1.1 Condition for submodularity:

$$P(\beta,\gamma) + P(\alpha,\delta) - P(\beta,\delta) - P(\alpha,\gamma) \geq 0.$$

$\forall\ \alpha, \beta, \gamma, \delta$ such that $\beta > \alpha$ and $\delta > \gamma$.

Quadratic Function: $P(w_m, w_n) = c\,(w_m - w_n)^2$

$$\therefore\ P(\beta,\gamma) + P(\alpha,\delta) - P(\beta,\delta) - P(\alpha,\gamma)$$

$$= c(\beta^2 + \gamma^2 - 2\beta\gamma) + c(\alpha^2 + \delta^2 - 2\alpha\delta) - c(\beta^2 + \delta^2 - 2\beta\delta) - c(\alpha^2 + \gamma^2 - 2\alpha\gamma)$$

$$= c(-2\beta\gamma - 2\alpha\delta + 2\beta\delta + 2\alpha\gamma)$$

$$= 2\beta c(\delta - \gamma) - 2\alpha c(\delta - \gamma)$$

$$= 2c(\delta - \gamma)(\beta - \alpha)$$

Since, $\beta > \alpha$ and $\delta > \gamma$, then $2c(\delta - \gamma)(\beta - \alpha) \geq 0$

~~Since c ≥ 0 and~~

~~Assumi~~

$\therefore\ P(w_m, w_n) = c\,(w_m - w_n)^2$ is submodular.

[Proved]