

Hypothesis Testing

Q. (1) The dean from UCLA is concerned that student's grade point averages have changed dramatically in recent years. The graduating senior's mean GPA over the 5 year is 2.75. The dean randomly samples 256 seniors from last graduating class and finds that their mean GPA is 2.85 with a sample std of 0.65

(a) what would the null & Alternate Hypothesis for this scenario?

Ans \rightarrow SE (sample std error) is not given

H_0 = GPA did not change dramatically over the years

H_1 = GPA change dramatically over the time

(b) what would be the standard error for this scenario?

Ans \Rightarrow ~~SE = 0.65~~
 $\sigma_p \hat{=} \sigma_s$ Hence $\sigma_p = 0.65$

(c) Describe critical Regions for $\alpha = 0.05$

Ans \Rightarrow Since scenario asked ^{GPA} has changed (it can be decreased or increase) so two tailed test

$$CI = 95\%$$

$$\alpha = 0.05 \Rightarrow 5\%$$



1d) Test the null hypothesis & explain your decision

$$N = 256$$

$$\begin{aligned} Z_s &= \frac{\bar{x}_{\text{is}} - \mu_{\text{pop}}}{\frac{\sigma}{\sqrt{N}}} \\ &= \frac{2.85 - 2.75}{\frac{0.65}{\sqrt{256}}} \\ Z_s &= \frac{2.85 - 2.75}{\frac{0.65}{\sqrt{256}}} = 2.46 \end{aligned}$$

$$\boxed{Z_s = 2.46}$$

Reject Null Hypothesis as it falls under critical region. CR is reject region.

Hence, GPA has not been changed dramatically over the years.

(2) The College bookstore tells prospective student that average cost of its textbook is Rs. 52 with std dev. of ~~Rs~~ 4.50 rupees. A group of small statistics student think that the average cost is higher. To test the bookstore's claim against their alternative, the student will select a random sample of size 100. Assume that the mean from their sample is 52.80 perform a hypothesis test, at the 5% CI & state your decision.

$$\underline{\text{Ans}}: \mu_{\text{pop}} = 52, \bar{x}_{\text{is}} = 52.80, \sigma_{\text{pop}} = 4.50, N = 100$$

Hypothesis

H_0 = Average cost is not higher

H_1 = Average cost is higher

$$Z = \frac{\bar{x} - \mu_{pop}}{\frac{\sigma_{pop}}{\sqrt{N}}}$$

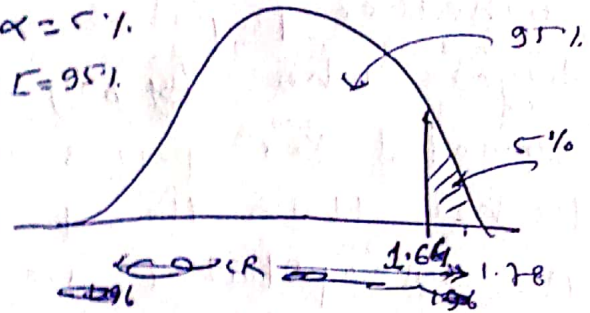
$$= \frac{52.6 - 52}{\frac{4.50}{\sqrt{100}}}$$

$$Z = 1.78$$

Since students are saying it's higher
then a right tailed test

$$\alpha = 5\%$$

$$(Z = 1.64)$$



Since $1.78 > 1.64$ Reject Null Hypothesis
Average cost is higher.

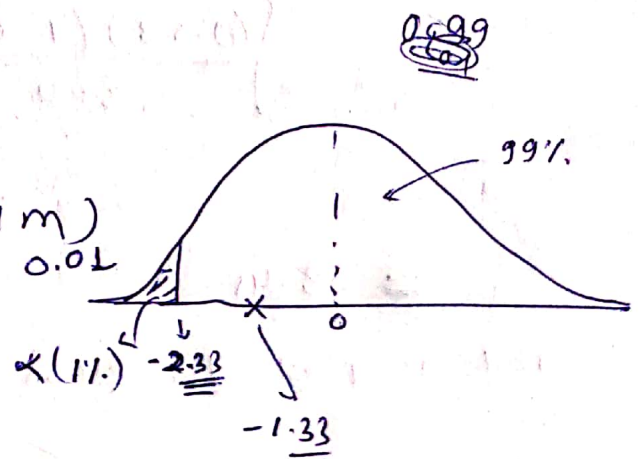
(3) A certain chemical pollutant in the river has been constant for several years with a mean of 34 ppm and standard deviation of 8 ppm. A group of factory representative whose companies discharge liquids into the river is now claiming that they have lowered the average with improved filtration devices. A group of environmentalists will test to see if this is true at the 1% level of significance. Assume that their sample of size 50 gives a mean of 32.5 ppm. Perform a Hypothesis test at 1% significance.

Ans $\rightarrow H_0: \bar{X} = 34 \text{ ppm}$

$H_A: \bar{X} < 34 \text{ ppm}$ (claim)

$$Z = \frac{32.5 - 34}{\frac{8}{\sqrt{50}}}$$

$$= -1.33$$



H_0
Accept Null Hypothesis.

Company's claim is not accurate.

(4) Carry out one tailed test to determine whether population proportion of travel's check buyers who buy atleast \$2500 in check when sweepstakes prizes are offered is at least 10% higher than the proportion of such buyers when no sweepstakes are on.

Population 1: With sweepstakes

$$N_1 = 300, X_1 = 120, S_1 = 0.53$$

Population 2: No sweepstakes

$$N_2 = 700, X_2 = 140, S_2 = 0.20$$

Ans

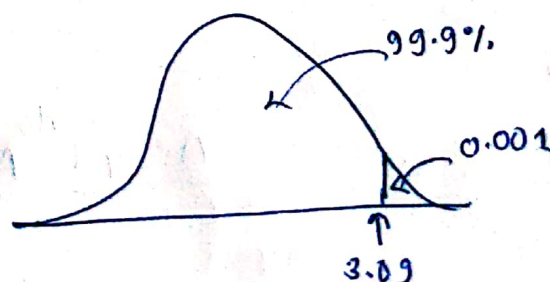
$$H_0: p_1 - p_2 \leq 0.10$$

$$H_A: p_1 - p_2 > 0.10$$

$$Z = \frac{(S_1 - S_2) - 0}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$$

$$= \frac{(0.53 - 0.20) - 0.10}{\sqrt{\frac{(0.53)(1-0.53)}{300} + \frac{0.20(0.80)}{700}}} = \frac{0.23}{\sqrt{0.00146}} = 3.45$$

Not accepted for even 0.01%, i.e. 0.001 alpha



or

$$Z_{\text{value of } \alpha(0.9990)} = 3.09$$

reject Null Hypothesis

(7) The school nurse thinks the avg height of 7th graders has increased. The avg height of a 7th grader five years ago was 145 cm with a std deviation of 20 cm. She takes a random sample of 200 students & finds that avg height of her sample is 147 cm. Are 7th graders now taller than they before? Conduct a single-tailed hypothesis test using a 0.05 significance level to evaluate the null & alternative hypothesis.

Ans

H_0 = Height of 7th grader ≤ 145 ~~has increased~~ ~~145~~

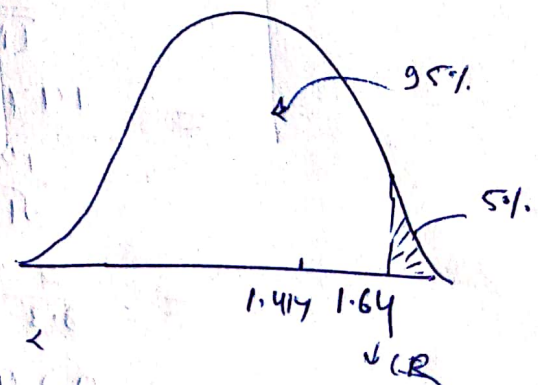
H_1 = Height of 7th grade has ~~not~~ increased > 145

$\sigma_{pop} = 20$, $N = 200$, $\alpha = 0.05 \neq 5\%$

$\mu_s = 147$, $\mu_{pop} = 145$

Testing the Hypothesis

$$\begin{aligned} Z_s &= \frac{\mu_s - \mu_{pop}}{\sigma_p / \sqrt{N}} \\ &= \frac{\mu_s - \mu_{pop}}{\sigma_p / \sqrt{N}} \\ &= \frac{147 - 145}{\frac{20}{\sqrt{200}}} \\ &= 1.414 \end{aligned}$$



Accept NULL Hypothesis

18) A farmer is trying out a planting technique that he hopes will increase the yield on his pea plants. The average no. of pods on one of his pea plants is 145 pods with a std. of 100 pods. This year, after trying his new planting technique, he takes a random sample of 144 of his plants and finds the avg no. of pods to be 147. He wonders whether or not this is a statistically increase. What are his hypothesis and test statistic?

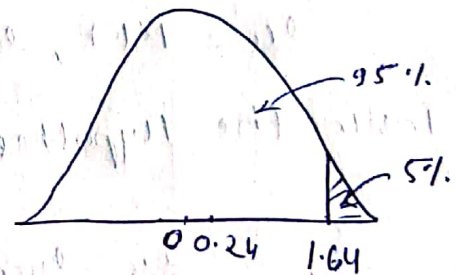
$$\Rightarrow H_0: \mu = 145$$

$$H_1: \mu > 145$$

$$\sigma = 100, \mu_s = 147, \mu_{pop} = 145$$

$$N = 144$$

$$\begin{aligned} Z_s &= \frac{\mu_s - \mu_{pop}}{\frac{\sigma}{\sqrt{N}}} \\ &= \frac{147 - 145}{\frac{100}{\sqrt{144}}} \\ &= \frac{2}{8.3} \\ &= 0.2409 \end{aligned}$$



Accept the H_0

So mean yield on farmer's pea plant has not increased

(9) You have just taken ownership of pizza shop. The previous owner told you that you would save money if you bought the mozzarella cheese in a 4.5 pound slab. Each time you purchase a slab of cheese, you weigh to ensure that you are receiving 72 ounces of cheese. The result of 7 random measurements are 70, 69, 73, 68, 71, 69 & 71 ounces. Are these diffⁿ due to chance or is the distributor giving you less cheese you deserve.

(a) state the Hypothesis

Ans \Rightarrow

H_0 = mean weight of cheese = μ = 72

H_1 : mean weight of cheese = $\mu \neq 72$

(b) calculate the test statistic

mean of sample $\bar{x} = 70.143$
 $s = 1.676$

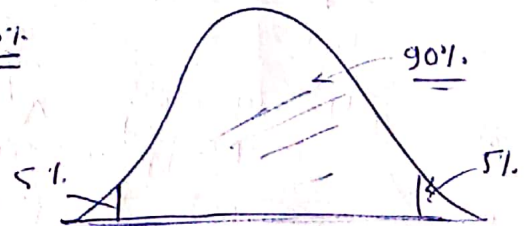
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{70.143 - 72}{\frac{1.676}{\sqrt{7}}}$$

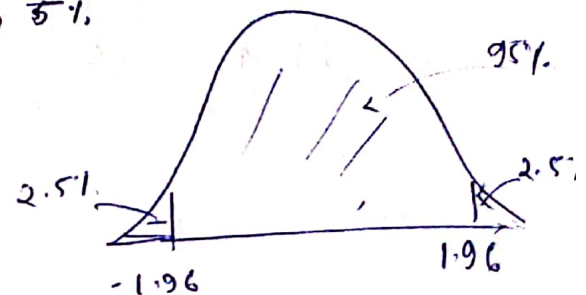
$$t \approx -2.9315$$

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}$$

1) 10%



2) 5%



Reject H_0 in 5%, 10% conf level

3) 1%

