Using ANOVA Models to Compare Efficacy of Randomized Complete Block Design vs. Completely Random Design

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Abstract

In this paper, we investigate the efficacy of an RCBD design vs. a CRD design, specifically by measuring a decrease in variability. In real life applications of statistics, especially in agriculture, nuisance factors exist in our data which we may not care for as they relate to our experiment and our response variable. Thus, blocking is an effective means of creating homogeneous experimental units to generate more precise estimates. For this experiment, we used agricultural data measuring Nitrogen levels under treatments to conduct ANOVA of a CRD design, then on an RCBD design (blocking established based on which fields were used). Subsequently we observed values that signaled decreased variance from the RCBD design. Our results showed a scaled decrease of 0.57x from Mean Square Error of the CRD to the Mean Squared Error of the RCBD, and the Mean Squared for Blocks is large relative to Error (suggesting significant difference between blocks). We conclude from these results that an RCBD improves the precision with which treatment means are compared, thus supporting the use of RCBD to improve the quality of statistical tests done in industries that often have controllable lurking factors at play.

Introduction

In conducting an experimental design, there are various means of reducing error, and a crucial way that has applications in real life trials is conducting Randomized Complete Block Designs. By 'blocking', wherein an experimenter groups experimental units based on recognizable similarities (though random blocking is also a tool used), he or she aims to decrease variability within each group, allowing for us to detect differences caused by treatments more clearly. Often in experiments, we wish to isolate solely the factors we are testing against the response variable, thus any reduction of other noise is essential to an accurate test. We refer to these other sources of noise as nuisance factors, because they are design factors that may impact the response variable, though we are not interested in them. The alternate, purely randomized design for an experiment is a CRD (completely randomized design), where in "the treatments are assigned completely at random so that each experimental unit has the same chance of receiving any one treatment." (A Statistical Manual For Forestry Research) RCBD differs in that blocks are more homogenous experimental unit groups, but treatments are "allotted randomly to the experimental units within each block." (R Codes for Randomized Complete Block Design)

Overall, there are three core elements to a good design: replication, randomization, and local control (R Codes for Randomized Complete Block Design)

Replication refers to the necessity for repetition of treatments, randomization refers to treatments getting an equal probability of selection to experimental units, and local control factors in the need for blocking given possible nuisance factors (e.g., in agriculture, an industry that heavily relies on statistical tests, there are many sources of noise that must be addressed when running tests related to crop yields, chemical levels, etc.)

"R Codes for Randomized Complete Block Design" itself is a research paper written that researches the efficacy of a RCBD using Grain Yield of Rice variety IR8 with six different rates of seeding, specifically through a pre-written R function that outputs critical summary values.

Before we delve into my experimental design project, we also want to discuss the use of fixed vs. random blocking. In a paper by Philip Dixon of Iowa State University titled "Should Blocks Be Fixed or Random", he argues that blocks should be a fixed effect, unless there are compelling reasons for them to be treated as a random effect - his reasoning involves the fact that blocks are rarely a random sample from some larger population and random blocking provides a treatment effect estimator that is more likely to be biased, among other reasons (Source 3). Thus, in my experiment we will be using fixed blocking on my dataset (though we would like to note that the original dataset uses random blocking that disregards the importance of discerning between field 1 and 2, as introduced below in Materials and Methods).

To analyze the difference between CRD and RCBD we will be eliminating blocking for the CRD and subsequently seeing both design types ANOVA summary values to see if RCBD is an overall improvement as per a decrease of variability.

Below are calculations for ANOVA CRD and RCBD tables, though we will be using R script to forgo the calculations for Sum of Squares and Mean Square.

ANOVA of CRD (A Statistical Manual For Forestry Research)

Source of variation	Degree of freedom (df)	Sum of squares (SS)	Mean square $\left(MS = \frac{SS}{df}\right)$	Computed F
Treatment	t - 1	SST	MST	$\frac{MST}{MSE}$
Error	n - t	SSE	MSE	
Total	n - 1	$SSTO$		

ANOVA of RCBD (A Statistical Manual For Forestry Research)

Source of variation	Degree of freedom (df)	Sum of squares (SS)	Mean square $\left(MS = \frac{SS}{df}\right)$	Computed F
Replication	r - 1	SSR	MSR	
Treatment	t - 1	SST	MST	MST/MSE

Error	(r - 1)(t - 1)	SSE	MSE	
Total	rt - 1	SSTO		

Material and Methods

Using a dataset provided by "Agricultural Statistics Support" we will be conducting a comparison between CRD and RCBD with Nitrogen levels as a response variable for crops which have undergone 6 treatments over two fields. My experimental design is as follows:

1. Conduct ANOVA on a fitted model using Nitrogen and Treatment, with Nitrogen as the response variable (with blocking removed from the dataset to mimic CRD)
2. Generate a summary statement, noting significance levels of factors, Mean Squared Error, and residual plots
3. Conduct ANOVA on a fitted model using Nitrogen, Treatment, and Blocks (to enforce a RCBD)
4. Generate a summary statement, noting significance levels, Mean Squared Error, and residual plots
5. Using the results gathered to compare CRD and RCBD to make a determination on variance

To conduct the necessary statistical procedures, we will be using R through RStudio, using the following functions:

aov() - to generate a model with ANOVA

summary() - to gather ANOVA model statistics

plot() - to gather residual plots

Results and Discussion

The R code we used is written below:

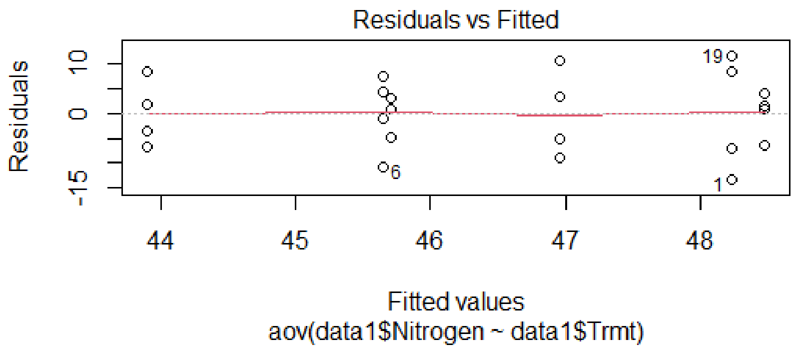
```
# # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # #
# #

# Final Project - Analyzing the Effectiveness of Blocking

# Using agricultural data documenting Nitrogen levels
# given 6 treatments on crops and 2 fields, we first generate a
CRD

# Simply contains Treatments and Nitrogen levels
data1$Trmt <- as.factor(data1$Trmt)

data1

model1 <- aov(data1$Nitrogen~data1$Trmt)

summary(model1)

plot(model1)

# Important values to note are data1$Trmt of 0.959, suggesting
# that at a significance level of 0.05
# the Nitrogen levels were not significantly different among
treatments
# Also, Residuals Mean Sq is 61.20

# Next, we conduct the expected RCBD

data2$Trmt <- as.factor(data2$Trmt)

data2
```

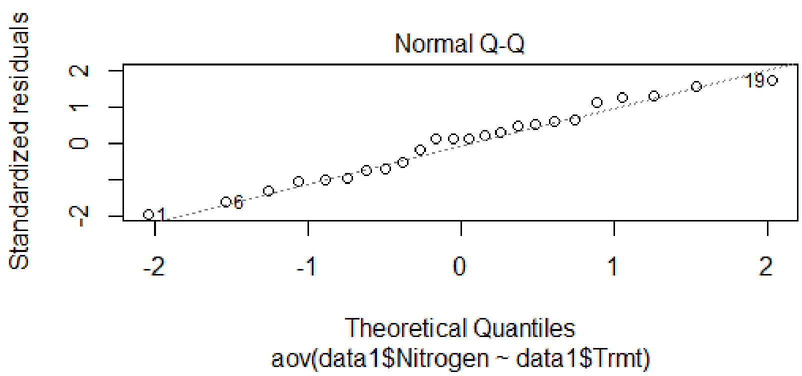
```
model2 <- aov(data2$Nitrogen ~ data2$Block + data2$Trmt)

summary(model)

plot(model2)

# There is a significant difference among Blocks.
# Residuals Mean Sq is much lower than that of CRD

# # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # #
# #
```

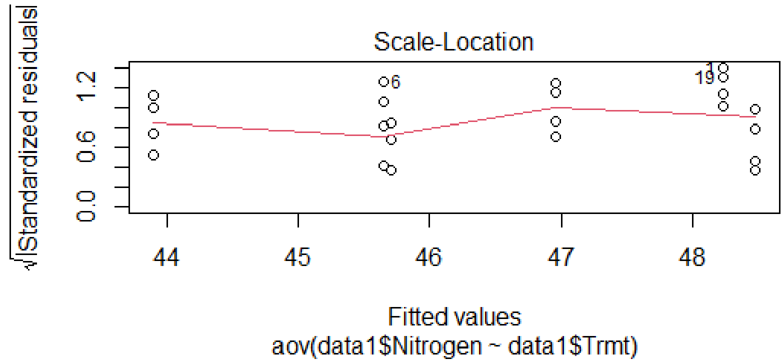
The ANOVA statistics for the CRD are as follows:

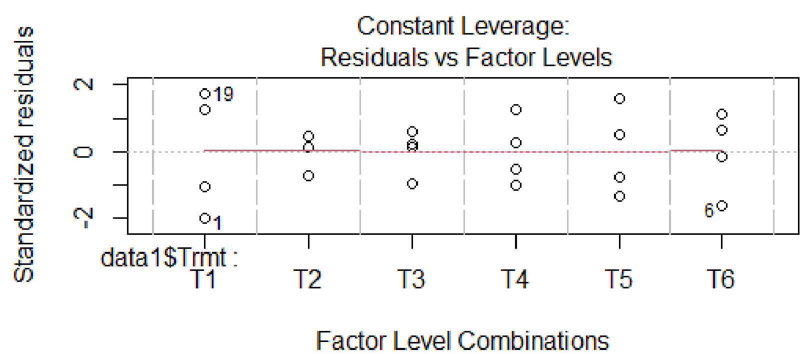
```
            Df Sum Sq Mean Sq F value

data1$Trmt   5   60.8   12.16   0.199

Residuals   18 1101.6   61.20

            Pr(>F)

data1$Trmt   0.959

Residuals
```

Residual plots are provided below:


Residuals vs Fitted
Residuals
10
0
-15
19
6
1
44
45
46
47
48
Fitted values
aov(data1$Nitrogen ~ data1$Trmt)


Normal Q-Q
Standardized residuals
2
1
0
-2
1
6
19
-2
-1
0
1
2
Theoretical Quantiles
aov(data1$Nitrogen ~ data1$Trmt)





As listed, the Mean Squared Error is 61.20, which we will compare to the RCBD Mean Squared Error.

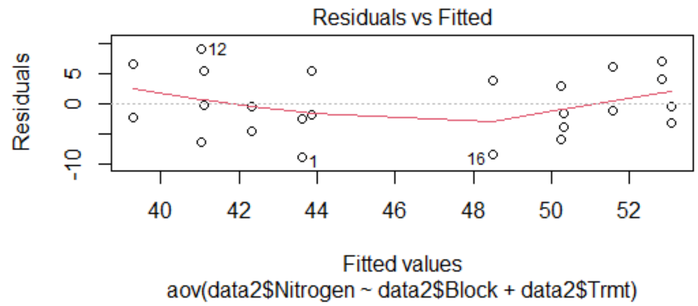
A Pr(>F) of 0.959 is cause to not reject the null hypothesis - treatment levels do not affect Nitrogen

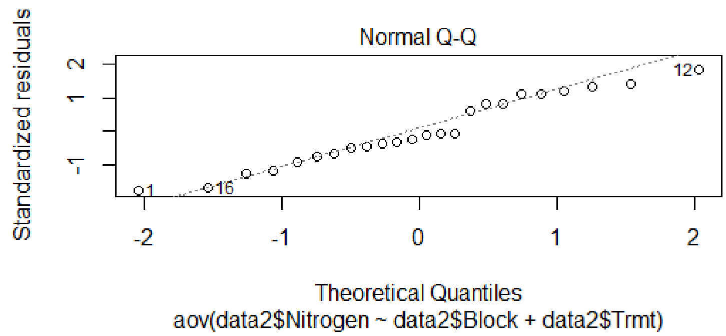
As per the residual plots, normality is maintained (especially the Q-Q plot, wherein the dataset exhibit points that fall along a straight diagonal line)

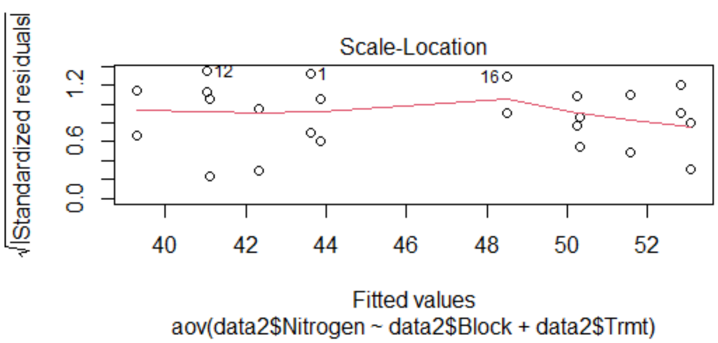
The ANOVA statistics for the RCBD are as follows:

```
# # # # # # # # # # # # # # # # # # #

             Df Sum Sq Mean Sq F value
data2$Block  1  509.7   509.7  14.638
data2$Trmt   5   60.8    12.2   0.349
Residuals   17  591.9    34.8
              Pr(>F)
data2$Block 0.00135 **
data2$Trmt  0.87568
Residuals

# # # # # # # # # # # # # # # # # # #
```

Residual plots are provided below:







As listed, the Mean Squared Error is 34.8, a Pr(>F) of 0.00135 for data2$Block suggests that there is significant difference among treatments. Another indicator of the blocks' significant difference is the blocks' Mean Square being large relative to error.

As per the residual plots, normality is maintained (especially the Q-Q plot, wherein the dataset exhibit points that fall along a straight diagonal line)

It is important to note that the mean square for error has decreased by a scale of ~0.57, decreasing from 61.2 to 34.8 (for CRD to RCBD). As per our course textbook, "All of the variability due to blocks is now in the error term. This makes it easy to see why we sometimes call the RCBD a noise-reducing design technique; it effectively increases the signal-to-noise ratio in the data, or it improves the precision with which treatment means are compared." (Pages 121-122)

The results provided do confirm more accurate results from RCBD as opposed to CRD, as per a significantly lower MSE, which can be attributed to the formation of homogeneous blocks. Overall, RCBD serves as a better estimate of treatment effects. In the context of our data, fields 1 vs. 2 had a large impact on Nitrogen, a phenomenon applicable to real-life industry. Such methods are critical in agricultural statistics, which provides many numerical data related to harvests, yields, treatments, etc.; also, there are many variable factors that can be accounted for through agricultural statistical tools, like blocking (e.g., sunlight, fields, water, etc.) To make conclusions of statistical tests is critical for agriculturalists, as they use such quantitatives in dictating how to conduct their growth in subsequent seasons.

Literature Cited

Jayaraman, K. (n.d.). 4. Design and Analysis of Experiments. A Statistical Manual For Forestry Research. Retrieved April 24, 2022, from https://www.fao.org/3/x6831e/x6831e07.htm

Jeelani, M. I., Nazir, N., Sharma, M. K., Bhat, A., Ghul, M., & Kumar, B. (2018). R Codes for Randomized Complete Block Design. International Journal of Information Science and System, 6. Retrieved April 24, 2022.

Dixon, Philip (2016). "Should Blocks Be Fixed Or Random?," Conference on Applied Statistics in Agriculture. https://doi.org/10.4148/2475-7772.1474

Appendices

Appendix I: Datasets (courtesy of Agricultural Statistics Support)

CRD Dataset

Trmt,Nitrogen

T1,34.90

T2,40.80

T3,42.00

T4,37.10

T5,37.90

T6,34.80

T1,41.20

T2,46.60

T3,49.40

T4,45.80

T5,41.90

T6,50.10

T1,56.90

T2,46.60

T3,52.60

T4,40.20

T5,57.60

T6,44.50

T1,59.90

T2,48.80

T3,49.90

T4,52.50

T5,50.40

T6,53.20

RCBD Dataset

Block,Trmt,Nitrogen

1,T1,34.90

1,T2,40.80

1,T3,42.00

1,T4,37.10

1,T5,37.90

1,T6,34.80

1,T1,41.20

1,T2,46.60

1,T3,49.40

1,T4,45.80

1,T5,41.90

1,T6,50.10

2,T1,56.90

2,T2,46.60

2,T3,52.60

2,T4,40.20

2,T5,57.60

2,T6,44.50

2,T1,59.90

2,T2,48.80

2,T3,49.90

2,T4,52.50

2,T5,50.40

2,T6,53.20

Contributions:

Given the constraints we (Aakash and Angelique) faced with being a two member team, we were able to work effectively to ensure the project is done in a timely manner, adhering to the many guidelines set for us. Aakash contributed to the analysis/interpretation of results and paper writing and Angelique contributed to narrowing in on the idea, searching for appropriate research to utilize, and paper writing.

3 Questions:

1. Short Answer: How does a CRD differ from an RCBD? Is one considered more effective than the other?
2. Multiple Choice: What is the minimum number of blocks required in an RCBD?
3. True/False: Lower MSE correlates to lower variance