SSC 384 7 Bayesian Statistical Methods **Spring 2013**

Bayesian Statistical Methods
HW 1

Due: Jan 29, 2013

- Note the Gamma density has the following form:

$$p(y) = \frac{1}{\Gamma(a)b^a} y^{a-1} \exp(-y/b), y > 0$$

- This homework is to emphasize simple analytic computations useful in Bayesian inference and as well as doing inference in very simple settings.

1. Consider a realization, Y from a binomial distribution, $Bin(n, \theta)$ with a $Beta(a, b)$ prior on θ. Assume $n = 20$ and we observe $Y = 3$. Consider various choices of (a, b) including $\{(1, 1), (10, 10), and (10, 1)\}$. Use WinBugs to sample from the posteriors or write your own algorithm to sample from the posteriors in R (note the rbeta function in R can be used to sample from Beta distributions). Comment on differences between the posteriors based on the priors.

2. Consider the setting of problem 1. Suppose we observed a previous study based on 5 patients where the response rate was 20%. Explain how we could bring this information (if desired) into our Beta prior.

3. Derive Jeffreys' prior for the parameter in the Binomial model.

4. Assume we have an independent sample of n Poisson responses with mean θ and we specify a Gamma prior on θ with parameters (a, b). Show that the posterior distribution of θ is a Gamma distribution and identify the parameters of the Gamma distribution.

5. Derive the posterior for $1/\sigma^2$ where the data are n independent normal random variables with known mean, μ and the prior on $1/\sigma^2$ is a Gamma distribution with parameters, a and b. Based on the form of the posterior, demonstrate how we could bring in prior information on $1/\sigma^2$ similar to what we did for the Beta prior in Problem 2.

6. Consider an iid sample of size $n = 10$ from a normal distribution with mean μ and variance σ^2. Assume the realized sample is $(3.7, 3.4, 5.5, 5.0, 5.4, 6.6, 4.8, 4.4, 5.1, 5.4)$. Assume noninformative priors as discussed in class (and specify your choices). Sample from the posterior distribution. Either do this in WinBUGS or write your own sampler in R.