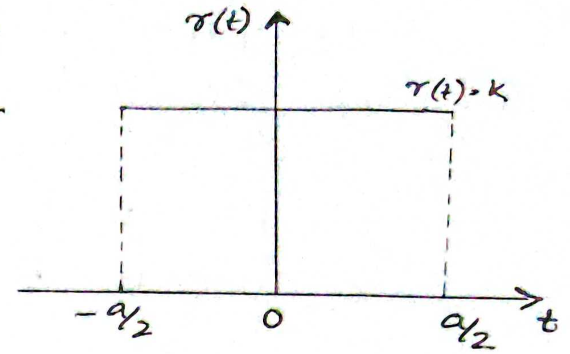
1. a) $$r(t) = \begin{cases} k, & |t| \leq a/2 \\ 0, & |t| > a/2 \end{cases}$$

Therefore, $$r(t) = \begin{cases} k & \text{if } -a/2 \leq t \leq \frac{a}{2} \\ 0 & \text{if } t < -a/2 \text{ or } t > a/2 \end{cases}$$



Now, $R(\omega) = \mathcal{F}(r(t))$

$$= \int_{-\infty}^{\infty} r(t)\, e^{-j2\pi\omega t}\, dt. \qquad (j = \sqrt{-1})$$

$$= \int_{-\infty}^{-a/2} r(t)\, e^{-j2\pi\omega t}\, dt + \int_{-a/2}^{a/2} r(t)\, e^{-j2\pi\omega t}\, dt$$

$$+ \int^{\infty} r(t)\, e^{-j2\pi\omega t}\, dt$$

$$= 0 + k \int_{-a/2}^{a/2} e^{-j2\pi\omega t}\, dt + 0$$

$$= -\frac{k}{j2\pi\omega}\, e^{-j2\pi\omega t} \Big|_{-a/2}^{a/2}$$

$$= \frac{k}{\pi\omega} \left(\frac{e^{j(\pi\omega a)} - e^{-j(\pi\omega a)}}{2j} \right)$$

$$= \frac{k}{\pi\omega a} \cdot a \sin(\pi\omega a)$$

$$= ka \operatorname{sinc}(\omega a).$$

1.b) The Fourier Transform of an image breaks down the image function into a sum of constituents sine waves. The given image is an magnitude spectra of the original image. The ripples in both X and Y directions are kind of symmetric. The center contains the low frequency components and in the FT it’s a red blob indicating high pixel values at the centre in the original image.

Therefore, the original image has a square of white pixels in the center with a black background.

1.d) The given matrix A is a full rank matrix and rank(A) = 5 > 1 since the columns of A are linearly independent. Hence, A is not separable.

1.e) The given matrix A has rank 1 since the maximum number of linearly independent columns is 1. Hence, A is separable.

One factorization of A is given by singular value decomposition.

```
A can be factorized as: u * s * vh where,
u = [[-0.48038 -0.87059 -0.10629  0.       0.     ]
 [ 0.16013 -0.03758 -0.41587  0.89443  0.     ]
 [ 0.       0.       0.      -0.       1.     ]
 [ 0.80064 -0.48477  0.35209  0.      -0.     ]
 [-0.32026  0.07516  0.83174  0.44721  0.     ]]
s = [[55.50676  0.       0.       0.       0.     ]
 [ 0.       0.       0.       0.       0.     ]
 [ 0.       0.       0.       0.       0.     ]
 [ 0.       0.       0.       0.       0.     ]
 [ 0.       0.       0.       0.       0.     ]]
vh = [[ 0.78756 -0.22502 -0.11251 -0.45004 -0.33753]
 [-0.61616 -0.28389 -0.14195 -0.56779 -0.44409]
 [ 0.00941  0.24341  0.1217   0.48682 -0.82998]
 [ 0.      -0.85869 -0.18867  0.47651  0.     ]
 [ 0.      -0.26866  0.95749 -0.10504  0.     ]]
```

1.f) Initialization:

0	0	0	0	0	0	0	0
∞	∞	∞	0	0	∞	∞	∞
∞	∞	∞	0	0	∞	∞	∞
∞	∞	∞	0	0	∞	∞	∞
∞	∞	0	0	0	0	∞	∞
∞	0	0	∞	∞	0	0	∞
0	0	∞	∞	∞	∞	0	0
0	∞	∞	∞	∞	∞	∞	0

Forward Pass:

0	0	0	0	0	0	0	0
1	1	1	0	0	1	1	1
2	2	2	0	0	1	2	2
3	3	3	0	0	1	2	3
4	4	0	0	0	0	1	2
5	0	0	1	1	0	0	1
0	0	1	2	2	1	0	0
0	1	2	3	3	2	1	0

Backward Pass:

0	0	0	0	0	0	0	0
1	1	1	0	0	1	1	1
2	2	1	0	0	1	2	2
3	2	1	0	0	1	2	3
2	1	0	0	0	0	1	2
1	0	0	1	1	0	0	1
0	0	1	2	2	1	0	0
0	1	2	3	3	2	1	0