

2.2 Derivative of Log-likelihood:

$$L = \sum_{i=1}^I y_i \log\left(\frac{1}{1 + e^{-w^T x_i}}\right) + \sum_{i=1}^I (1 - y_i) \log\left(\frac{e^{-w^T x_i}}{1 + e^{-w^T x_i}}\right)$$

Simplifying the equation, we have

$$L = \sum_{i=1}^I [y_i \log(1) - y_i \log(1 + e^{-w^T x_i})] + \sum_{i=1}^I [(1 - y_i) \log(e^{-w^T x_i}) - (1 - y_i) \log(1 + e^{-w^T x_i})]$$

$$L = - \sum_{i=1}^I y_i \log(1 + e^{-w^T x_i}) + \sum_{i=1}^I [\log(e^{-w^T x_i}) - y_i \log(e^{-w^T x_i}) - \log(1 + e^{-w^T x_i}) + y_i \log(1 + e^{-w^T x_i})]$$

$$L = \sum_{i=1}^I \log(e^{-w^T x_i}) - y_i \log(e^{-w^T x_i}) - \log(1 + e^{-w^T x_i})$$

Taking derivative w.r.t w

$$\frac{\partial L}{\partial w} = \sum_{i=1}^I \frac{1}{e^{-w^T x_i}} (-x_i \cdot e^{-w^T x_i}) - y_i \cdot \frac{1}{e^{-w^T x_i}} (-x_i \cdot e^{-w^T x_i}) - \frac{1}{1 + e^{-w^T x_i}} (-x_i \cdot e^{-w^T x_i})$$

$$\frac{\partial L}{\partial w} = - \sum_{i=1}^I \left[(1 - y_i) x_i - \frac{x_i \cdot e^{-w^T x_i}}{1 + e^{-w^T x_i}} \right]$$

$$\frac{\partial L}{\partial w} = - \sum_{i=1}^I \left(-y_i + 1 - \frac{e^{-w^T x_i}}{1 + e^{-w^T x_i}} \right) x_i$$

$$\frac{\partial L}{\partial w} = - \sum_{i=1}^I \left(-y_i + \frac{1 + e^{-w^T x_i} - e^{-w^T x_i}}{1 + e^{-w^T x_i}} \right) x_i$$

$$\frac{\partial L}{\partial w} = - \sum_{i=1}^I \left(-y_i + \frac{1}{1 + e^{-w^T x_i}} \right) x_i = \boxed{- \sum_{i=1}^I \left(\frac{1}{1 + \exp(-w^T x_i)} - y_i \right) x_i}$$