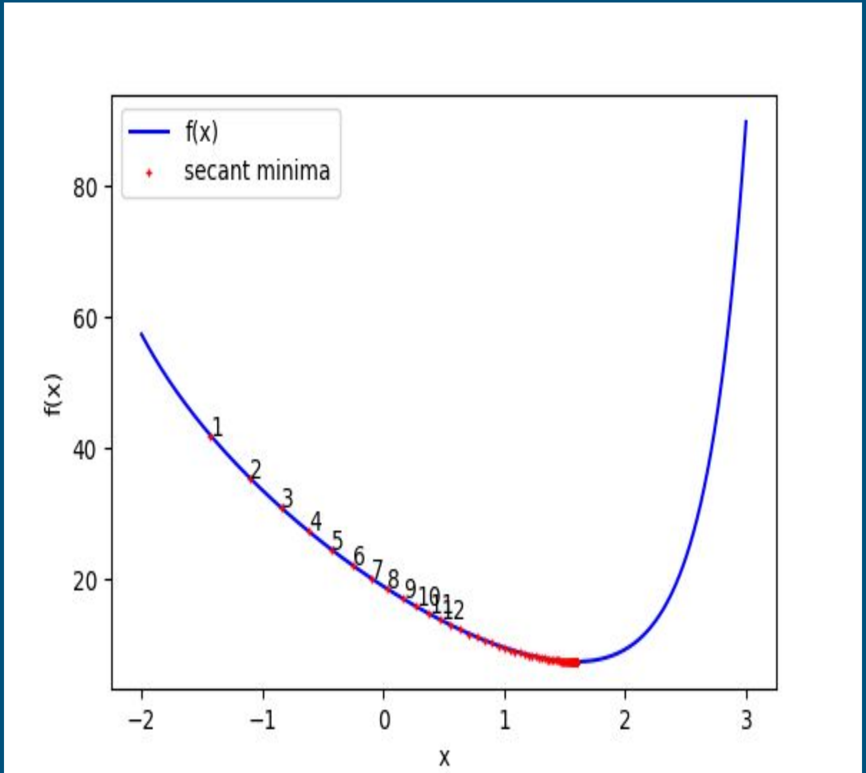


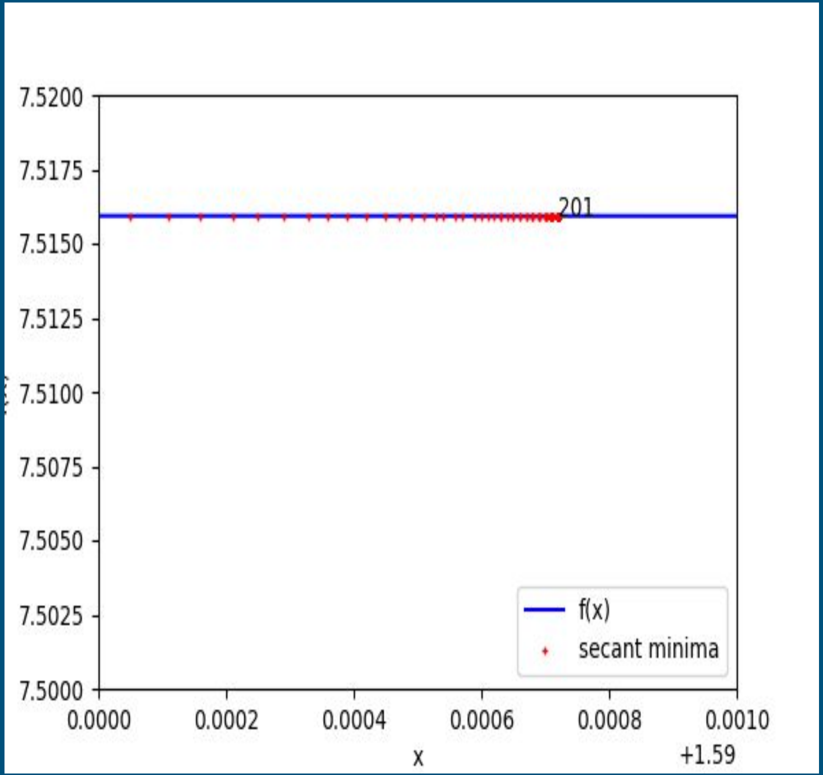


Minimize

$f(x) = 2(x-3)^2 + e^{0.5x^2}$ in interval (-2, 3)

Epsilon = 0.000001
Minima = 1.59072
Total 201 iterations.

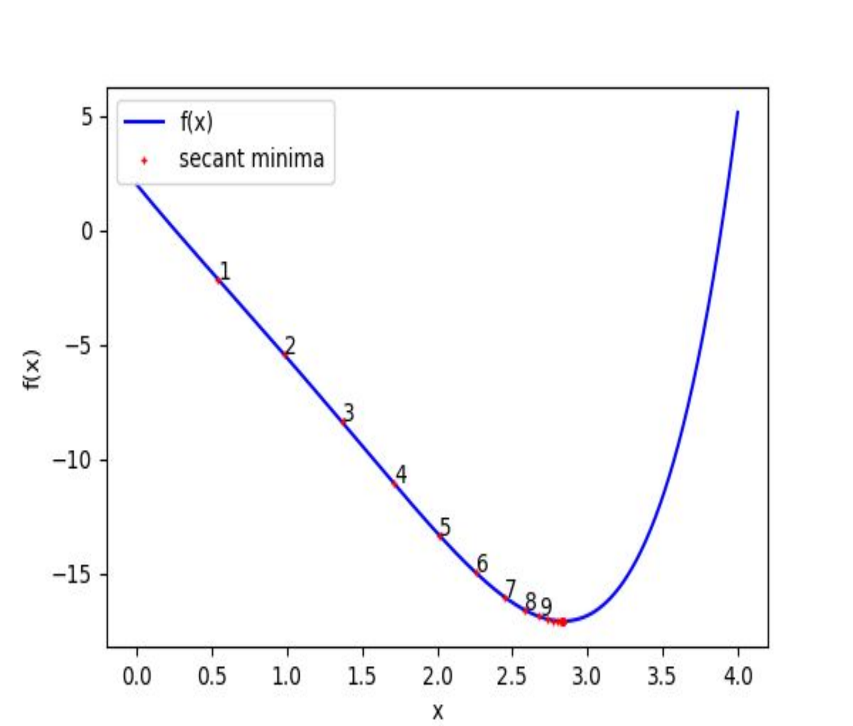


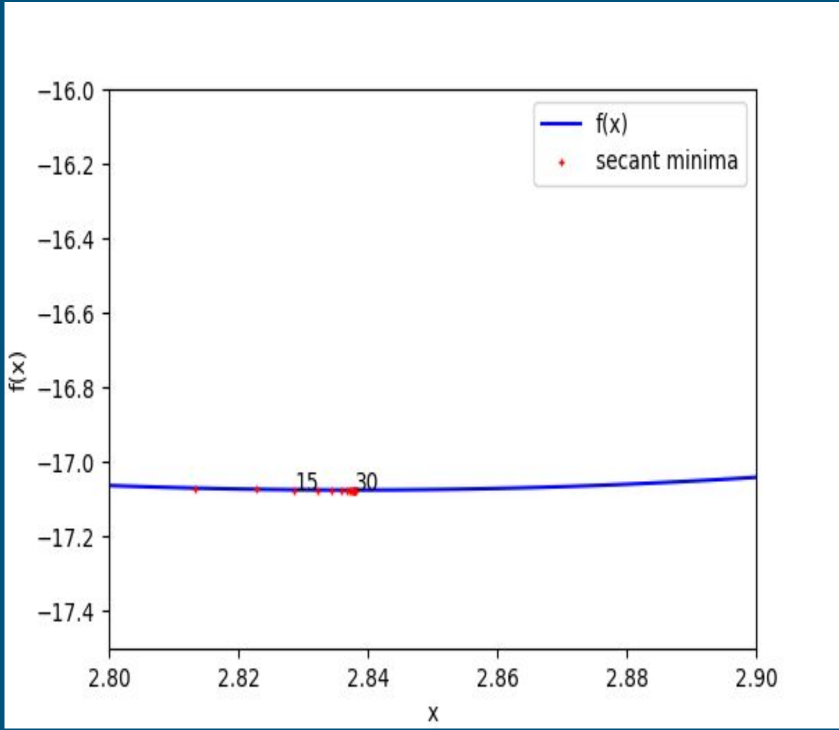


7. Minimize

$$f(x) = 2e^x - x^3 - 10x \quad \text{in interval } (0, 4)$$

Epsilon = 0.0001
Minima = 2.83797
Total 30 iterations.

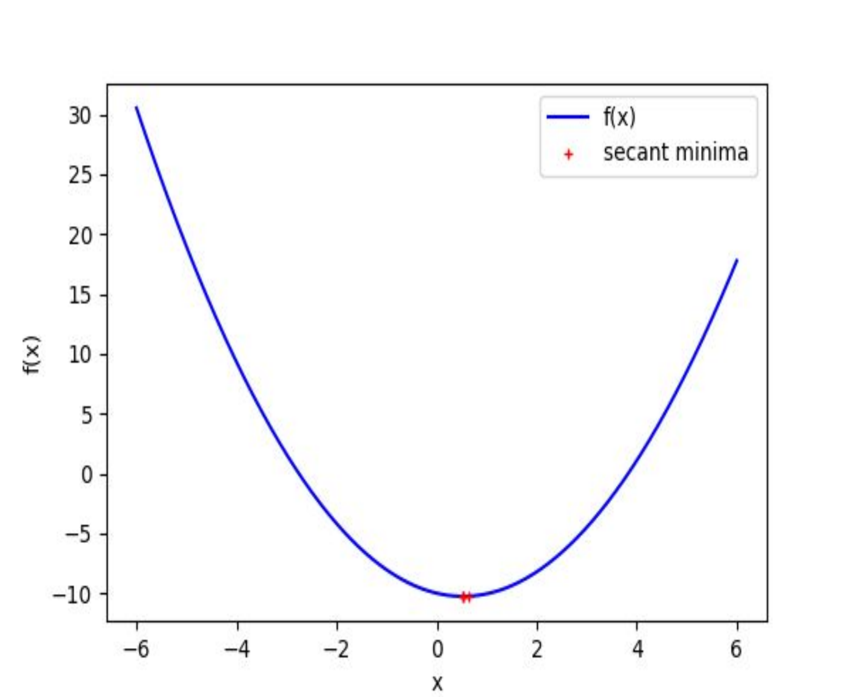


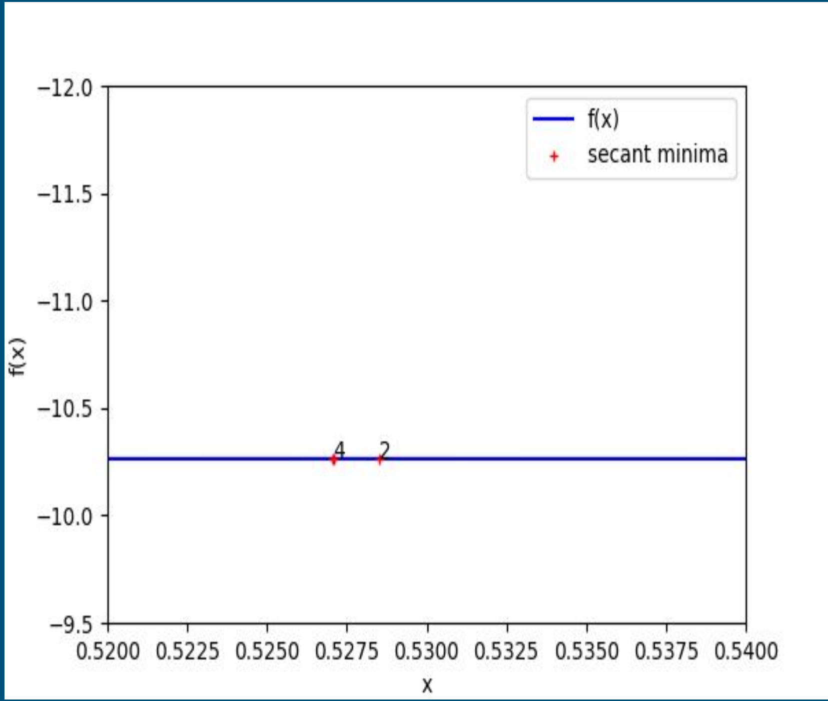


8. Minimize

$$f(x) = x^2 - 10e^{(0.1x)} \quad \text{in interval } (-6, 6)$$

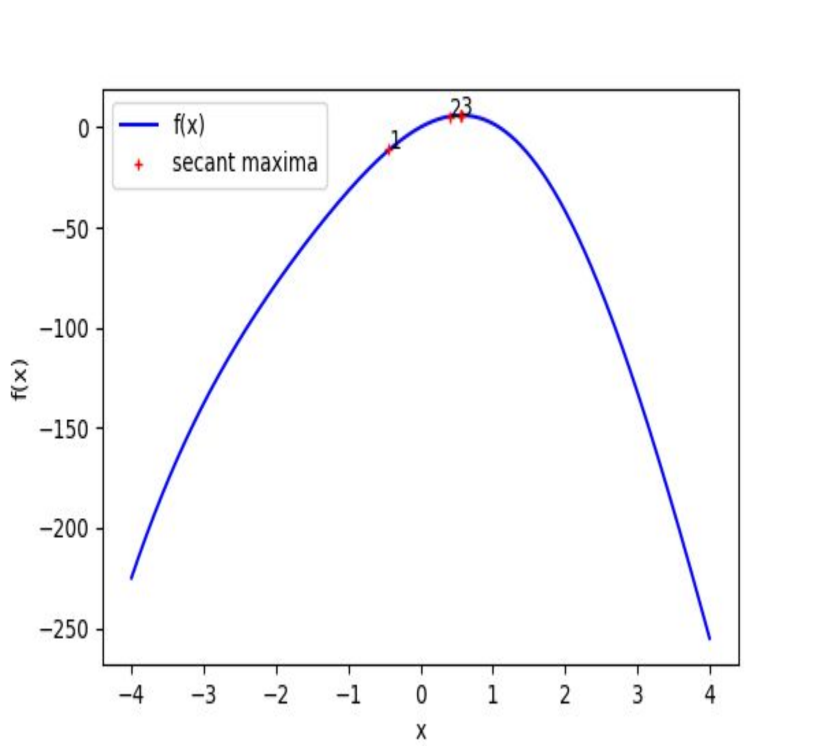
Epsilon = 0.00001
Minima = 0.52706
Total 4 iterations.

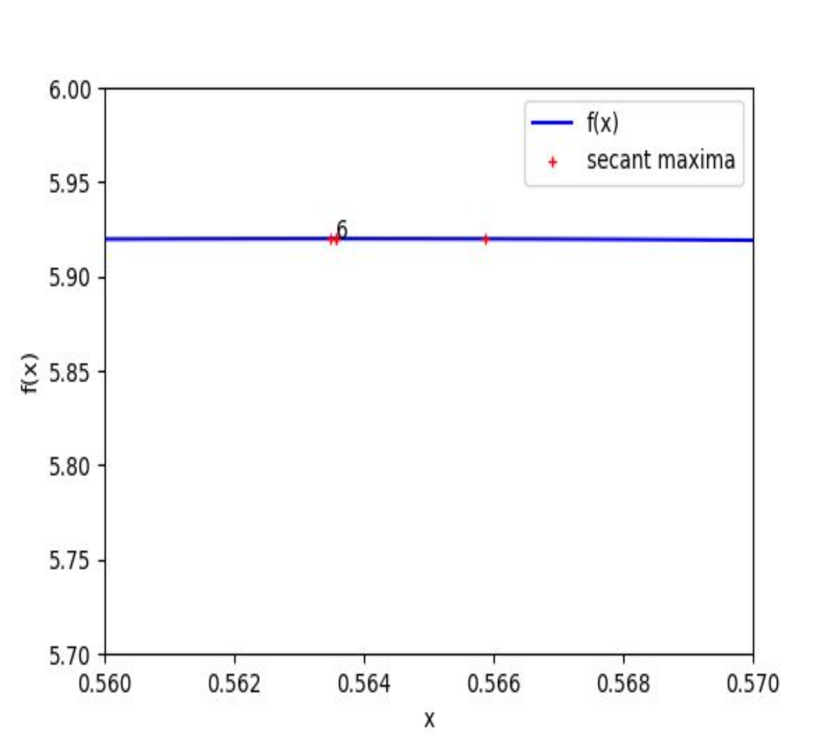




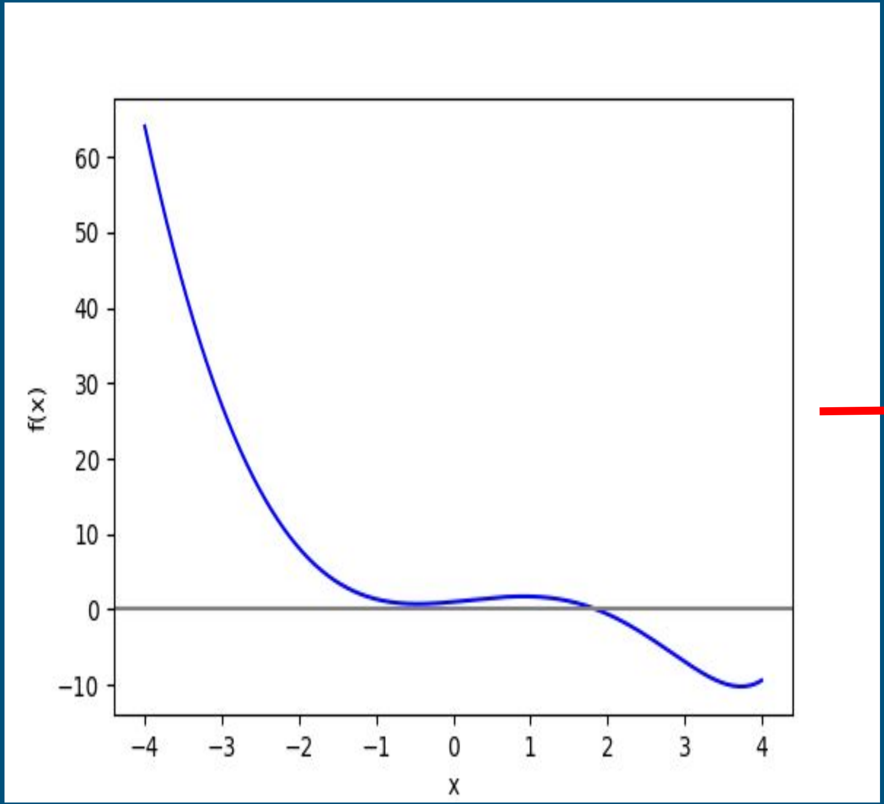
9. Maximize
$f(x) = 20\sin x - 15x^2$ in interval (-4, 4)

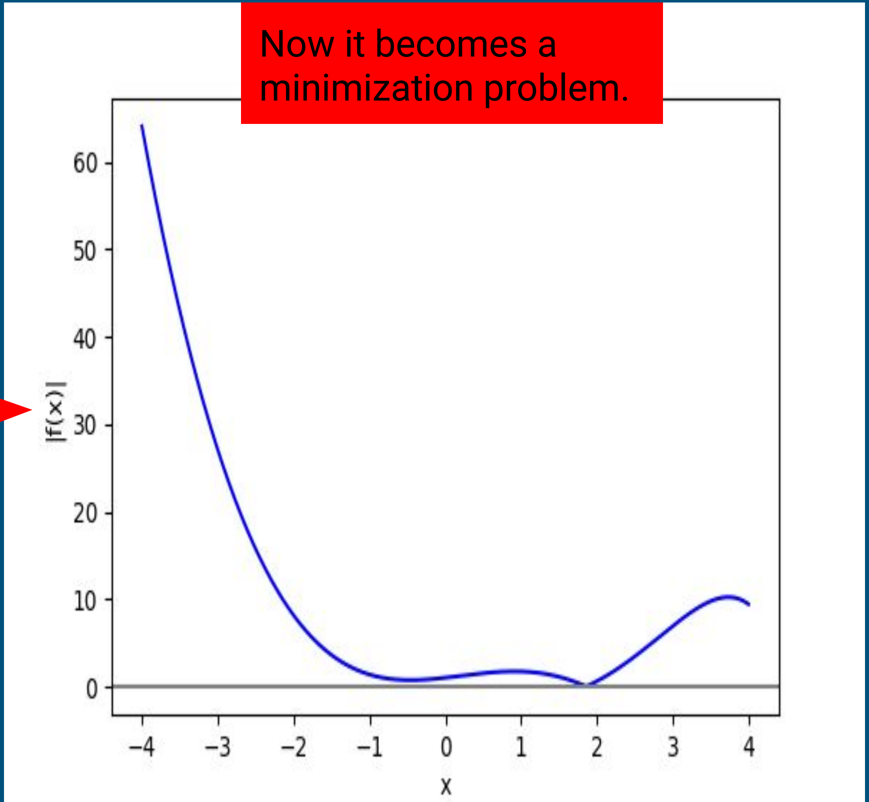
Epsilon = 0.000001
Maxima = 0.563569
Total 6 iterations.





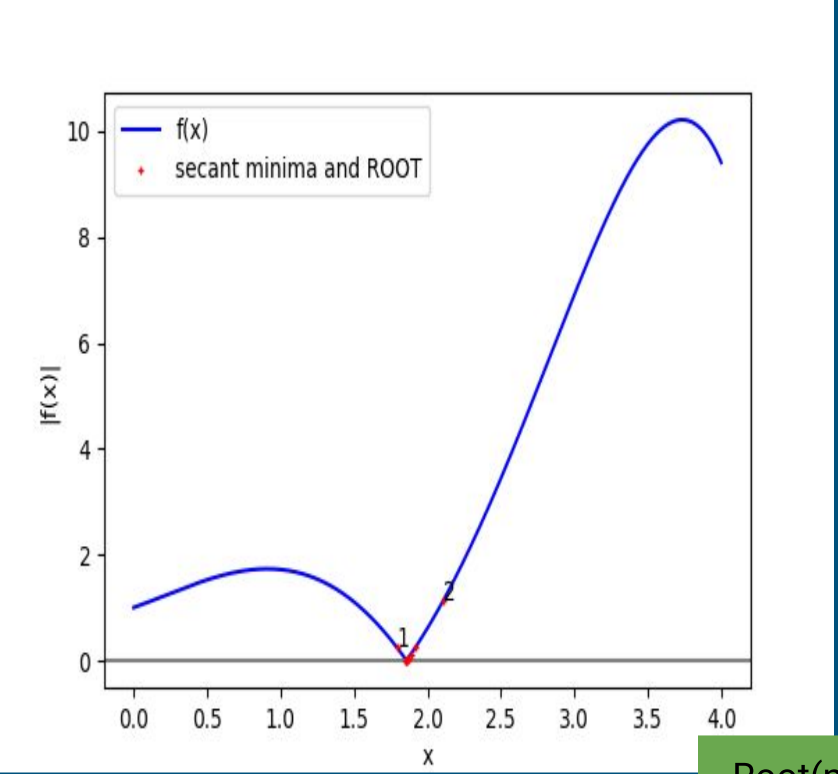
10. Find at least one root of the following function
$f(x) = e^x - x^3$

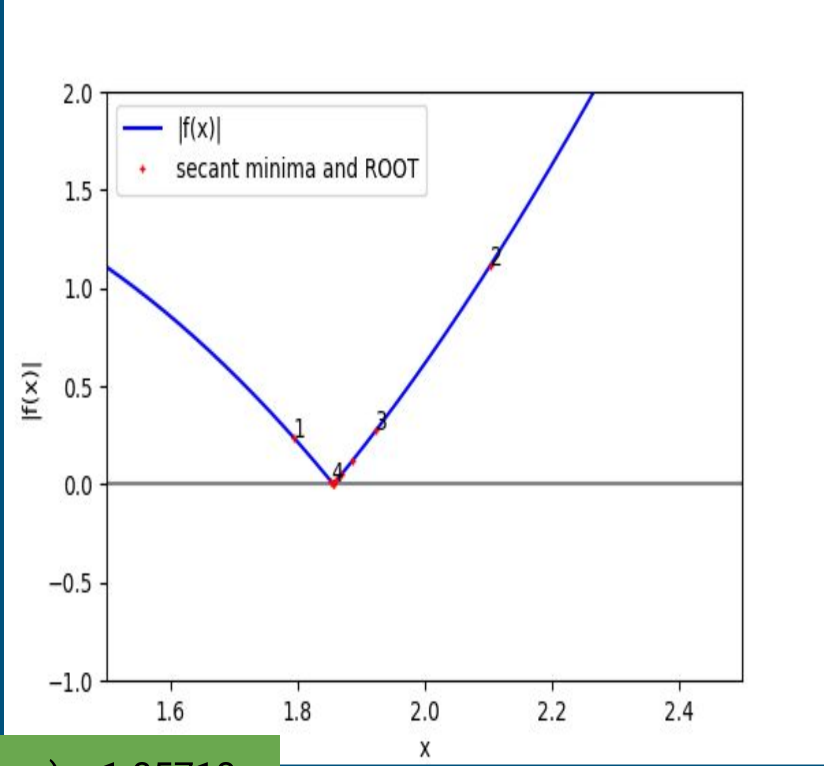




Bracketing Method: Bounding Phase Method
x(0) = 1.2, Delta = 0.1, epsilon = 0.0001.

The minima lies between:
a = 1.5 and b = 2.7.

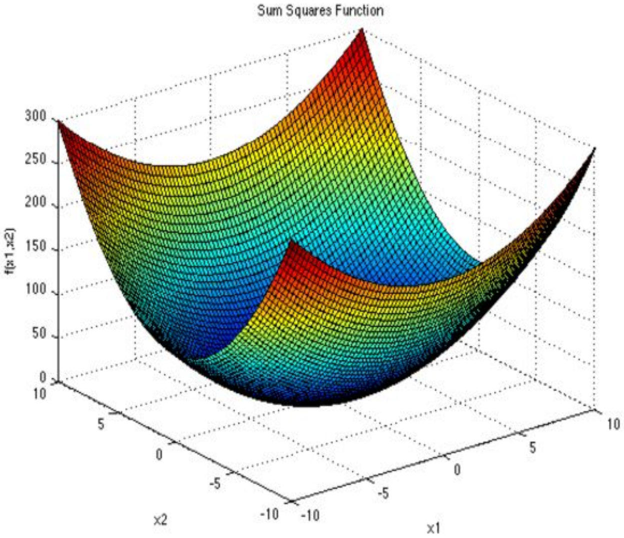




Root(minima) = 1.85718.
Total 11 iterations.

PHASE_2

SUM SQUARES FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d} i x_i^2$$

Total Iterations = 5, Dimension = 5.

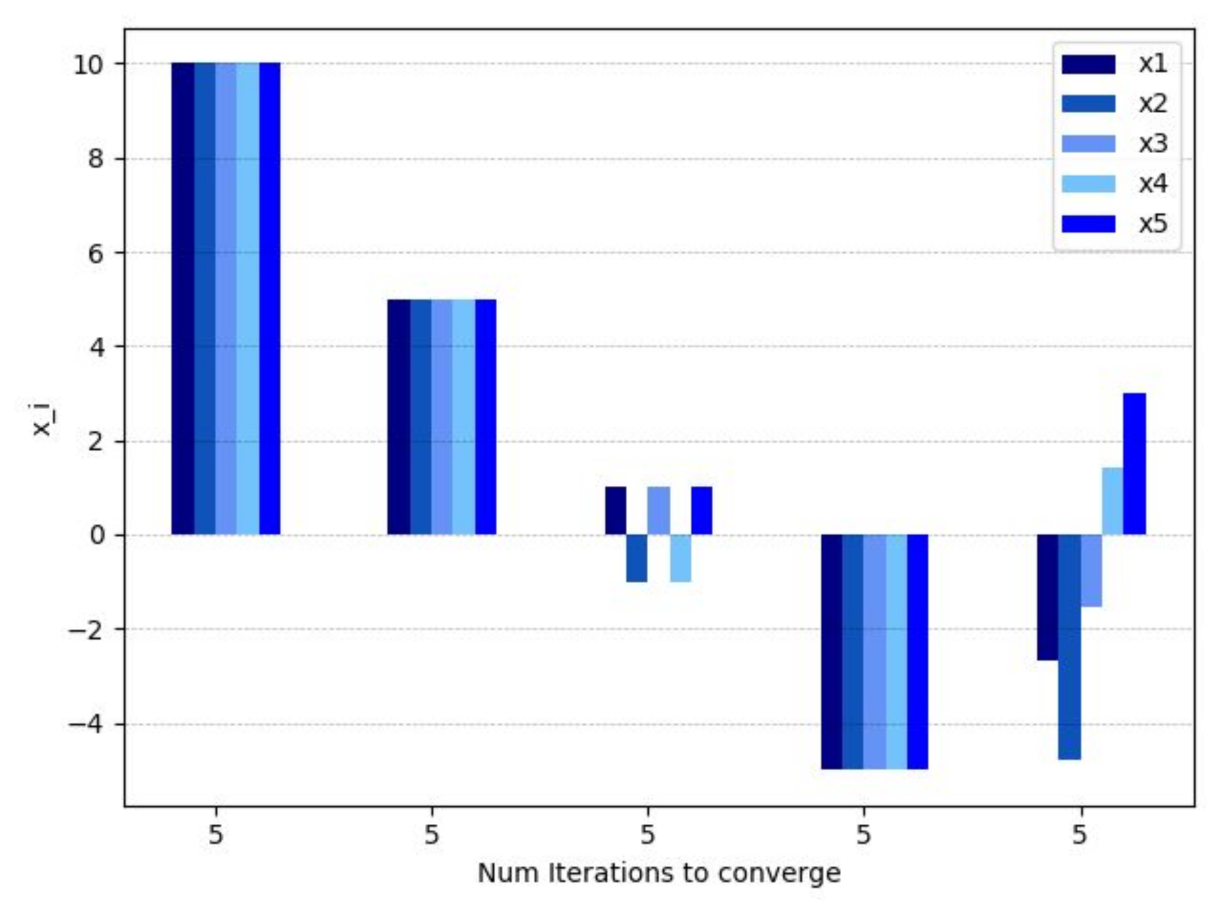
```
Iteration Number -- 5
0
0
0
0
0

0.5     0       0       0       0
0       0.25    0       0       0
0       0       0.166   0       0
0       0       0       0.125   0
0       0       0       0       0.1
```

Inverse of Hessian

Analysis

Only 1 minima x* = (0,0,0,0,0) = Global Minima

F(x*)=0

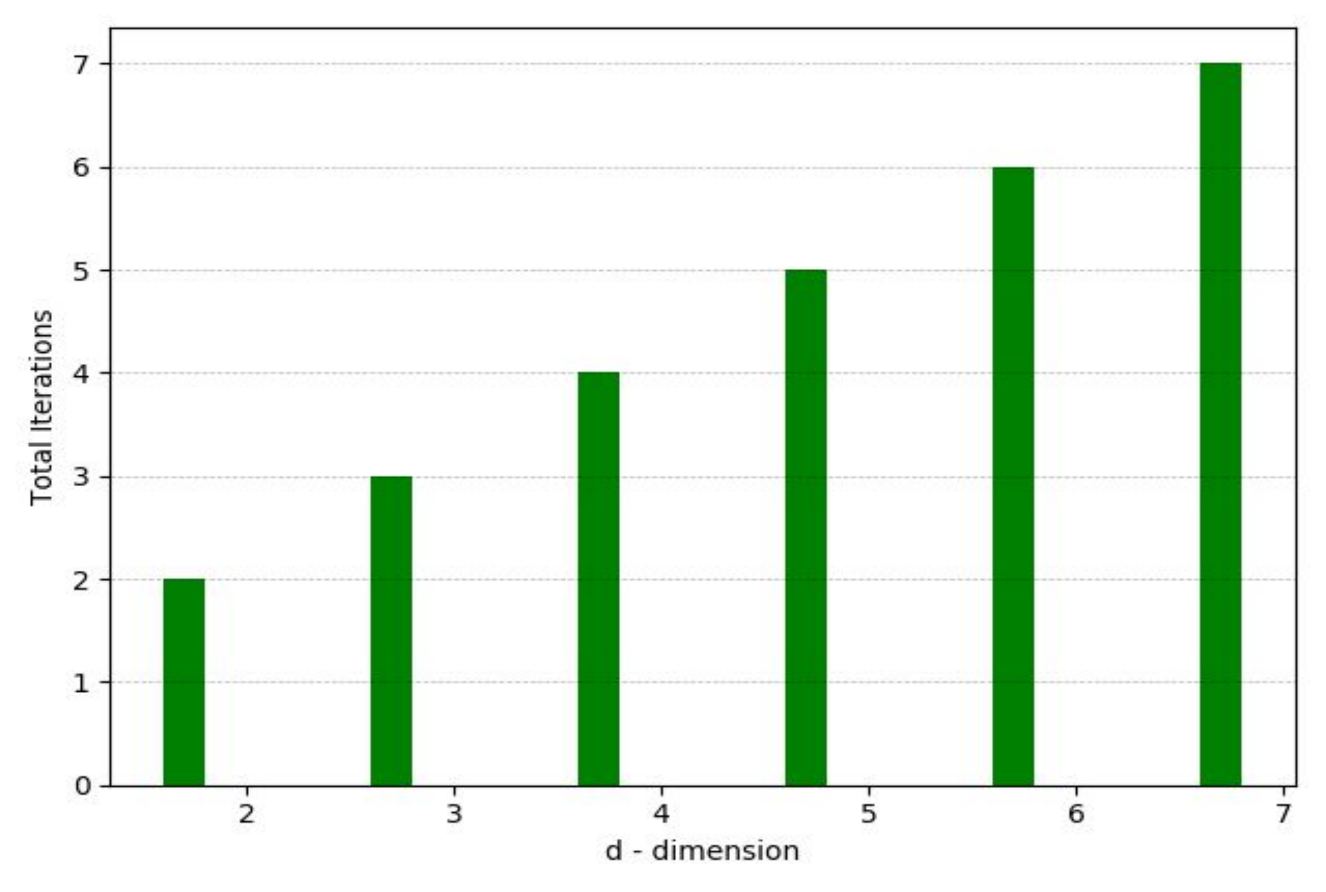


Total Gradient Evaluation = 6

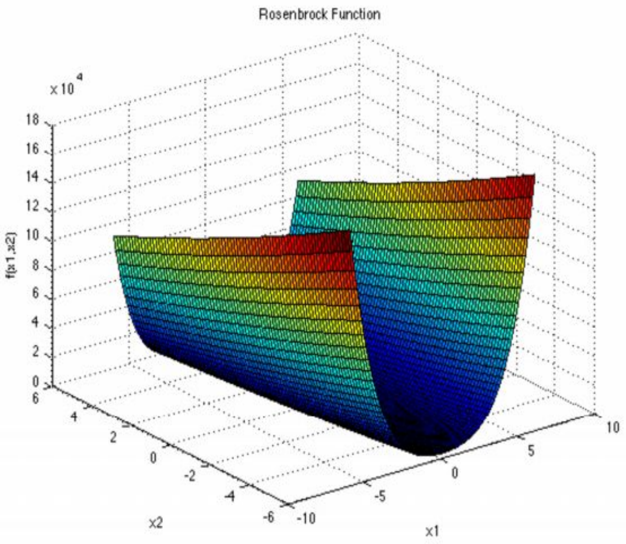
*Converged for all possible value of (x1, x2, x3, x4, x5) in [-10, 10] as initial guess.

No HESSIAN Evaluations.

Initial Guess x0 = (1,1,1,..)



ROSENBROCK FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$$

Total Iterations = 18, Dimension = 3

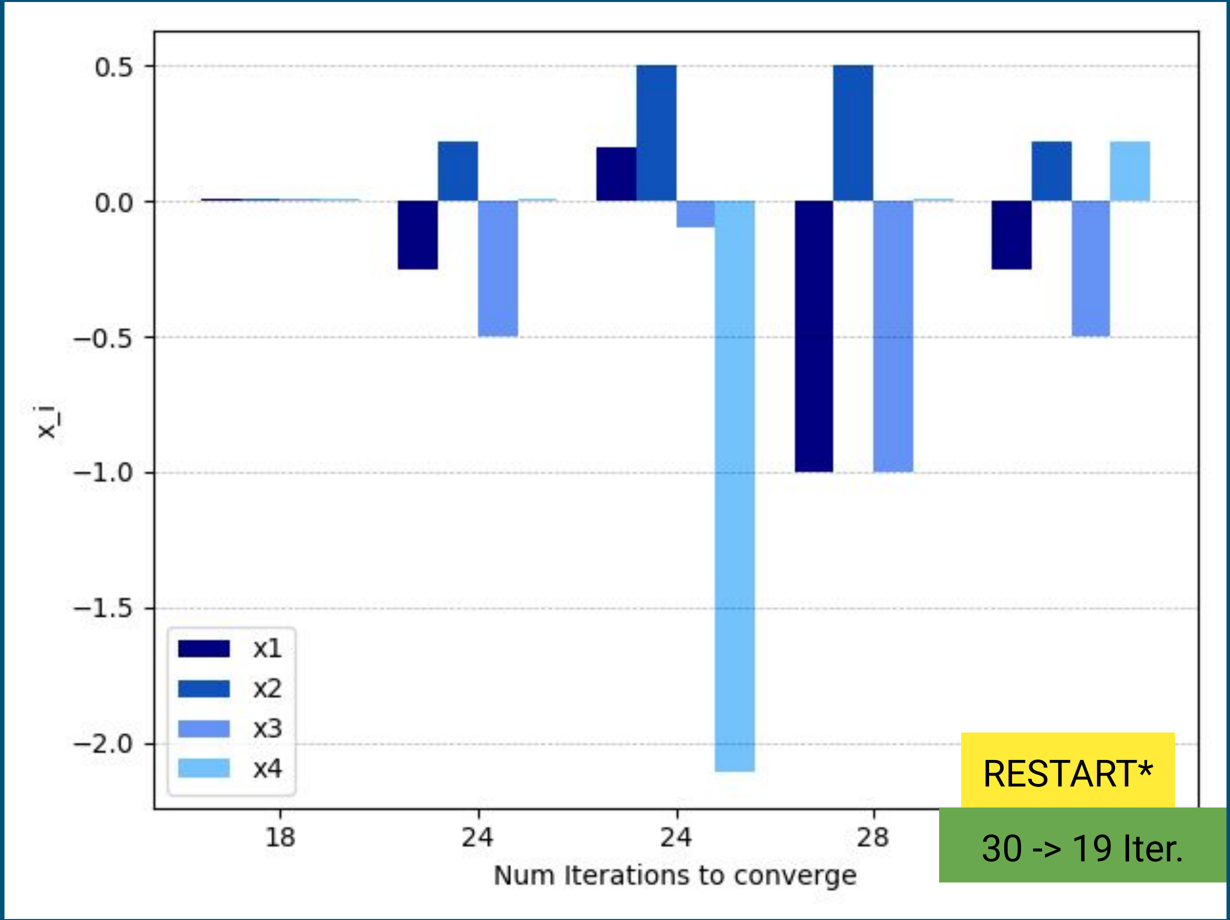
```
Iteration Number -- 18
0.99987
0.99975
0.99951

0.092   0.181   0.362
0.181   0.363   0.725
0.362   0.725   1.454
```

Inverse of Hessian

Analysis

Global Minima x* = (1,1,1)

F(x*)=0

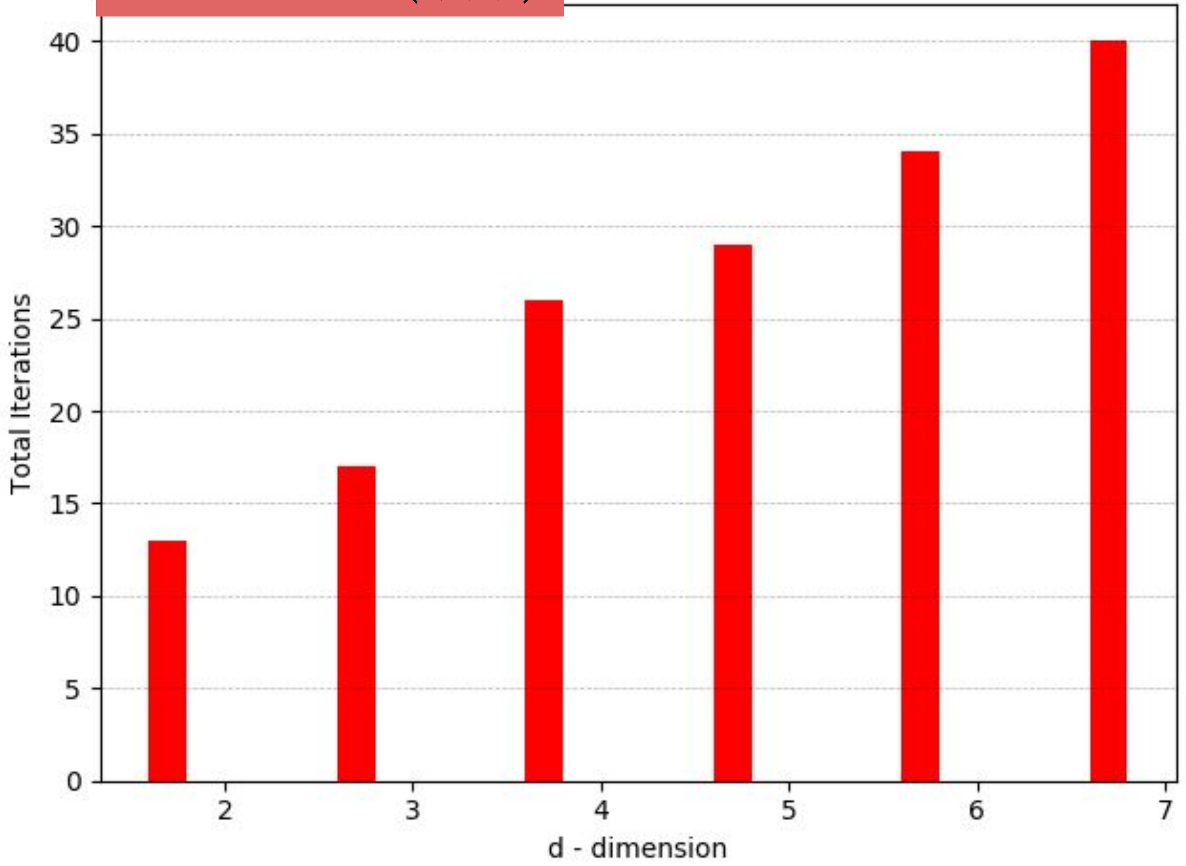


RESTART*

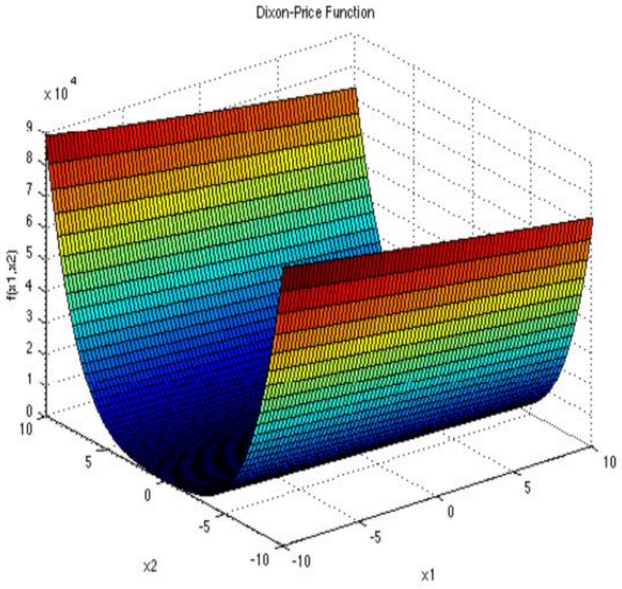
30 -> 19 Iter.

Time Consuming

Initial Guess x0 = (1,1,1,..)



DIXON-PRICE FUNCTION



$$f(\mathbf{x}) = (x_1 - 1)^2 + \sum_{i=2}^{d} i\left(2x_i^2 - x_{i-1}\right)^2$$

Total Iterations = 7, Dimension = 4.

```
Iteration Number -- 7
0.99999
0.7071
0.5946
0.54525

0.494    0.175    0.072    0.034
0.175    0.093    0.038    0.017
0.072    0.038    0.045    0.021
0.034    0.017    0.021    0.035
```

Inverse of Hessian

Total Gradient Evaluation = 9

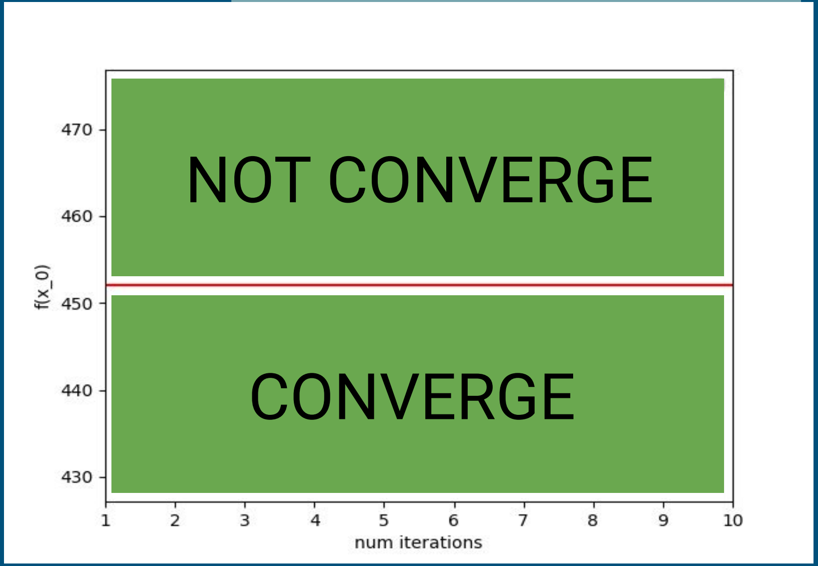
Analysis

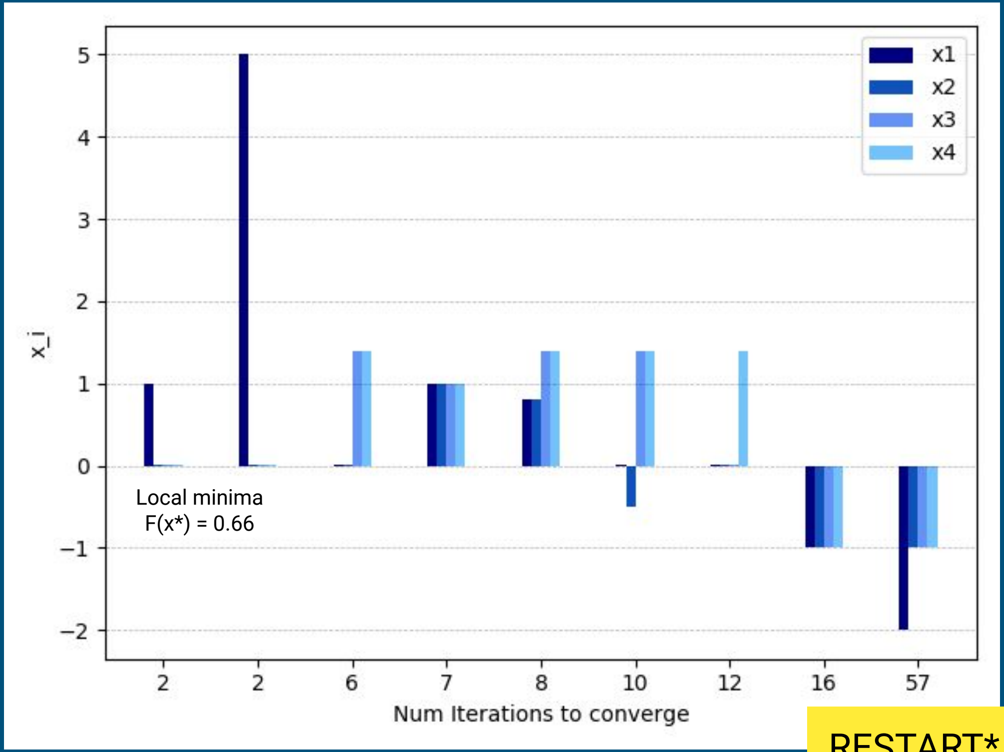
Objective Function symmetric wrt X4 so there are 2 global minima

x1* = (0.999, 0.707,0.594, 0.545); x2* = (0.999, 0.707,0.594, -0.545)

F(x1*)=F(x2*)=0

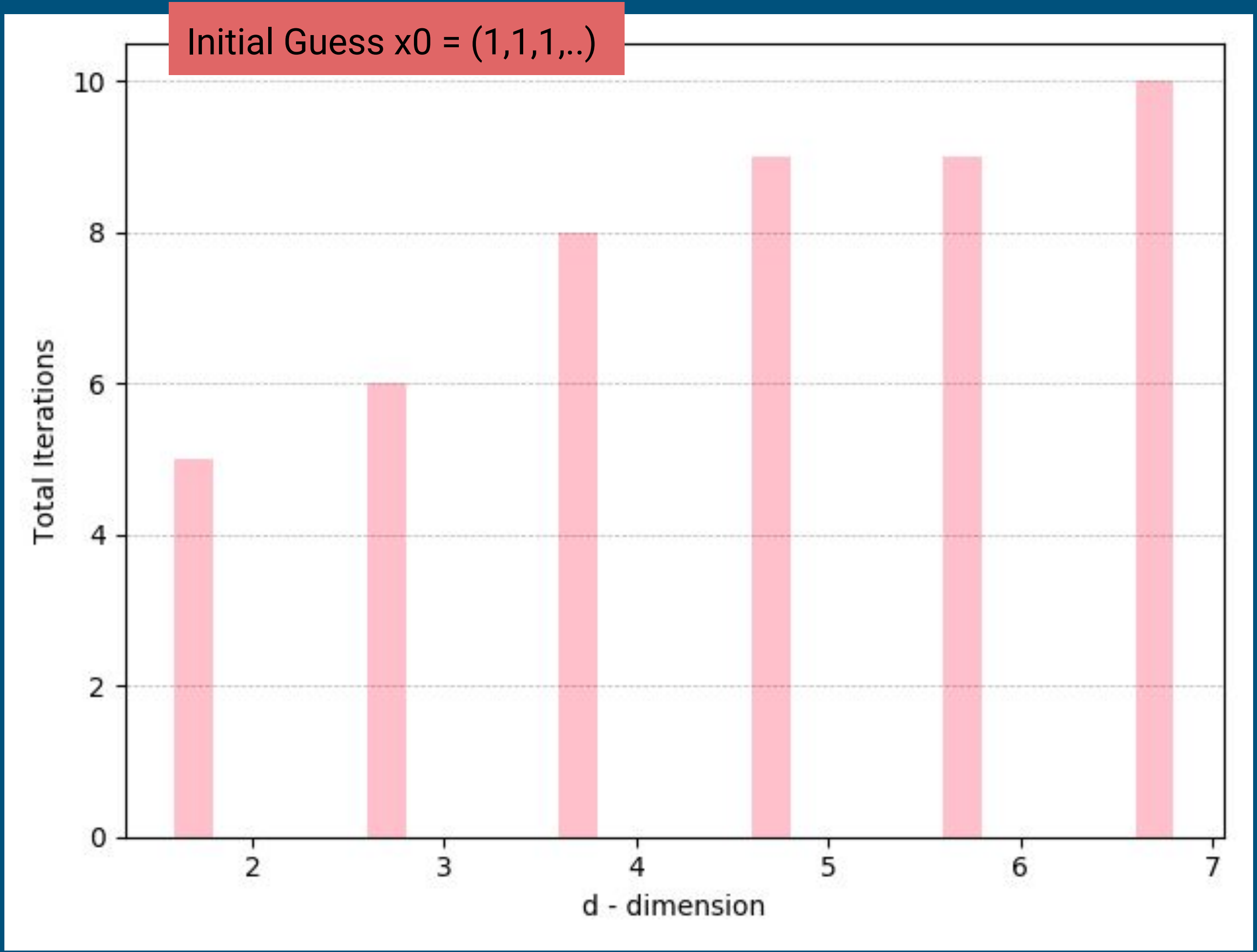
Local Optima x_* = (0.333,0,0,0)



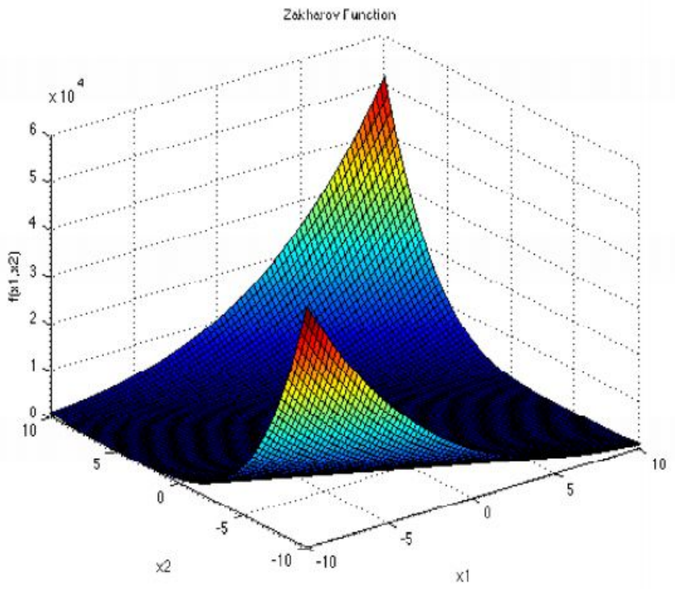


RESTART*

57 -> 11 Iter.


Initial Guess x0 = (1,1,1,..)
Total Iterations
d - dimension
0
2
4
6
8
10
2
3
4
5
6
7

ZAKHAROV FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d} x_i^2 + \left(\sum_{i=1}^{d} 0.5 i x_i\right)^2 + \left(\sum_{i=1}^{d} 0.5 i x_i\right)^4$$

Total Iterations = 3, Dim. = 3.

```
Iteration Number -- 2
-0.00012
6e-05

Iteration Number -- 3
0
0

0.444   -0.111
-0.111  0.277
```

Inverse of Hessian

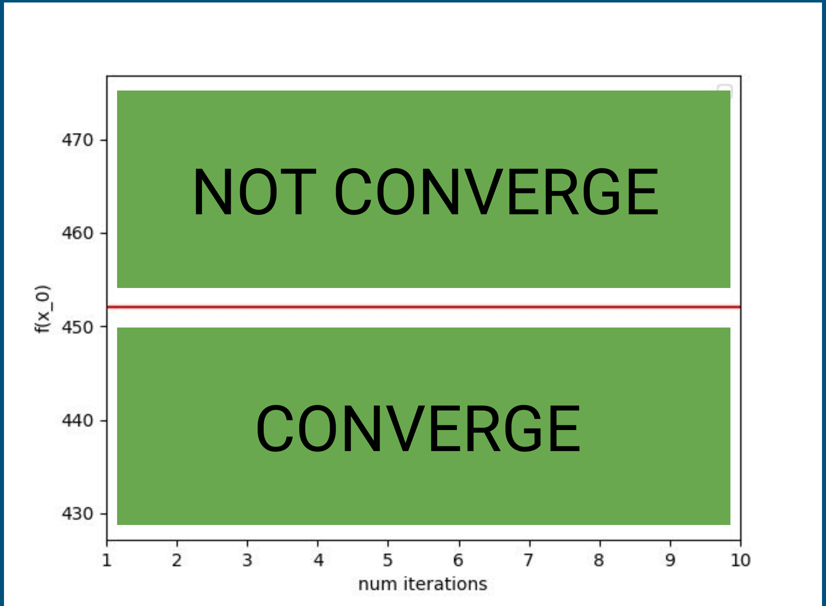
Total Gradient Evaluation = 5

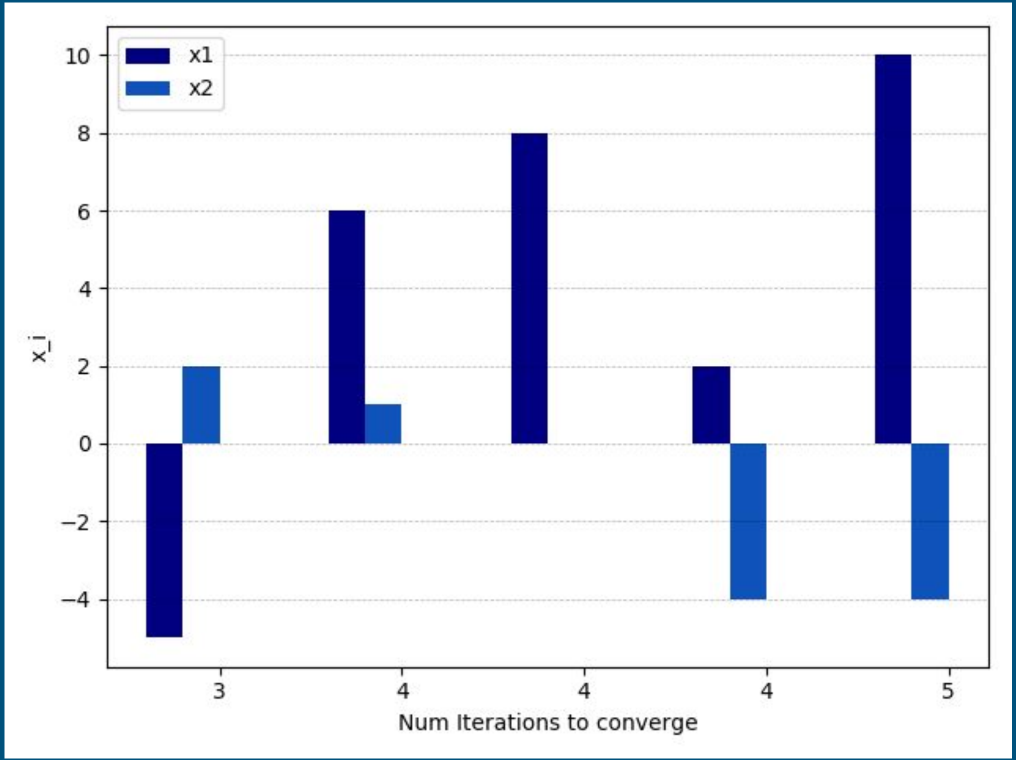
Analysis

Only 1 minima x* = (0,0) = Global Minima

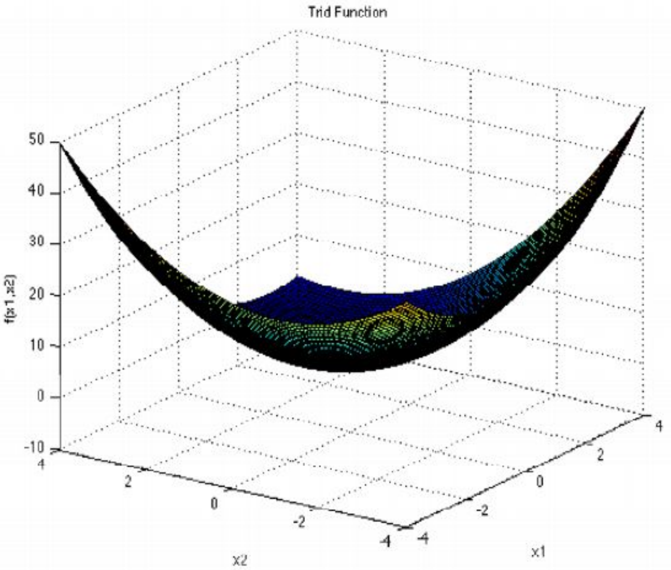
F(x*)=0

[-5, 10]





TRID FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d}(x_i - 1)^2 - \sum_{i=2}^{d} x_i x_{i-1}$$

Total Iterations = 6, Dimension = 6.

```
Iteration Number -- 6
6
9.99999
12
12
10
5.99999

0.857   0.714   0.571   0.428   0.285   0.142
0.714   1.428   1.142   0.857   0.571   0.285
0.571   1.142   1.714   1.285   0.857   0.428
0.428   0.857   1.285   1.714   1.142   0.571
0.285   0.571   0.857   1.142   1.428   0.714
0.142   0.285   0.428   0.571   0.714   0.857
```

Inverse of Hessian

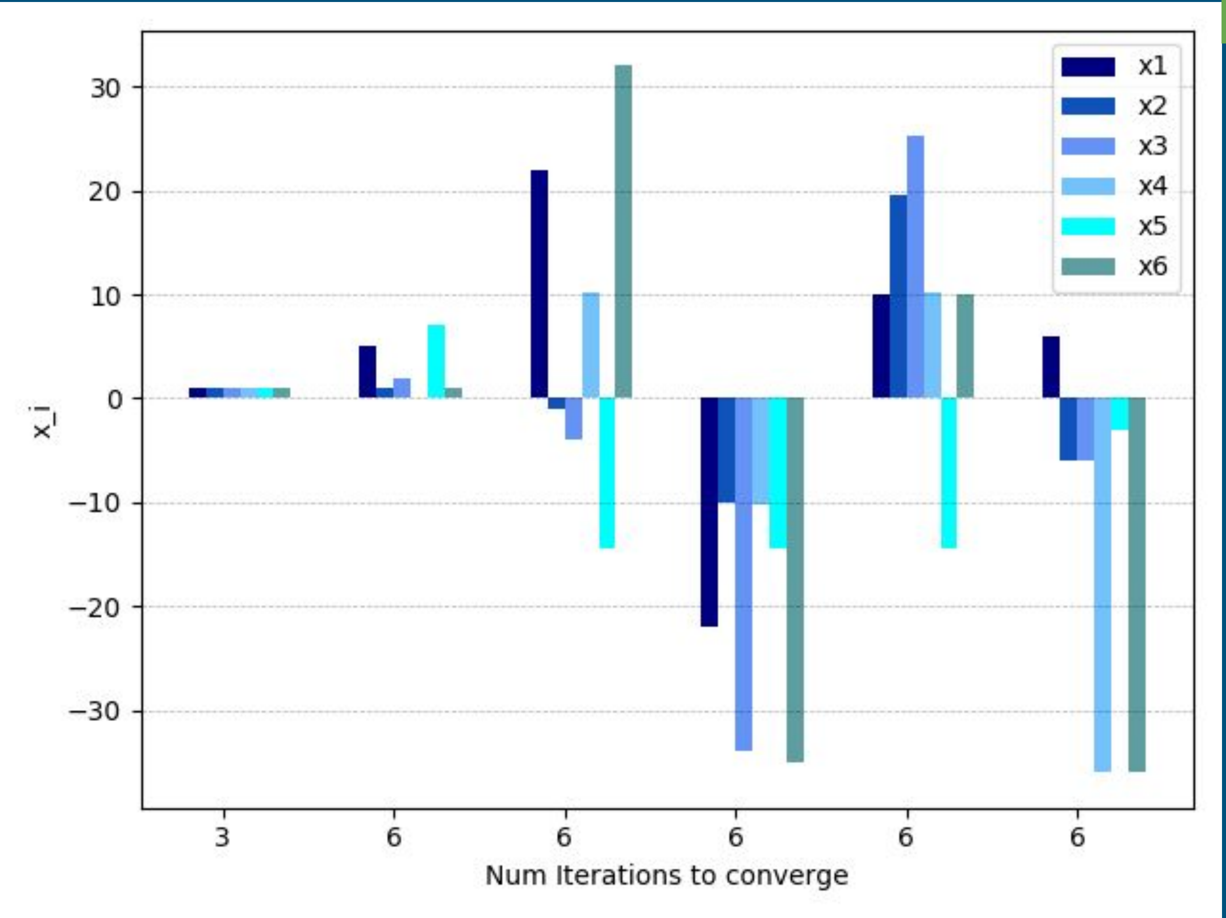
Total Gradient Evaluation = 8

Analysis

Only 1 minima x* = (6,10,12,12,10,6) = Global Minima

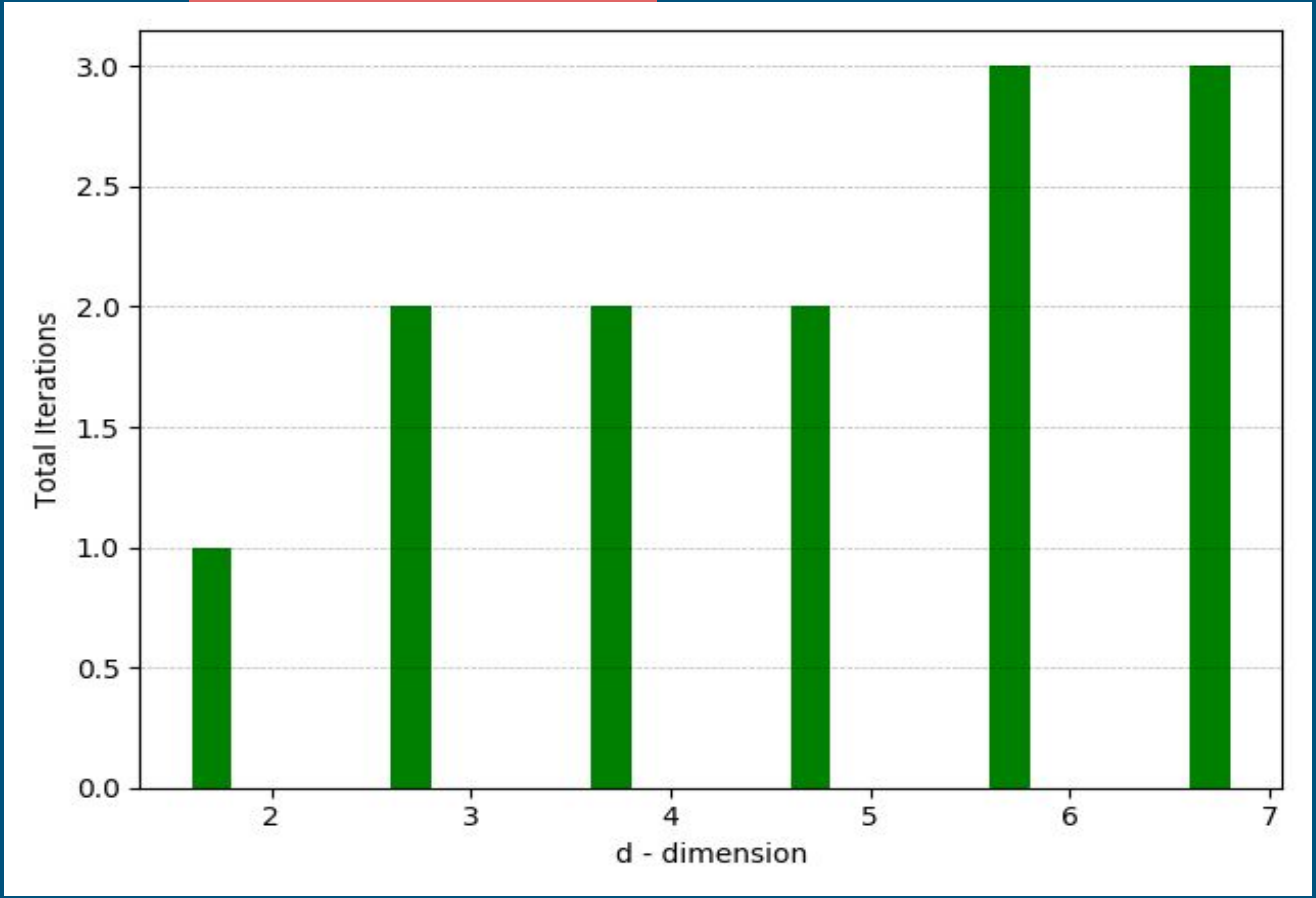
[-36, 36]

F(x*)=-50



*Converged for all possible value of (x1, x2, x3, x4, x5, x6) in [-36, 36] as initial guess.

Initial Guess x0 = (1,1,1,..)



PHASE_3

Phase_1

Bracketing Method → Bounding Phase method.

Accurate Method → Secant method.

Phase_2

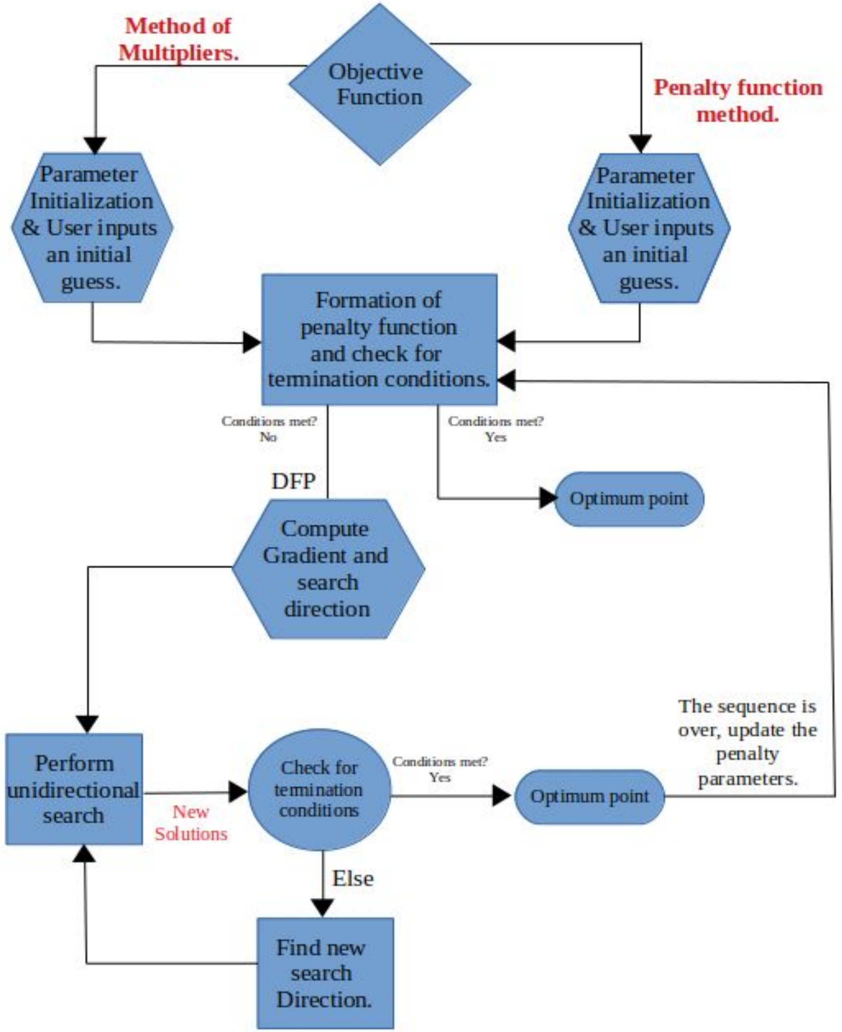
Multivariable optimization using DFP method.

Phase_3

Refashioning of Unconstrained Optimization techniques for Constrained Optimization using Variable Metric Method.

1. Penalty Function Method.
2. Method of Multipliers.

Flowchart of the Algorithm.



Computational Complexity of the Algorithm.

$$O = numseq * (JN + KN + (n_{dfp} + 1)(n_{bracket} + n_{secant}N) + n_{dfp}N)$$

Himmelblau Function

$$\text{Minimize} \quad (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

subject to

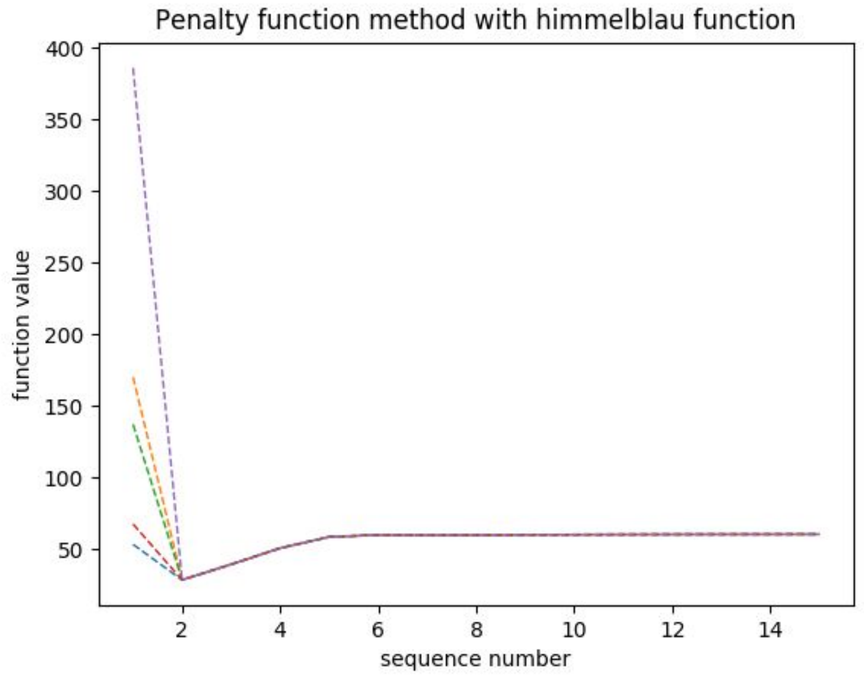
$$(x_1 - 5)^2 + x_2^2 - 26 \geq 0, \qquad x_1, x_2 \geq 0.$$

Penalty function method:

Initial Guess	Obtained optima	Number of Sequences	Function value	F_global
3.001 0.115	0.897945 2.93661	9	60.1153	60.372
0 0	0.835535 2.93429	14	60.3343	60.372
0.22 0.9	0.897945 2.93661	9	60.1153	60.372
0.22 2.9	0.897945 2.93661	9	60.1153	60.372
2.62 4.9	0.897945 2.93661	9	60.1153	60.372
Function Value				
Best	Worst	Median	Mean	Std. Dev.
60.3343	60.1153	60.1153	60.1591	0.087

R = 0.07
C = 1.55

Convergence Plots:

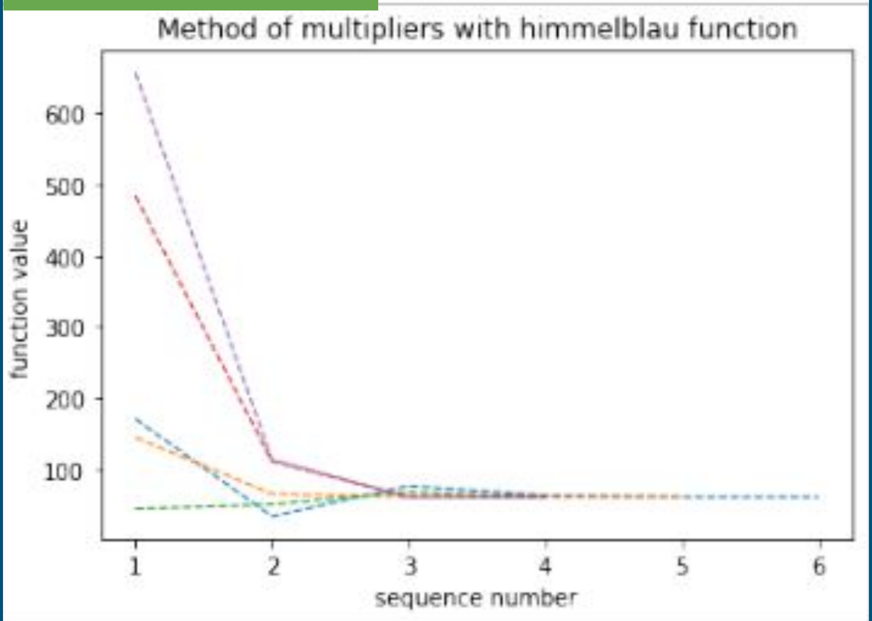


Method of Multipliers:

Initial Guess	Obtained optima	Number of Sequences	Function value	F_global
3.001 0.115	0.828717 2.93394	4	60.4013	60.372
0 0	0.834829 2.94209	8	60.3881	60.372
0.22 0.8	0.832493 2.93855	5	60.3824	60.372
4.25 2.66	0.832961 2.94157	4	60.4165	60.372
4.88 2.66	0.833486 2.94249	4	60.4197	60.372
Function Value				
Best	Worst	Median	Mean	Std. Dev.
60.4197	60.3824	60.4013	60.4015	0.014

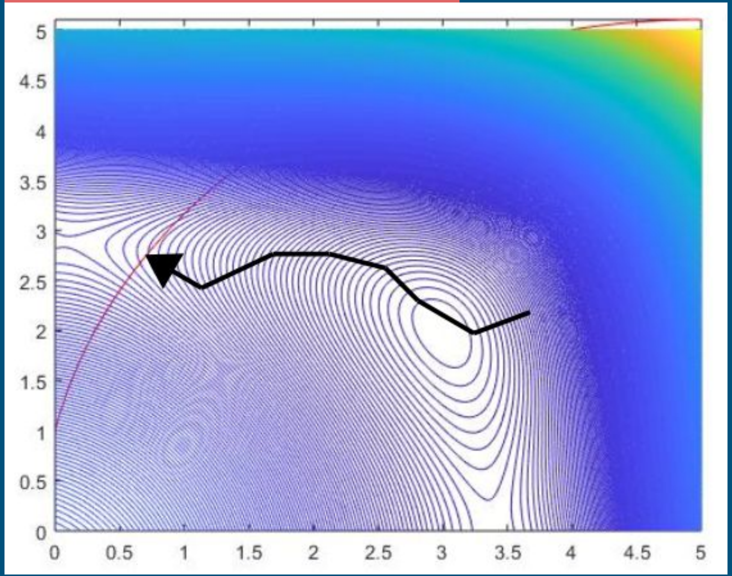
R = 0.1

Convergence Plots:

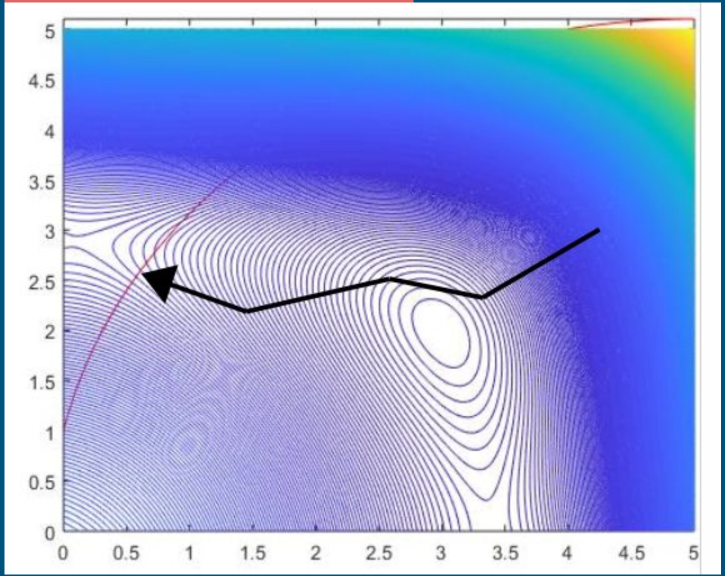


Comparison Contour Plots:

Penalty function method:



Method of Multipliers:



Penalty function method:

Initial Guess	Obtained optima	Number of Sequences	Function value	F_global
20 33	13.6401 0.0	6	-7951.38	-7973.0
65 2	13.8511 0.0	18	-7942.89	-7973.0
99 55	13.0084 0.0	3	-7925.55	-7973.0
22.0 22.2	14.8116 0.0	10	-7888.61	-7973.0
52.03 1.66	13.8515 0.0	28	-7942.87	-7973.0
14.055 13.0	13.6433 0.0	6	-7951.36	-7973.0
14 0.3	15.8741 0.0	26	-7797.32	-7973.0
33.89 56.98	13.6521 0.0	13	-7943.19	-7973.0
100 0	13.8515 0.0	27	-7942.87	-7973.0
66 32	13.6435 0.0	5	-7951.5	-7973.0
Function Value				
Best	Worst	Median	Mean	Std. Dev.
-7951.5	-7797.32	-7942.88	-7923.753	41.8189

Problem_1

$$\min f(x) = (x_1 - 10)^3 + (x_2 - 20)^3,$$

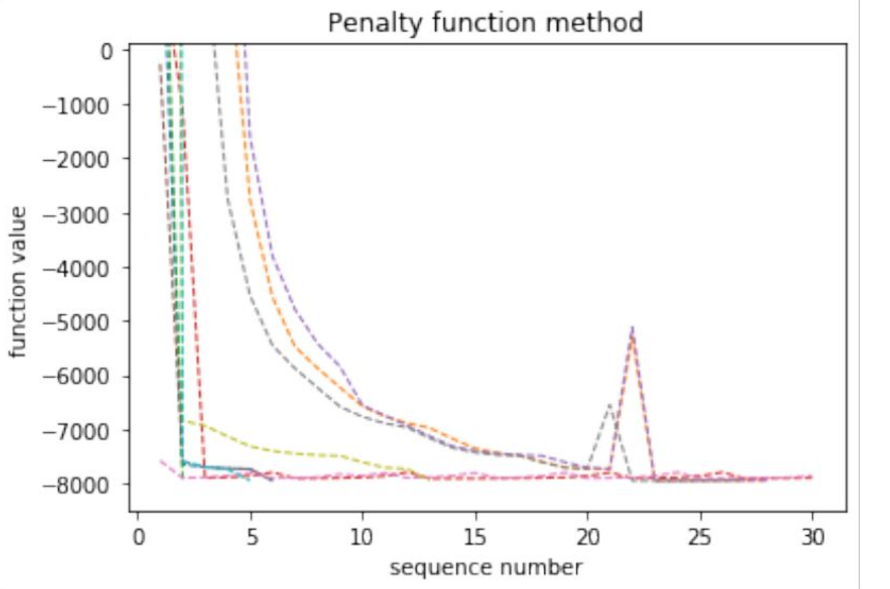
$$\textit{subject to } g_1(x) = (x_1 - 5)^2 + (x_2 - 5)^2 - 100 \geq 0,$$

$$g_2(x) = 82.81 - (x_1 - 5)^2 - (x_2 - 5)^2 \leq 0,$$

$$13 \leq x_1 \leq 100, \qquad 0 \leq x_2 \leq 100.$$

- Number of variables: 2 variables.
- The global minima: x* = (14.095,0.84296), f(x*) = -6961.81388.

Convergence Plots:

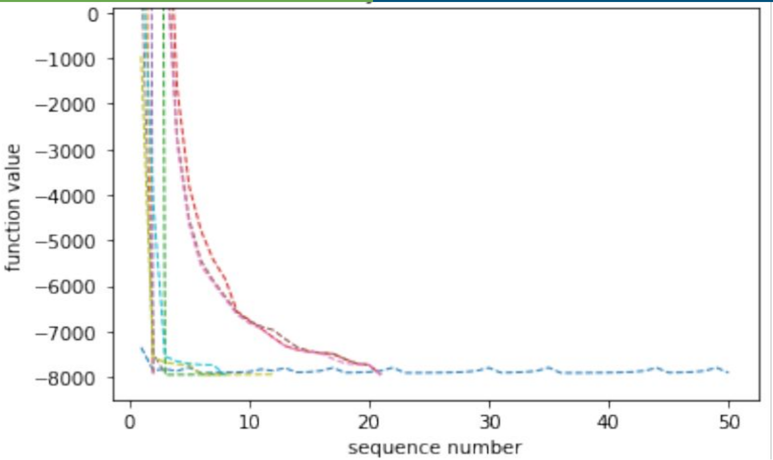


R = 0.05
C = 2

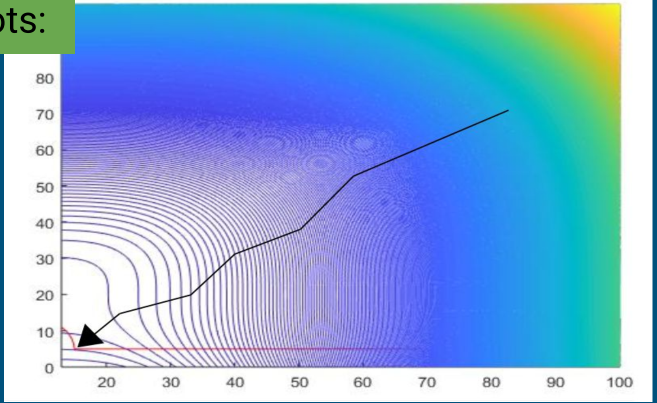
Method of Multipliers:

Initial Guess	Obtained optima	Number of Sequences	Function value	F_global
22.22 11.11	13.0946 0.0	3	-7932.77	-7973.0
100 0	13.5994 0.0	21	-7953.57	-7973.0
14.0 0.5	14.5455 0.0	30	-7908.44	-7973.0
30.33 12.05	13.0097 0.0	5	-7960.9	-7973.0
50 50	13.8473 0.0	8	-7943.06	-7973.0
87.099 0.99	13.6039 0.0	21	-7953.41	-7973.0
48.00 15.66	13.0097 0.0	4	-7960.9	-7973.0
99.66 33.99	13.6071 0.0	21	-7953.3	-7973.0
13 10	13.8516 0.0	12	-7942.91	-7973.0
23.32 12.21	13.6604 0.0	8	-7950.23	-7973.0
Function Value				
Best	Worst	Median	Mean	Std. Dev.
-7960.9	-7908.44	-7951.764	-7945949	14.937

Convergence Plots:



Contour Plots:



R (penalty parameter) = 0.1

Problem_2

$$\max f(x) = \frac{\sin^3(2\pi x_1)\sin(2\pi x_2)}{x_1^3(x_1+x_2)},$$

$$\text{subject to } g_1(x) = x_1^2 - x_2 + 1 \le 0,$$

$$g_2(x) = 1 - x_1 + (x_2-4)^2 \le 0,$$

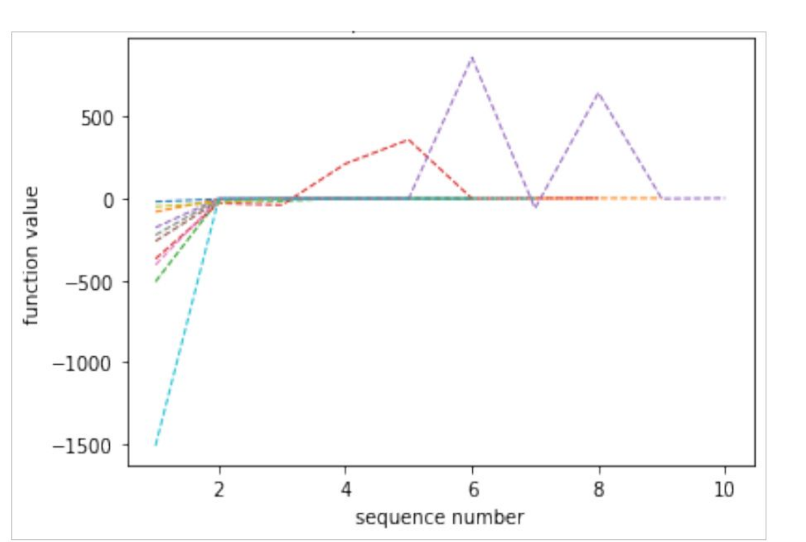
$$0 \le x_1 \le 10, \quad 0 \le x_2 \le 10$$

- Number of variables: 2 variables.
- The global minima: x∗ = (1.2279713, 4.2453733) , f (x∗) = 0.095825.

Penalty function method:

Initial Guess	Obtained optima	Number of Sequences	Function value	F_global
1.1 1.1	1.23116 4.2544	7	2.87713	2.87072
6.1 1.5	1.73675 4.75387	8	1.22582	2.87072
6.1 0.1	1.73675 4.75381	4	1.22581	2.87072
5.1 3.1	1.73672 4.75376	4	1.22581	2.87072
0.5 0.5	1.73686 4.75357	5	1.2258	2.87072
1.0 7.5	1.23166 4.25408	10	2.87707	2.87072
2.1 2.9	1.73676 4.7539	7	1.22582	2.87072
1.5 2.06	1.23112 4.25453	8	2.87714	2.87072
1.05 8.95	1.73675 4.75386	4	1.22581	2.87072
1.0001 1.9999	1.73678 4.75388	5	1.22581	2.87072
Function Value				
Best	Worst	Median	Mean	Std. Dev.
2.87714	1.2258	1.22581499	1.721202	0.75672

Convergence Plots:

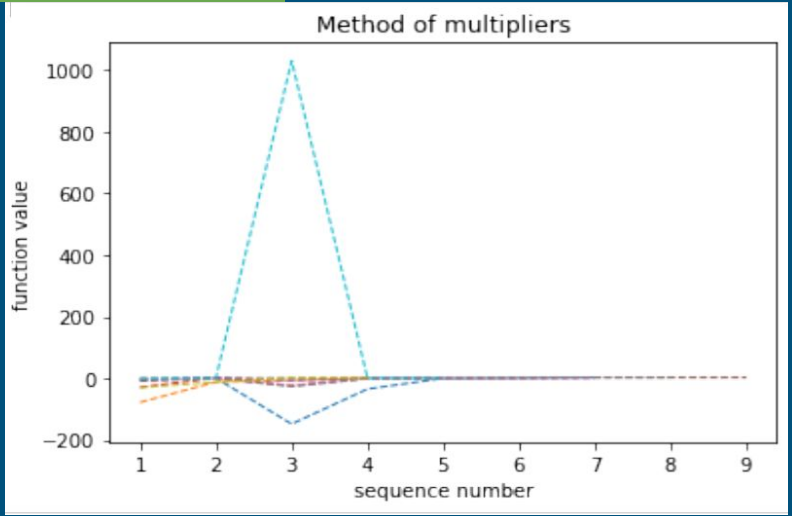


R = 0.05
C = 10

Method of Multipliers:

Initial Guess	Obtained optima	Number of Sequences	Function value	F_global
1.0001 1.9999	1.49213 3.2997	6	0.0001667	2.87072
1.1 1.1	1.23113 4.2545	9	2.87714	2.87072
5 5	1.00007 3.99168	6	0	2.87072
4.25 1.47	1.23101 4.25443	8	2.87714	2.87072
0.33 0.66	1.23106 4.25452	10	2.87714	2.87072
3.55 0.78	1.73676 4.75379	4	1.22581	2.87072
4.5 2.5	1.73676 4.7539	5	1.22582	2.87072
0.5 7.66	1.00009 3.99803	6	0	2.87072
2.654 1.999	1.73673 4.75381	3	1.22582	2.87072
6 1	1.73675 4.7539	7	1.22581	2.87072
Function Values				
Best	Worst	Median	Mean	Std. Dev.
2.87714	0.0	1.22582	1.4919	1.1534

Convergence Plots:



R = 1.15

Penalty function method:

```
0.5
Sequence number 9
initial function value is 2873.74
341.096
1016.69
1130.01
184.978
673.891
396.445
786.084
698.119
final function value is 2533.09
aakash@aakash-HP-Notebook:~/cpp$
```

Problem_3

$$\min f(x) = x_1 + x_2 + x_3$$

$$\text{subject to } g_1(x) = -1 + 0.0025(x_4 + x_6) \leq 0,$$

$$g_2(x) = -1 + 0.0025(-x_4 + x_5 + x_7) \leq 0,$$

$$g_3(x) = -1 + 0.01(-x_6 + x_8) \leq 0,$$

$$g_4(x) = 100x_1 - x_1x_6 + 833.33252x_4 - 83333.333 \leq 0,$$

$$g_5(x) = x_2x_4 - x_2x_7 - 1250x_4 + 1250x_5 \leq 0,$$

$$g_6(x) = x_3x_5 - x_3x_8 - 2500x_5 + 1250000 \leq 0,$$

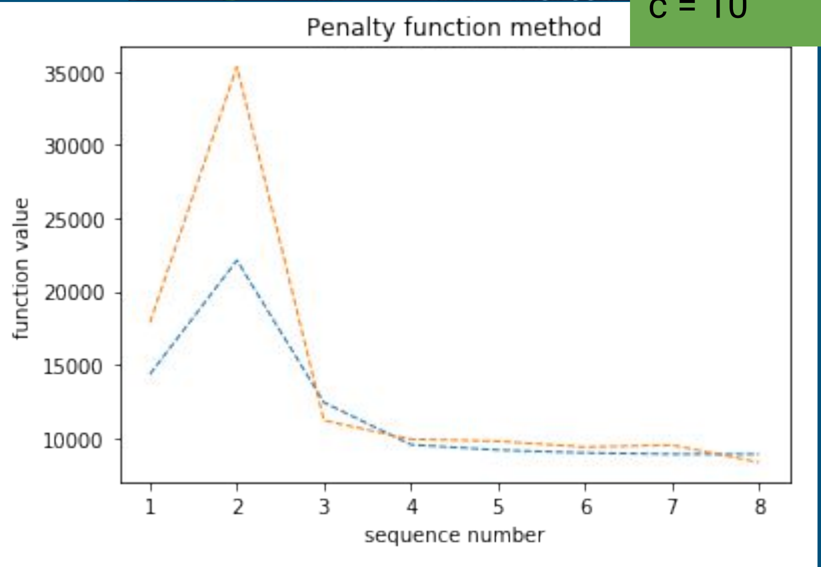
$$100 \leq x_1 \leq 10000$$

$$1000 \leq x_i \leq 10000, i = 2, 3$$

$$10 \leq x_i \leq 1000, i = 4, 5, \dots, 8$$

- Number of variables: 8 variables.
- The global minima: x∗ = (579.3167, 1359.943, 5110.071, 182.0174, 295.5985, 217.9799, 286.4162,395.5979), f (x∗) = 7049.3307.

R = 0.000000005
c = 10



x(0) = (354,1111,1233,400,400,400,400,322)

Best F_value = 3400