

* Distribution Assignment *

(1) The avg monthly sales of 2000 firms are normally distributed with mean Rs. 38,000 and standard deviation of Rs. 10,000.

Find:

(i) the number of firms with sales of over Rs. 50,000

Ans Probability (P) $(X > 50,000) = 1 - P(X < 50,000)$

$$Z_{\text{score}} = \frac{50,000 - 38,000}{10,000} = \left(\frac{\mu - \bar{x}}{\sigma} \right)$$

$$= 1.2$$

consult z-table for probability associated with 1.2

$$P(X < 50,000) = 0.8849$$

$$P(X > 50,000) = 1 - P(X < 50,000) = 1 - 0.8849 = 0.1151$$

$$\text{No. of firms with sales over 50,000} = 2000 \times 0.1151$$

$$= 230$$

(ii) The percentage of firms with sales between 38,500 & 41,000

Ans $P(38,500 < X < 41,000) = P(X < 41,000) - P(X < 38,500)$

$$P(X < 41,000) =$$

$$Z_{\text{score}} = \frac{41,000 - 38,000}{10,000} = 0.3$$

$$P(X < 41,000) = \text{Area } (0.3)$$

$$= 0.6179$$

$$P(X < 38,500) =$$

$$Z_{\text{score}} = \frac{38,500 - 38,000}{10,000} = 0.05$$

$$P(X < 38,500) = 0.5199$$

Now \rightarrow No. of firms $= (2000 \times 0.6179) - (2000 \times 0.5199)$

$$= 1235.8 - 1039.8$$

$$= 196$$

(3) A test is conducted which consists of 20 MCQ with every question having 4 options. Determine the probability of a person answering exactly 5 wrong answers.

Ans To get exactly 5 answers wrong we have to calculate exactly 15 and right or define success as further.

$$\begin{aligned}
 n &= 20 \\
 r &= 15 \\
 n-r &= 5 \\
 \text{Probability of success} &= \frac{1}{4} \\
 \text{failure} &= 1 - \frac{1}{4} = \frac{3}{4} \\
 &= {}^{nCr} (p)^r (q)^{n-r} \\
 &= {}^{20C5} (p)^{15} (q)^5 \\
 &= {}^{20C5} \left(\frac{1}{4}\right)^{15} \left(\frac{3}{4}\right)^5 \\
 &= \frac{20 \times 19 \times 18 \times 17 \times 16}{5 \times 4 \times 3 \times 2 \times 1} \left(\frac{1}{4}\right)^{15} \left(\frac{3}{4}\right)^5 \\
 &\approx 0.0000034
 \end{aligned}$$

(4) In an observational astronomy experiment, let the average rate of photons reaching the telescope is 4 photons per second (Poisson Random variable with mean of 4). Find the prob. that no photon reaches the telescope in a given second.

Ans ~~P(X=0)~~ $P(X_0) \rightarrow$ No photons reaches the telescope in given second

$$\begin{aligned}
 P(X_0) &= \frac{e^{-\mu} \mu^x}{x!} \\
 &= \frac{e^{-4} (4)^0}{0!} \\
 &= \frac{e^{-4} \times 1}{1} \\
 &= \frac{1}{e^4}
 \end{aligned}$$

$\mu \rightarrow$ mean
 $\mu = 4$
 $x \rightarrow$ No. of success

$P(X_0) \approx 0.0183156$

(15) The number of calls coming per minute into a customer care support center is Poisson random variable with mean 3.

(a) Find the probability that no calls come in a given 1-minute period

Ans

$$\lambda = 3$$

for no calls $x = 0$

$$\begin{aligned} P(X_0) &= \frac{e^{-\lambda} \lambda^x}{x!} \\ &= \frac{e^{-3} 3^0}{0!} \\ &= \frac{1}{e^3} \end{aligned}$$

$$P(X_0) = 0.04978$$

(b) Assume that the no. of calls arriving in two different minutes are independent. Find the probability that at least two calls will arrive in a given two minute period.

Ans

(6) If a production line has a 20% defective rate. Calculate the probability of obtaining the first defective product after 3 good products. What is the avg no. of inspections to obtain the first defective.

Ans Products produced by a machine has 20% defective rate. after 3rd is 4th its defective

$$P(X=4) = P(\text{1st 3 non-defective}) P(\text{4th defective})$$

$$= (0.8)^3 (0.2)$$

$$= 0.08192$$

$$(2) E(X) = \frac{1}{p}$$

Avg. No. of inspection to obtain first defective

$$E(X) = \frac{1}{0.2} = 5$$

(7) The probability that a student is accepted to a prestigious college is 0.3. If 5 students from the same school apply, what is the probability that at most 2 are accepted.

Ans $P(X \leq 2) = P(X_0) + P(X_1) + P(X_2)$

← no. ← 1 → 2

$$n = 5, \quad p(\text{probability of success}) = 0.3$$

$$q = 1 - p = 0.7$$

$$P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

(8) The maximum weight that an elevator in an apartment complex can accommodate is 800 kg. The avg adult weight be about 70 kg with a variance of 200. What is the probability that the lift safely reaches the ground when there are 10 diff adults in the lift? What if there are 12 adults?

Ans

$$\text{Variance } \sigma^2 = 200$$

$$\text{standard deviation } \sigma = \sqrt{200} = 14.1421$$

$$\mu = 70$$

$$Z = \frac{x - \mu}{\sigma} = \frac{80 - 70}{14.142} = \frac{10}{14.142} = 0.707 = 0.7794$$

$$Z = \frac{x - \mu}{\sigma} = \frac{66.66 - 70}{14.142} = -0.24 = 0.4052$$

$$10 \text{ adults} = 78\%, \quad 12 \text{ adults} = 40\%$$

(9) A student, to test his luck, went to an examination unprepared. It was MCQ type examination with 2 choices for each question. There are 50 questions of which 20 are to be answered correctly to pass the test. What is the probability that he clears exam, if each question has 4 choices instead of two, what is the probability?

Ans :-

Probability of getting

right ans when 2 choice is $= 1/2$

$$P = 1/2, \quad Q = 1/2, \quad r = 20, \quad n = 50$$

$$= {}^nC_r (P)^r (Q)^{n-r}$$

$$= {}^{50}C_{20} (1/2)^{20} (1/2)^{50-20}$$

$$= {}^{50}C_{20} (1/2)^{50}$$

$$= 4.71 \times e^{13} \times 8.88 \times e^{-16}$$

$$= \frac{4.71 \times 8.88}{e^3}$$

(10) A company manufactures LED bulbs with a failure rate of 30%. If 9 randomly select 6 bulbs, what is the probability that exactly 2 are faulty?

Ans

Probability of getting defective bulb is = 30%
= 0.3.

Exactly two bulbs are defective

$$\begin{aligned}P(X=2) &= {}^nC_r (p)^r (q)^{n-r} \\&= {}^6C_2 (0.3)^2 (0.7)^{6-2} \\&= {}^6C_2 (0.3)^2 (0.7)^4 \\&= 15 \times 0.09 \times 0.24 \\&= 0.324 \\&= \underline{\underline{32.41\%}}\end{aligned}$$

(11) For a writer, the efficiency of typing is 6 errors per hour entering 77 words per minute. What is the probability of error of 2 errors in 322 word reports?

Ans