Bayesian Statistical Methods: Homework 2

Due on Feb 14, 2013

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Problem 1

Reading Assignment.

Problem 2

I began by doing a test run of 1000 iterations with no burn in or thinning. From the traces of μ and σ, I saw that both had converged within a few iterations. So, I burned 100 iterations because running the model is not computationally expensive. Then, I ran the model again and observed significant autocorrelation of μ within about 15 itterations, so I selected to thin the results by 15. Fig. 1 and Fig. 2 are the resulting posterior summaries after 10,000 iterations.

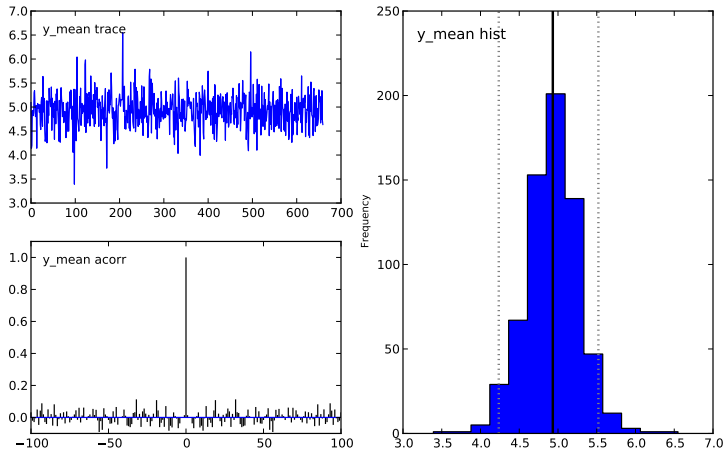


Figure 1: Posterior summary for μ using PyMC

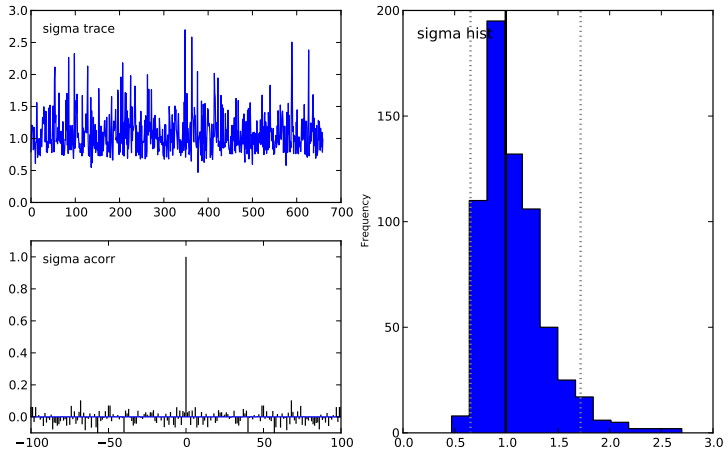


Figure 2: Posterior summary for σ using PyMC

Problem 3

Write out the steps for your slice sampler, use h1 tails to boost U?

Problem 4

The following code is a Metropolis-Hastings algorithm written in Python for the data given in Problem 2 with $\sigma = 1$. A poster summary for μ is given in Fig. 3 for 10000 iterations. Roughly 10% of the samples were accepted.

```
from __future__ import division
import numpy as np
import scipy.stats
import matplotlib as plt
from pylab import *

def mh_sample(n, data, mu, walk_sig):
    # n is the number of iterations
    # data is what we are conditioning on
    # mu is an initial value for the chain of samples
    # walk_sig is the variance of the random walk

    # Initialize arrays
    x_list = np.zeros(n)
    n_list = np.zeros(n)
    x_list[0] = mu
    n_list[0] = 1

    # Variance is 1 for now
    sigma = 1.0

    # Pointer for last accepted value
    acc = 0

    for i in range(n):
        # Random Walk to generate next value of x
        z = x_list[acc] + np.random.normal(0.0,walk_sig**2,1)

        # Compute Denominator of MH alogrithm
        den = 1
        for j in range(0,data.shape[0]):
            den = den * np.exp((-(data[j]-x_list[acc])**2)/(2.0*sigma**2))

        # Compute Numerator of MH alogrithm
        num = 1
        for j in range(0,data.shape[0]):
            num = num * np.exp((-(data[j]-z)**2)/2.0)

        # Compute the probability of a move, alpha
        alpha = min(1, num/den)
```

Problem 4 continued on next page...

```
        # Accept / Reject
        if alpha == 1:
            x_list[acc+1] = z
            n_list[acc+1] = i+1
            acc = acc + 1
        elif alpha < 1:
            p_a = np.random.binomial(1, alpha, 1)
            if p_a == 1:
                x_list[acc+1] = z
                n_list[acc+1] = i+1
                acc = acc + 1

    return x_list, n_list, acc
```

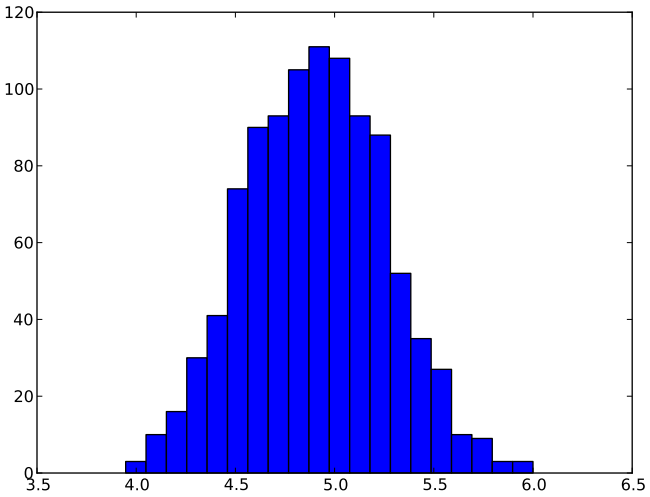


Figure 3: Posterior summary for μ using Metropoils-Hastings

Problem 5

The following code is a Slice Sampling algorithm written in Python for the data given in Problem 2 with $\sigma = 1$. A poster summary for μ is given in Fig. 4 for 10000 iterations.

```
from __future__ import division
import numpy as np
import scipy.stats
import matplotlib as plt
from pylab import *

def slice_sample(n, data, mu):
```

```
    # n is the number of iterations
    # data is what we are conditioning on
    # mu is an initial value for the chain of samples

    # Initialize arrays
    x_list = np.zeros(n+1)
    x_list[0] = mu
    sigma = 1.0

    width = 1.0

    for i in range(n):
        # Calculate h(x) at data
        h_x = 1
        for j in range(0,data.shape[0]):
            h_x = h_x * np.exp((-(data[j]-x_list[i])**2)/(2.0*sigma**2))

        # Sample from 0 to h(x) to get U
        U = np.random.uniform(0, h_x, 1)

        # Find bounds x_L and x_R
        x_L = x_list[i] - width
        x_R = x_list[i] + width
        left_bound = False
        right_bound = False
        while left_bound == False:
            h_x_L = 1.0
            for j in range(0,data.shape[0]):
                h_x_L = h_x_L * np.exp((-(data[j]-x_L)**2)/(2.0*sigma**2))
            if h_x_L < U:
                left_bound = True
            elif h_x_L > U:
                x_L = x_L - width

        while right_bound == False:
            h_x_R = 1.0
            for j in range(0,data.shape[0]):
                h_x_R = h_x_R * np.exp((-(data[j]-x_R)**2)/(2.0*sigma**2))
            if h_x_R < U:
                right_bound = True
            elif h_x_R > U:
                x_R = x_R + width

        # Sample from uniform on interval x_L to x_R
        x_list[i+1] = np.random.uniform(x_L, x_R, 1)

    return x_list
```

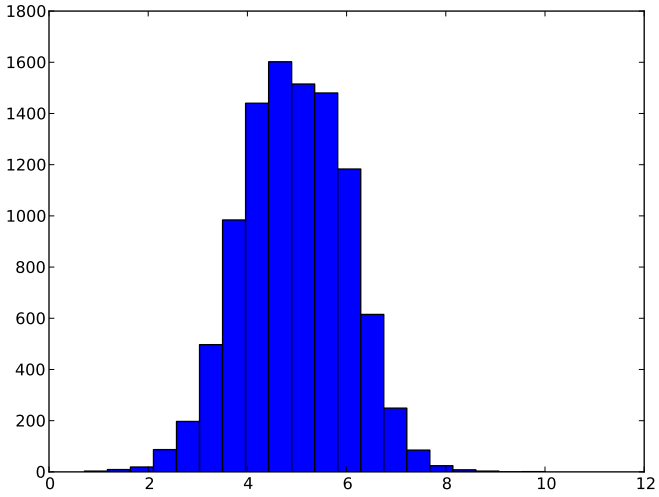


Figure 4: Posterior summary for μ using Slice Sampling

Problem 6

I used the same methods stated in Problem 2 to choose the Burn In and Thinning for each case. The main differences to take note of between the posteriors are that the means sampled from the Informed and Uniform priors are both slightly higher than that of the diffuse Gamma prior. Additionally, the posterior generated from the Uniform prior has a larger tail on the right hand side, due to the fact that the Uniform prior was given the range from 0 to 20.

Table 1: Comparison of Specified Priors

Prior	Iterations	Burn In	Thinning
Diffuse Gamma(0.001,0.001)	50,000	10,000	20
Informed Gamma(1.11,1.61)	30,000	10,000	20
Uniform(0,20)	30,000	4,000	15

Compute probability that lambda is greater than 0.5. Need to get full sample data somehow and run a script on it. P(lambda¿0.5) = Number of samples over 0.5 / Total number of samples. do this for each prior also.

Problem 7

For each Prior: Differences in posterior, what was the thin and burn?

Compute the probability that the difference of the means is greater than zero. Compute the posterior probability that the ratio of the standard deviations is greater than 1.

Is this a good diagnostic?

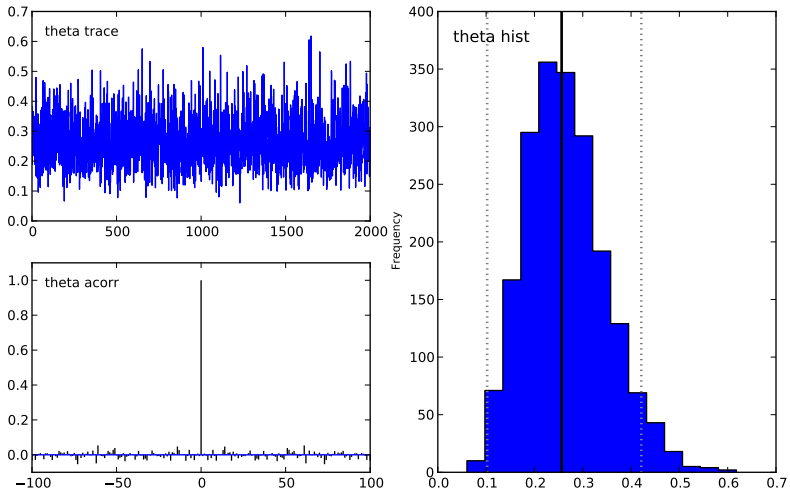


Figure 5: Posterior summary for θ using a diffuse gamma prior

Table 2: Burn In and Thinning for Informed and Uninformed Priors

Priors	Burn In	Thinning
Informed	20,000	15
Uninformed	20,000	25

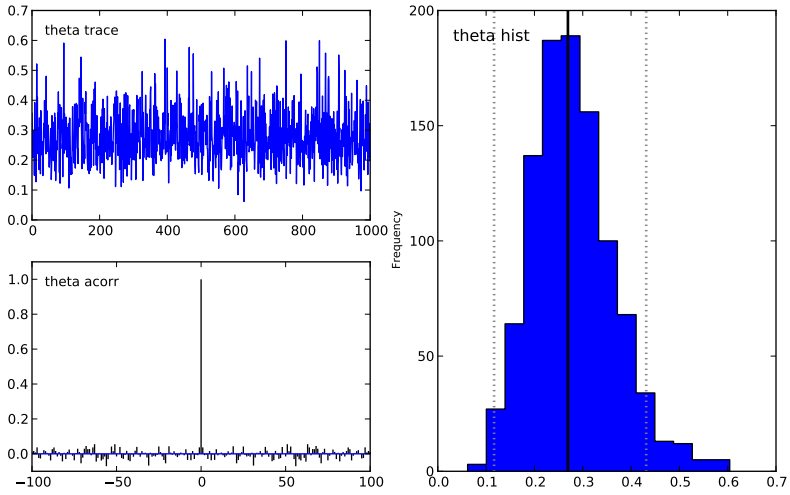


Figure 6: Posterior summary for θ using an informed gamma prior

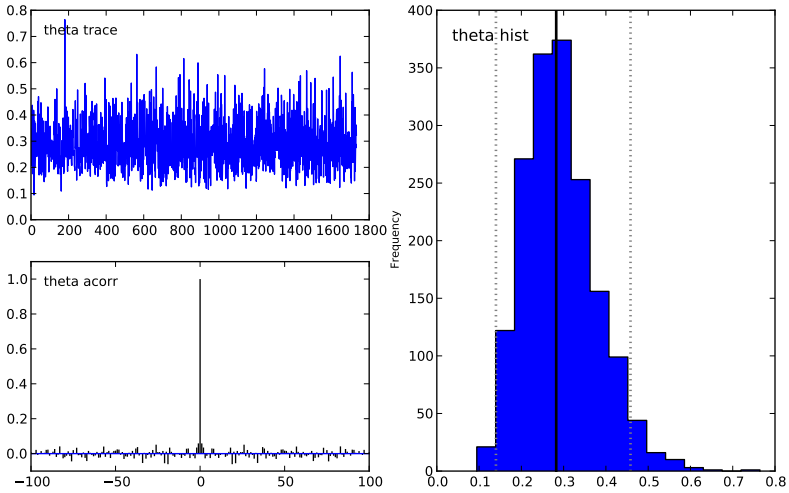
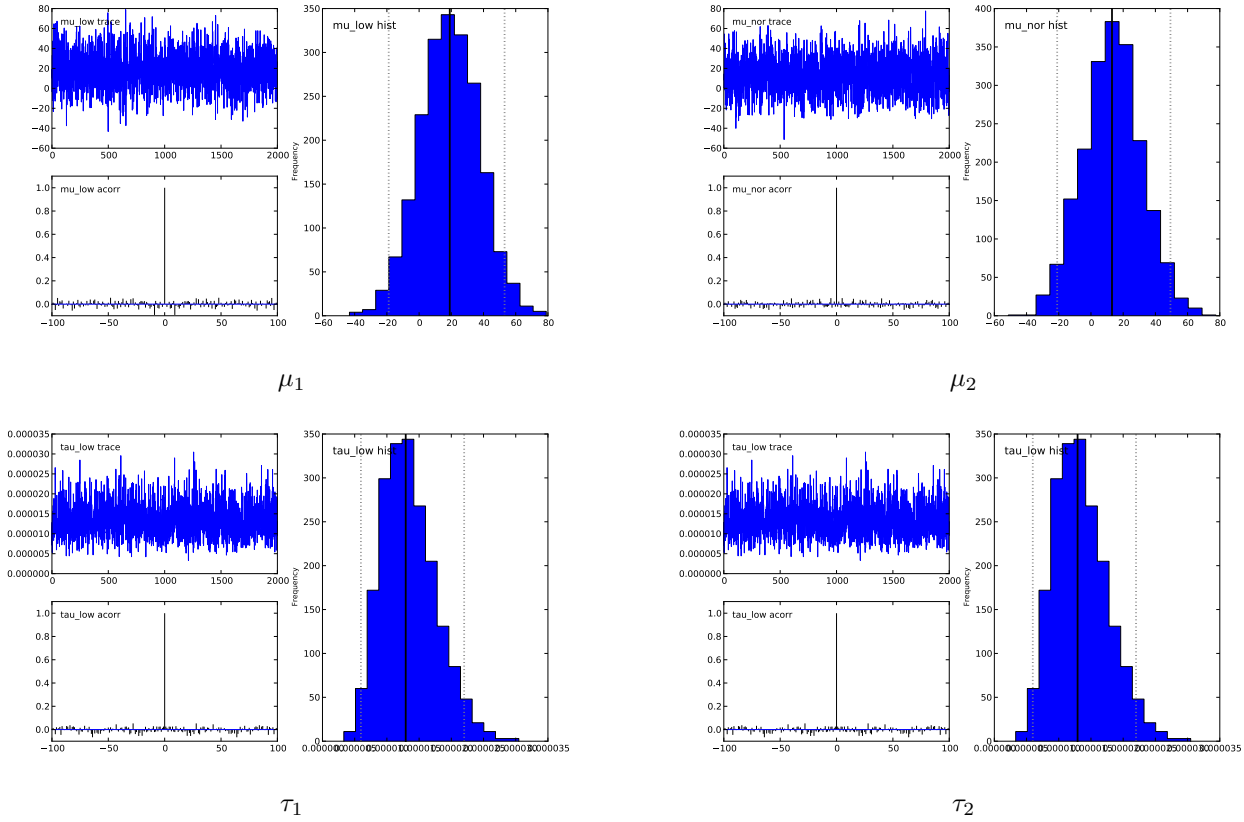


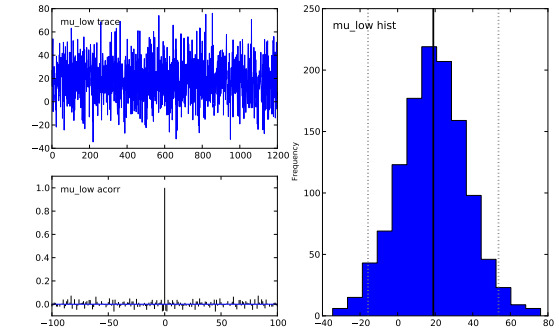
Figure 7: Posterior summary for θ using a uniform prior



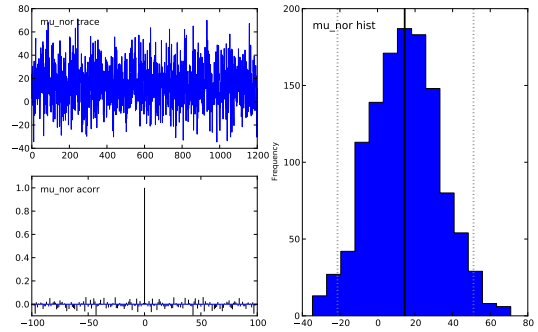
μ_1 μ_2

τ_1 τ_2

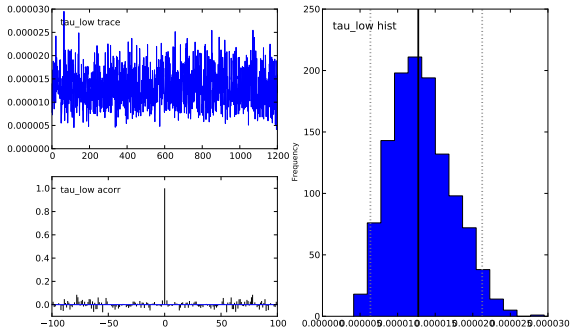
Table 3: Posteriors with Informed Priors



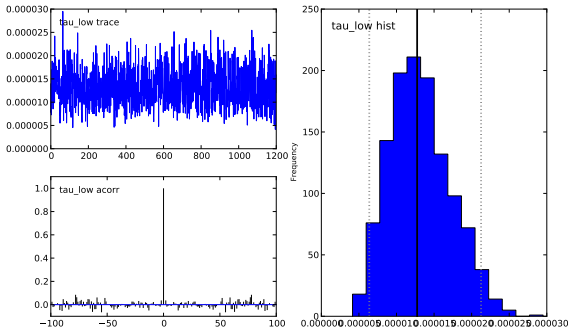
μ_1



μ_2



τ_1



τ_2

Table 4: Posteriors with Uninformed Priors