* Probability Assignment *

1. Two dices are rolled at once. Find out the probability for sum of numbers being even and one of the dice shows 6.

Ans :-

Sample space (Total No. of possibilities of rolling two dices) = 36

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6) *
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6) *
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2) *	(6,3)	(6,4) *	(6,5)	(6,6) *

$P(S_{even}) = 5/36$ (Probability of sum of No. being even & one of the dice shows 6)

2. Two dices are rolled at once. Find out the probability for sum of numbers being less that 7.

Ans :- By observing table from Q_1.

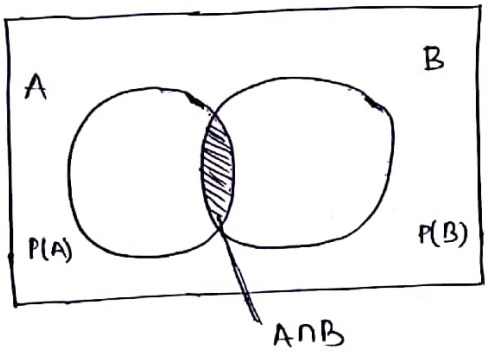
probability of sum of No. being less than 7 = $\frac{\text{total possibilities}}{\text{sample space}}$

$= \frac{5+4+3+2+1}{36} = \frac{15}{36} = \frac{5}{12}$

$P(S_{<7}) = 5/12$

3. You toss a fair coin 3 times: Given that you have observed at least one heads, what is the probability that you observe at least two heads.

Ans:-



$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

* Prob. of A given B is already occured

Given P(B) = Prob. of observing at least one heads when it is tossed a fair coin 3 times

$$P(B) = 1 - P(TTT) = 1 - \frac{1}{8} = \frac{7}{8}$$

↑ Total No. of possibilities

P(A) = Prob. of at least two heads

$$P(A) = P(HHT) + P(HTH) + P(THH) + P(HHH)$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8}$$

Thus we can write

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$ → common set P(A) & P(B)

$$= \frac{P(A) \cap P(B)}{P(B)}$$

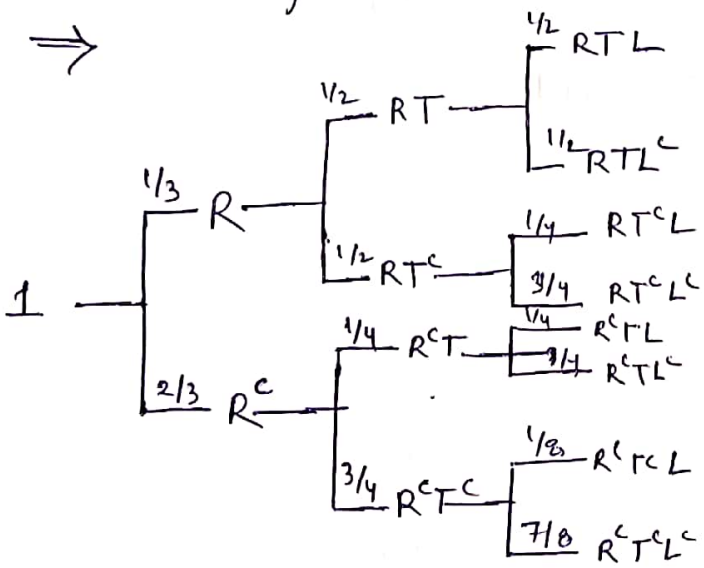
$$= \frac{4/8}{7/8}$$

$$\boxed{P(A/B) = 4/7}$$

4. In my town, it is rainy 1/3 of the days. Given that it is rainy, there will be heavy traffic with prob 1/2, and given that it is not rainy, there will heavy traffic with probability 1/4. If it is rainy and there is heavy traffic, I arrive late for work with prob. 1/2. On the other hand, the prob. of being late is 1/8 if it is not rainy & there is no heavy traffic. In other situations (rainy & no traffic, not rainy & traffic) the prob. of being late is 0.25. You pick a random day. What is the prob. that it is not raining & there is heavy traffic & I am not late?

(a) What is the probability that it's not raining & there is heavy traffic & I am not late?

$\Rightarrow$



R → Rain, R^c → No Rain
T → Traffic, T^c → No Traffic
L → Late, L^c → No Late

(a) Ans :- $P(R^cTL^c) = 2/3 \times 1/4 \times 3/4 = 3/16 = 1/8$

(b) Ans :- What is probability that I am late

$= P(RTL) + P(RT^cL) + P(R^cTL) + P(R^cT^cL)$

$= (1/3 \times 1/2 \times 1/2) + (1/3 \times 1/2 \times 1/4) + (2/3 \times 1/4 \times 1/4) + (2/3 \times 3/4 \times 1/8) = 11/48$

(c) Ans :- $P(\text{Late}) = 11/48$. $P(R \cap L) = P(R,T,L) + P(R,T^c,L)$

$= (1/2 \times 1/2 \times 1/3) + 1/24 = 1/8$

$P(R/L) = \frac{P(R \cap L)}{P(L)} = \frac{1}{8} \times \frac{48}{11} = 6/11$

5. A box contains 3 coins: two regular coins and one fake 2 headed coin ($P(\text{Heads}) = 1$), you pick a coin at random and toss it.

(a) What is the probability that it lands heads up?

Ans ⇒ Let C_1 be the event for Regular coin
C_2 be the event for fake coin

H|H|A

$P(H/C_1) = 1/2$ (Prob. of head in Regular coin)
$P(H/C_2) = 1$ (Prob. of head in fake coin)

$$P(H) = P(H|C_1)\,P(C_1) + P(H|C_2)\,P(C_2)$$

$$\boxed{P(H) = \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(1 \times 1/3\right) = 2/3}$$

(b) You pick a coin at random & toss it & gets heads. What is the prob. that it is the 2 headed coin?

$$\Rightarrow P(C_2/H) = \frac{P(H/C_2)\,P(C_2)}{P(H)}$$

$$\boxed{P(C_2/H) = \frac{1 \cdot 1/3}{2/3} = 1/2}$$

(6) Suppose that, of all the customers at a coffee shop

(a) 70% purchase a cup of coffee
(b) 40% purchase a piece of cake
(c) 20% purchase both a cup of coffee & a piece of cake.

Given that a randomly chosen customer has purchased a piece of cake, what is the prob. that he/she has also purchased a cup of coffee?

Ans ⇒

$P(\text{coffee}) = 0.7$, $P(\text{Cake}) = 0.4$, $P(\text{coffee} \cap \text{cake}) = 0.2$

Prob. of purchasing coffee, given customer has purchased a piece of cake

$$P(\text{coffee}/\text{cake}) = \frac{P(\text{coffee} \cap \text{cake})}{P(\text{cake})}$$

$$P(\text{coffe}/\text{cake}) = \frac{0.2}{0.4} = 1/2$$

11. A is known to tell the truth in 5 case out of 6 and he states that a white ball was drawn from a bag containing 8 black & 1 white ball.

Find the probability that the white ball was drawn

Ans ⇒

Probability of drawing white ball $P(W) = 1/9$,

$P(\vec{W}) = 1 - 1/9$ (RW)

$= 8/9$

Probability of A telling truth $P(T/W) = 5/6$

" Not telling truth $(T/\vec{W}) = 1/6$

$$P(W/T) = \frac{P(T/W) \cdot P(W)}{P(T/W) \cdot P(W) + P(T/\vec{W}) P(\vec{W})}$$

$$= \frac{5/6 \times 1/9}{(\frac{5}{6} \times \frac{1}{9}) + \frac{1}{6} \times 8/9}$$

$$P(W/T) = 5/13$$

12. A speaks truth 4 out of 5 time, A dice is tossed. A reports that it is 6. What are the chances that there actually was a 6?

Ans ⇒

$P(T/d_6) = 4/5$, $P(d_6) = 1/6$

$P(\bar{d_6}) = 1 - 1/6 = 5/6$

$P(T/\vec{d_6}) = 1 - 4/5$

$= 1/5$

(Bayes theorem)

$$P(d_6/T) = \frac{P(T/d_6) \cdot P(d_6)}{P(T/d_6) \cdot P(d_6) + P(T/\vec{d_6})\, P(\vec{d_6})}$$

$$= \frac{\frac{4}{5} \times 1/6}{\frac{4}{5} \times \frac{1}{6} + \frac{1}{5} \times \frac{5}{6}}$$

$$= \frac{\frac{4}{30}}{\frac{4}{30} + \frac{5}{30}} = \frac{\frac{4}{30}}{\frac{9}{30}} = 4/9$$

$$\boxed{P(d_6/T) = 4/9}$$