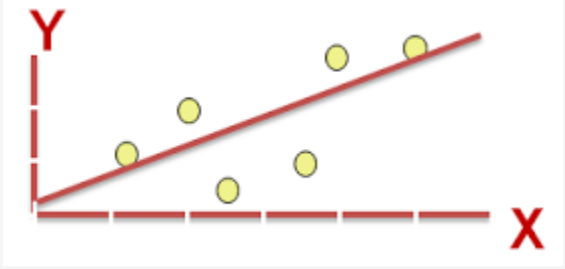
Linear Regression in R - R Tutorial

This article explains how to run linear regression in R. This tutorial covers assumptions of linear regression and how to treat if assumptions violate. It also covers fitting the model and calculating model performance metrics to check the performance of linear regression model. Linear Regression is one of the most popular statistical technique. It has been in use for more than 3 decades. It is widely accepted in almost every domain as it's easy to understand output of linear regression.

Linear Regression

It is a way of finding a relationship between a single, continuous variable called Dependent or Target variable and one or more other variables (continuous or not) called Independent Variables.



Linear Regression

It's a straight line curve. In the above figure, diagonal red line is a regression line which is also called best-fitting straight line. The distance between dots and regression line is errors. Linear regression aims at finding best fitting straight line by minimizing the sum of squared vertical distance between dots and regression line.

Variable Type

Linear regression requires the dependent variable to be continuous i.e. numeric values (no categories or groups).

Simple vs. Multiple Linear Regression

Linear regression can be simple linear regression when you have only one independent variable . Whereas Multiple linear regression will have more than one independent variable.

Regression Equation

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + \ldots\ldots\ldots + b_kX_k$$

Linear Regression Equation

Interpretation:

b_0 is the intercept the expected mean value of dependent variable (Y) when all independent variables (Xs) are equal to 0. and b_1 is the slope.
b1 represents the amount by which dependent variable (Y) changes if we change X_1 by one unit keeping other variables constant.

Important Term : Residual

The difference between an observed (actual) value of the dependent variable and the value of the dependent variable predicted from the regression line.

Algorithm

*Linear regression is based on **least square estimation** which says regression coefficients (estimates) should be chosen in such a way that it minimizes the sum of the squared distances of each observed response to its fitted value.*

Minimum Sample Size

Linear regression requires 5 cases per independent variable in the analysis.

Assumptions of Linear Regression Analysis

1. **Linear Relationship :** Linear regression needs a linear relationship between the dependent and independent variables.

2. **Normality of Residual :** Linear regression requires residuals should be normally distributed.

3. **Homoscedasticity** : Linear regression assumes that residuals are approximately equal for all predicted dependent variable values. In other words, it means constant variance of errors.

4. **No Outlier Problem**

5. **Multicollinearity :** It means there is a high correlation between independent variables. The linear regression model MUST NOT be faced with problem of multicollinearity.

6. **Independence of error terms - No Autocorrelation**

It states that the errors associated with one observation are not correlated with the errors of any other observation. It is a problem when you use time series data. Suppose you have collected data from labors in eight different districts. It is likely that the labors within each district will tend to be more like one another that labors from different districts, that is, their errors are not independent.

If you want to know how to check these assumptions and how to treat them if violated, check out this tutorial - **Checking the assumptions and Treatment to Violations of Assumptions**

Distribution of Linear Regression

Linear regression assumes target or dependent variable to be ***normally distributed****. Normal Distribution is same as Gaussian distribution. It uses identity link function of gaussian family*

Standardized Coefficients

The concept of standardization or standardized coefficients (aka estimates) comes into picture when predictors (aka independent variables) are expressed in different units. Suppose you have 3 independent variables - age, height and weight. The variable 'age' is expressed in years, height in cm, weight in kg. If we need to rank these predictors based on the unstandardized coefficient, it would not be a fair comparison as the unit of these variable is not same.

Standardized Coefficients (or Estimates) are mainly used to rank predictors (or independent or explanatory variables) as it eliminate the units of measurement of independent and dependent variables). We can rank independent variables with ***absolute value of standardized coefficients****. The most important variable will have maximum absolute value of standardized coefficient.*

Standardized Coefficient for Linear Regression Formula

Standardized Coefficient (X1)	= Unstandardized Coefficient (X1)	* (Standard Deviation of x1 / Standard deviation of y)

Standardized Coefficient for Linear Regression Model

Interpretation of Standardized Coefficient

A standardized coefficient value of 1.25 indicates that a change of one standard deviation in the independent variable results in a 1.25 standard deviations increase in the dependent variable.

Detailed Explanation : Standardized vs. Unstandardized Coefficient

Measures of Model Performance

1. R-squared

It measures the proportion of the variation in your dependent variable explained by all of your independent variables in the model. It assumes that every independent variable in the model helps to explain variation in the dependent variable. In reality, some variables don't affect dependent variable and they don't help building a good model.

$$r^2 = 1 - \frac{\text{SS Error}}{\text{SS Total}} = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$$

R-Squared Formula

In the numerator of equation above, yi-hat is the predicted value. Mean value of Y appears in denominator.

Rule :

Higher the R-squared, the better the model fits your data. In psychological surveys or studies, we generally found low R-squared values lower than 0.5. It is because we are trying to predict human behavior and it is not easy to predict humans. In these cases, if your R-squared value is low but you have statistically significant independent variables (aka predictors), you can still

generate insights about how changes in the predictor values are associated with changes in the response value.

Can R-Squared be negative?

Yes, it is when horizontal line explains the data better than your model. It mostly happens when you do not include intercept. Without an intercept, the regression could do worse than the sample mean in terms of predicting the target variable. It is not only because of exclusion of intercept. It can be negative even with inclusion of intercept.

Mathematically, it is possible when error sum-of-squares from the model is larger than the total sum-of-squares from the horizontal line.

R-squared = 1 - [(Sum of Square Error)/(Total Sum of Square)]

2. Adjusted R-squared

It measures the proportion of variation explained by only those independent variables that really affect the dependent variable. It penalizes you for adding independent variable that do not affect the dependent variable.

$$R^2_{adjusted} = 1 - \frac{(1 - R^2)(N - 1)}{N - p - 1}$$

where

R^2 = sample R-square

p = Number of predictors

N = Total sample size.

Adjusted R-Squared

Adjusted R-Squared is more important metrics than R-squared

Every time you add a independent variable to a model, the R-squared increases, even if the independent variable is insignificant. It never declines. Whereas Adjusted R-squared increases only when independent variable is significant and affects dependent variable.

3. RMSE (Root Mean Square Error)

It explains how close the actual data points are to the model's predicted values. It measures standard deviation of the residuals.

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

RMSE Calculation

In the formula above, yi is the actual values of dependent variable and yi-hat is the predicted values, n - sample size.

Important Point

RMSE has the same unit as dependent variable. For example, dependent variable is sales which is measured in dollars. Let's say RMSE of this sales model comes out 21. We can say it is 21 dollars.

What is good RMSE score?

There is no thumb rule regarding good or bad RMSE score. It is because it is dependent on your dependent variable. If your target variable lies between 0 to 100. RMSE of 0.5 can be considered as good but same 0.5 RMSE can be considered as a poor score if dependent variable ranges from 0 to 10. Hence there is so such good or bad RMSE by simply looking at the value.

Lower values of RMSE indicate better fit. RMSE is a good measure of how accurately the model predicts the response, and is the most important criterion for fit if the main purpose of the model is prediction.

The RMSE for your training and your test sets should be very similar if you have built a good model. If the RMSE for the test set is much higher than that of the training set, it is likely that you've badly over fit the data, i.e. you've created a model that works well in sample, but has little predictive value when tested out of sample

RMSE vs MAE

RMSE amplifies and severely punishes large errors as compared to mean absolute error (MAE).

R-Squared vs RMSE

R-squared is in proportion and has no units associated to target variable whereas RMSE has units associated to target variable. Hence, R-squared is a relative measure of fit, RMSE is an absolute measure of fit.

The code below covers the assumption testing and evaluation of model performance :

1. Data Preparation
2. Testing of Multicollinearity
3. Treatment of Multicollinearity
4. Checking for Autocorrelation
5. Checking for Outliers
6. Checking for Heteroscedasticity
7. Testing of Normality of Residuals
8. Forward, Backward and Stepwise Selection
9. Calculating RMSE
10. Box Cox Transformation of Dependent Variable
11. Calculating R-Squared and Adj, R-squared manually
12. Calculating Residual and Predicted values
13. Calculating Standardized Coefficient

R Script : Linear Regression

Theory part is over. Let's implement linear regression with R -

Load required packages

```
library(ggplot2)
library(car)
library(caret)
library(corrplot)
```

Make sure the above listed packages are already installed and loaded into R. If they are not already installed, you need to install it by using the command **install.packages("package-name")**

Read Data

We will use **mtcars** dataset from cars package. This data was extracted from the Motor Trend US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles.

```
#Loading data
data(mtcars)

# Looking at variables
str(mtcars)
```

```
'data.frame': 32 obs. of  11 variables:

 $ mpg : num  21 21 22.8 21.4 18.7 18.1 14.3 24.4 22.8 19.2 ...

 $ cyl : num  6 6 4 6 8 6 8 4 4 6 ...

 $ disp: num  160 160 108 258 360 ...

 $ hp  : num  110 110 93 110 175 105 245 62 95 123 ...

 $ drat: num  3.9 3.9 3.85 3.08 3.15 2.76 3.21 3.69 3.92 3.92 ...

 $ wt  : num  2.62 2.88 2.32 3.21 3.44 ...

 $ qsec: num  16.5 17 18.6 19.4 17 ...

 $ vs  : num  0 0 1 1 0 1 0 1 1 1 ...

 $ am  : num  1 1 1 0 0 0 0 0 0 0 ...

 $ gear: num  4 4 4 3 3 3 3 4 4 4 ...

 $ carb: num  4 4 1 1 2 1 4 2 2 4 ...
```

Variable description of the above variables are listed below against their respective variable names.

Variable	Description
mpg	Miles/(US) gallon
cyl	Number of cylinders
disp	Displacement (cu.in.)
hp	Gross horsepower

drat	Rear axle ratio
wt	Weight (1000 lbs)
qsec	1/4 mile time
vs	V/S
am	Transmission (0 = automatic, 1 = manual)
gear	Number of forward gears
carb	Number of carburetors

Summarize Data

In this dataset, mpg is a target variable. **See first 6 rows of data** by using head() function.

```
> head(mtcars)

                   mpg cyl disp  hp drat    wt  qsec vs am gear carb

Mazda RX4         21.0   6  160 110 3.90 2.620 16.46  0  1    4    4

Mazda RX4 Wag     21.0   6  160 110 3.90 2.875 17.02  0  1    4    4

Datsun 710        22.8   4  108  93 3.85 2.320 18.61  1  1    4    1

Hornet 4 Drive    21.4   6  258 110 3.08 3.215 19.44  1  0    3    1

Hornet Sportabout 18.7   8  360 175 3.15 3.440 17.02  0  0    3    2

Valiant           18.1   6  225 105 2.76 3.460 20.22  1  0    3    1
```

To see the distribution of the variables, submit summary() function.

summary(mtcars)

```
> summary(mtcars)

      mpg             cyl              disp             hp

 Min.   :10.40   Min.   :4.000   Min.   : 71.1   Min.   : 52.0
```

```
 1st Qu.:15.43   1st Qu.:4.000   1st Qu.:120.8   1st Qu.: 96.5
 Median :19.20   Median :6.000   Median :196.3   Median :123.0
 Mean   :20.09   Mean   :6.188   Mean   :230.7   Mean   :146.7
 3rd Qu.:22.80   3rd Qu.:8.000   3rd Qu.:326.0   3rd Qu.:180.0
 Max.   :33.90   Max.   :8.000   Max.   :472.0   Max.   :335.0
      drat             wt             qsec             vs
 Min.   :2.760   Min.   :1.513   Min.   :14.50   Min.   :0.0000
 1st Qu.:3.080   1st Qu.:2.581   1st Qu.:16.89   1st Qu.:0.0000
 Median :3.695   Median :3.325   Median :17.71   Median :0.0000
 Mean   :3.597   Mean   :3.217   Mean   :17.85   Mean   :0.4375
 3rd Qu.:3.920   3rd Qu.:3.610   3rd Qu.:18.90   3rd Qu.:1.0000
 Max.   :4.930   Max.   :5.424   Max.   :22.90   Max.   :1.0000
       am              gear            carb
 Min.   :0.0000   Min.   :3.000   Min.   :1.000
 1st Qu.:0.0000   1st Qu.:3.000   1st Qu.:2.000
 Median :0.0000   Median :4.000   Median :2.000
 Mean   :0.4062   Mean   :3.688   Mean   :2.812
 3rd Qu.:1.0000   3rd Qu.:4.000   3rd Qu.:4.000
 Max.   :1.0000   Max.   :5.000   Max.   :8.000
```

Data Preparation

Make sure categorical variables are stored as factors. In the program below, we are

converting variables to factors.

```
mtcars$am   = as.factor(mtcars$am)
mtcars$cyl  = as.factor(mtcars$cyl)
mtcars$vs   = as.factor(mtcars$vs)
mtcars$gear = as.factor(mtcars$gear)
```

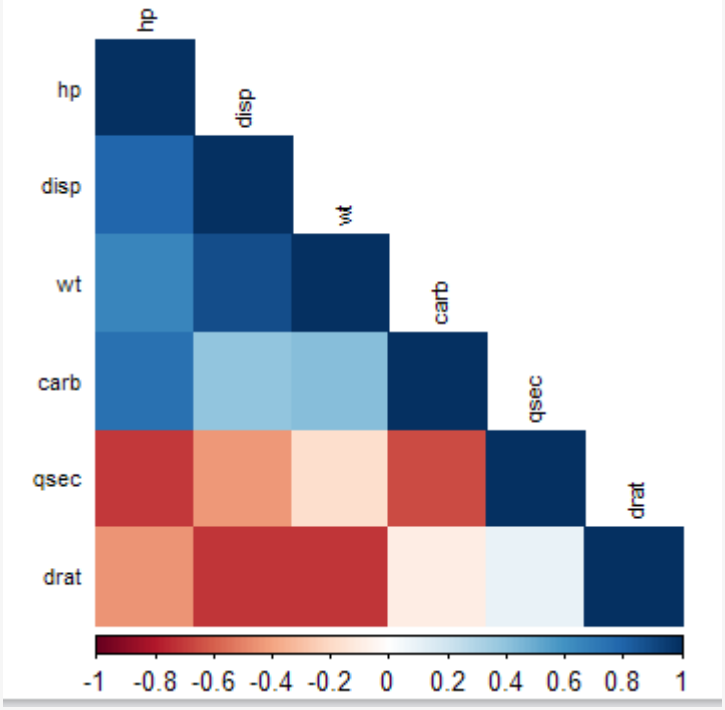
Identifying and Correcting Collinearity

In this step, we are identifying variables which are highly correlated to each other.

```
#Dropping dependent variable for calculating Multicollinearity

mtcars_a = subset(mtcars, select = -c(mpg))



#Identifying numeric variables

numericData <- mtcars_a[sapply(mtcars_a, is.numeric)]



#Calculating Correlation

descrCor <- cor(numericData)



# Print correlation matrix and look at max correlation

print(descrCor)

            disp         hp        drat         wt        qsec       carb

disp  1.0000000  0.7909486 -0.71021393  0.8879799 -0.43369788  0.3949769

hp    0.7909486  1.0000000 -0.44875912  0.6587479 -0.70822339  0.7498125

drat -0.7102139 -0.4487591  1.00000000 -0.7124406  0.09120476 -0.0907898

wt    0.8879799  0.6587479 -0.71244065  1.0000000 -0.17471588  0.4276059

qsec -0.4336979 -0.7082234  0.09120476 -0.1747159  1.00000000 -0.6562492
```

```
carb  0.3949769  0.7498125 -0.09078980  0.4276059 -0.65624923  1.0000000
```

```
# Visualize Correlation Matrix

corrplot(descrCor, order = "FPC", method = "color", type = "lower",
tl.cex = 0.7, tl.col = rgb(0, 0, 0))
```



Correlation Matrix

```
# Checking Variables that are highly correlated

highlyCorrelated = findCorrelation(descrCor, cutoff=0.7)



#Identifying Variable Names of Highly Correlated Variables

highlyCorCol = colnames(numericData)[highlyCorrelated]
```

```
#Print highly correlated attributes

highlyCorCol
```

```
[1] "hp"   "disp" "wt"
```

```
#Remove highly correlated variables and create a new dataset
dat3 = mtcars[, -which(colnames(mtcars) %in% highlyCorCol)]
dim(dat3)
```

```
[1] 32  8
```

There are three variables **"hp" "disp" "wt"** that found to be highly correlated. We have removed them to avoid collinearity. Now, we have 7 independent variables and 1 dependent variable.

Developing Regression Model

At this step, we are building multiple linear regression model.

```
#Build Linear Regression Model

fit = lm(mpg ~ ., data=dat3)



#Check Model Performance

summary(fit)



#Extracting Coefficients

summary(fit)$coeff

anova(fit)



par(mfrow=c(2,2))
```

```
plot(fit)
```

See the coefficients of Linear Regression Model and ANOVA table

Linear regression model tests the null hypothesis that the estimate is equal to zero. An independent variable that has a p-value less than 0.05 means we are rejecting the null hypothesis at 5% level of significance. It means the coefficient of that variable is not equal to 0. A large p-value implies variable is meaningless in order to predict target variable.

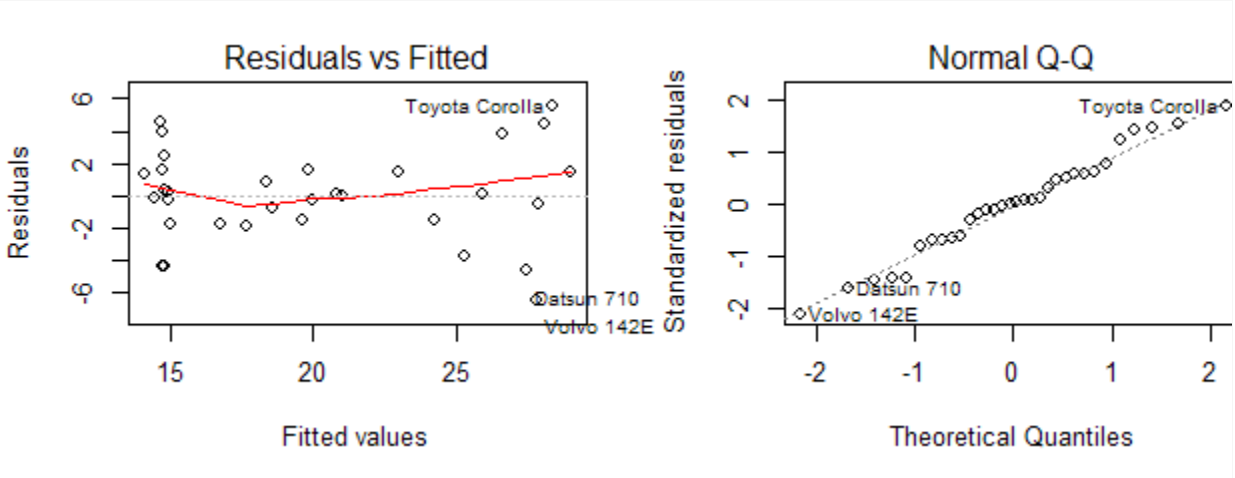
```
> summary(fit)

Call:

lm(formula = mpg ~ ., data = dat3)



Residuals:

    Min      1Q  Median      3Q     Max

-5.4850 -1.3058  0.1856  1.5278  5.2439



Coefficients:

            Estimate Std. Error t value Pr(>|t|)

(Intercept)  16.7823    19.6148   0.856    0.401

cyl6         -1.8025     2.6085  -0.691    0.497

cyl8         -3.5873     4.0324  -0.890    0.383

drat          1.4283     2.1997   0.649    0.523

qsec          0.1003     0.7729   0.130    0.898
```

```
vs1           0.7068     2.3291   0.303    0.764

am1           3.2396     2.4702   1.311    0.203

gear4         1.3869     3.0466   0.455    0.653

gear5         2.3776     3.4334   0.692    0.496

carb         -1.4836     0.6305  -2.353    0.028 *

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 3.004 on 22 degrees of freedom

Multiple R-squared:  0.8237, Adjusted R-squared:  0.7516

F-statistic: 11.42 on 9 and 22 DF,  p-value: 1.991e-06


> anova(fit)

Analysis of Variance Table


Response: mpg

          Df Sum Sq Mean Sq F value    Pr(>F)

cyl        2 824.78  412.39 45.7033 1.464e-08 ***

drat       1  14.45   14.45  1.6017   0.21890

qsec       1   2.83    2.83  0.3137   0.58108

vs         1   1.02    1.02  0.1132   0.73969

am         1  26.35   26.35  2.9198   0.10157

gear       2   8.15    4.07  0.4513   0.64254
```

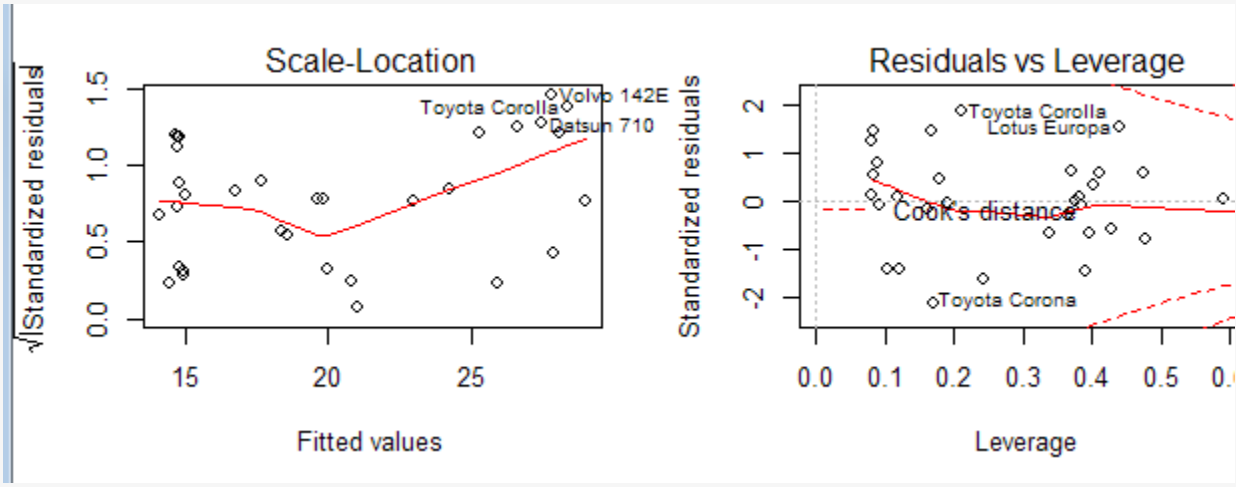
```
carb         1  49.96   49.96  5.5363   0.02798 *
Residuals 22 198.51    9.02
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The critical plots of linear regression model are shown below -

1. Residuals vs Fitted
2. Normal Q-Q
3. Scale Location
4. Residuals vs Leverage



Residuals and Normal Q-Q Plot



Scale- Location Plot

Calculating Model Performance Metrics

```
#Extracting R-squared value

summary(fit)$r.squared
```

[1] 0.8237094

```
#Extracting Adjusted R-squared value
summary(fit)$adj.r.squared
```

[1] 0.7515905

```
AIC(fit)
```

[1] 171.2156

```
BIC(fit)
```

[1] 187.3387

Higher R-Squared and Adjusted R-Squared value, better the model. Whereas, lower the AIC and BIC score, better the model.

Understanding AIC and BIC

AIC and BIC are measures of goodness of fit. They penalize complex models. In other words, it penalize the higher number of estimated parameters. It believes in a concept that a model with fewer parameters is to be preferred to one with more. In general, BIC

penalizes models more for free parameters than does AIC. Both criteria depend on the maximized value of the likelihood function L for the estimated model.

AIC value roughly equals the number of parameters minus the likelihood of the overall model. Suppose you have two models, the model with the lower AIC and BIC score is better.

Variable Selection Methods

There are three variable selection methods - **Forward, Backward, Stepwise.**
1. Starts with a single variable, then adds variables one at a time based on AIC ('Forward')
2. Starts with all variables, iteratively removing those of low importance based on AIC ('Backward')
3. Run in both directions ('Stepwise')

```
#Stepwise Selection based on AIC

library(MASS)

step <- stepAIC(fit, direction="both")

summary(step)




#Backward Selection based on AIC

step <- stepAIC(fit, direction="backward")

summary(step)




#Forward Selection based on AIC

step <- stepAIC(fit, direction="forward")

summary(step)
```

```
#Stepwise Selection with BIC

n = dim(dat3)[1]

stepBIC = stepAIC(fit,k=log(n))

summary(stepBIC)


> summary(stepBIC)

Call:

lm(formula = mpg ~ vs + am + carb, data = dat3)


Residuals:

    Min      1Q  Median      3Q     Max

-6.2803 -1.2308  0.4078  2.0519  4.8197


Coefficients:

            Estimate Std. Error t value Pr(>|t|)

(Intercept)  19.5174     1.6091  12.130 1.16e-12 ***

vs1           4.1957     1.3246   3.168  0.00370 **

am1           6.7980     1.1015   6.172 1.15e-06 ***

carb         -1.4308     0.4081  -3.506  0.00155 **

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Residual standard error: 2.962 on 28 degrees of freedom

Multiple R-squared:  0.7818, Adjusted R-squared:  0.7585

F-statistic: 33.45 on 3 and 28 DF,  p-value: 2.138e-09
```

Look at the estimates above after performing stepwise selection based on BIC. Variables have been reduced but Adjusted R-Squared remains same (very slightly improved). AIC and BIC scores also went down which indicates a better model.

AIC(stepBIC)
BIC(stepBIC)

Calculate Standardized Coefficients

Standardized Coefficients helps to rank predictors based on absolute value of standardized estimates. Higher the value, more important the variable.

```
#Standardised coefficients

library(QuantPsyc)

lm.beta(stepBIC)




#R Function : Manual Calculation of Standardised coefficients

stdz.coff <- function (regmodel)

{ b <- summary(regmodel)$coef[-1,1]

sx <- sapply(regmodel$model[-1], sd)

sy <- sapply(regmodel$model[1], sd)
```

```
beta <- b * sx / sy

return(beta)

}



std.Coeff = data.frame(Standardized.Coeff = stdz.coff(stepBIC))

std.Coeff = cbind(Variable = row.names(std.Coeff), std.Coeff)

row.names(std.Coeff) = NULL
```

Calculating Variance Inflation Factor (VIF)

Variance inflation factor measure how much the variance of the coefficients are inflated as compared to when independent variables are not highly non-correlated. It should be less than 5.

vif(stepBIC)

Testing Other Assumptions

```
#Autocorrelation Test

durbinWatsonTest(stepBIC)



#Normality Of Residuals (Should be > 0.05)

res=residuals(stepBIC,type="pearson")

shapiro.test(res)



#Testing for heteroscedasticity (Should be > 0.05)

ncvTest(stepBIC)



#Outliers – Bonferonni test
```

```
outlierTest(stepBIC)



#See Residuals

resid = residuals(stepBIC)



#Relative Importance

install.packages("relaimpo")

library(relaimpo)

calc.relimp(stepBIC)
```

See Actual vs. Prediction

```
#See Predicted Value
pred = predict(stepBIC,dat3)
#See Actual vs. Predicted Value
finaldata = cbind(mtcars,pred)
print(head(subset(finaldata, select = c(mpg,pred))))
```

```
                   mpg     pred

Mazda RX4         21.0 20.59222

Mazda RX4 Wag     21.0 20.59222

Datsun 710        22.8 29.08031

Hornet 4 Drive    21.4 22.28235

Hornet Sportabout 18.7 16.65583

Valiant           18.1 22.28235
```

Other Useful Functions

```
#Calculating RMSE

rmse = sqrt(mean((dat3$mpg - pred)^2))

print(rmse)



#Calculating Rsquared manually

y = dat3[,c("mpg")]

R.squared = 1 - sum((y-pred)^2)/sum((y-mean(y))^2)

print(R.squared)



#Calculating Adj. Rsquared manually

n = dim(dat3)[1]

p = dim(summary(stepBIC)$coeff)[1] - 1

adj.r.squared = 1 - (1 - R.squared) * ((n - 1)/(n-p-1))

print(adj.r.squared)



#Box Cox Transformation

library(lmSupport)

modelBoxCox(stepBIC)
```