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Autonomous Systems Lab 2

Section a

For this section we were asked "*to submit the formal description of the variables and constraints of your SAT model*". In our case we described variables as:

$$x_{i,j,k}$$

This states a boolean variable for each i representing the possible rows, j representing each possible column, and k representing each possible value. All of these are constrained to $\{1, 2, \dots, n\}$. Because we will later use our model as an input to a SAT solver, we need to express all constraints in conjunctive normal form (CNF). In our case out model consisted of 6 constraints:

1. Each cell has to have at least one value k:

$$C_1 = \bigwedge_{1 \le i \le n,\ 1 \le j \le n} (x_{i,j,1} \vee x_{i,j,2} \vee \dots \vee x_{i,j,n})$$

2. Each cell has to have at most one value k:

$$C_2 = \bigwedge_{1 \le i \le n,\ 1 \le j \le n,\ 1 \le k < k' \le n} (\neg x_{i,j,k} \vee \neg x_{i,j,k'})$$

3. Each row needs to have all possible values k':

$$C_3 = \bigwedge_{1 \le i \le n,\ 1 \le k \le n} (x_{i,1,k} \vee x_{i,2,k} \vee \dots \vee x_{i,n,k})$$

4. Each column needs to have all possible values k':

$$C_4 = \bigwedge_{1 \le j \le n,\ 1 \le k \le n} (x_{1,j,k} \vee x_{2,j,k} \vee \dots \vee x_{n,j,k})$$

5. Each block needs to have all possible values k':

$$C_5 = \bigwedge_{1 \le i' \le \sqrt{n},\ 1 \le j' \le \sqrt{n},\ 1 \le k \le n} \left(\bigvee_{(i,j) \in B_n(i',j')} x_{i,j,k} \right)$$

where:

$$B_n(i', j') = \{(i'\sqrt{n} + x, j'\sqrt{n} + y) \mid 0 \le x < \sqrt{n},\ 0 \le y < \sqrt{n}$$

6. The solution must respect all previous constraints:

$$C_6 = \wedge_{(i,j,k) \in H} \ (x_{i,j,k})$$