

# Quiz 1 Practice Problems: Cyclic Groups

Math 332, Spring 2010

These are not to be handed in. The quiz will be on Tuesday.

1. Find all generators of  $Z_6$ ,  $Z_8$ , and  $Z_{20}$ .
2. List all elements of the subgroup  $\langle 30 \rangle$  in  $Z_{80}$ .
3. If  $|a| = 60$ , what is the order of  $a^{24}$ ?
4. How many subgroups does  $Z_{20}$  have? List the possible generators for each subgroup.
5. Suppose that  $|a| = 24$ . Find a generator for  $\langle a^{21} \rangle \cap \langle a^{10} \rangle$ . In general, what is a generator for the subgroup  $\langle a^m \rangle \cap \langle a^n \rangle$ ?
6. In  $Z_{60}$ , list all generators for the subgroup of order 12.
7. Let  $G$  be a group and let  $a$  be an element of  $G$ .
  - a. If  $a^{12} = e$ , what can we say about the order of  $a$ ?
  - b. If  $a^m = e$ , what can we say about the order of  $a$ ?
8. List all elements of order 8 in  $Z_{8000000}$ . How do you know your list is complete?
9. Determine the subgroup lattice for  $Z_{p^2q}$ , where  $p$  and  $q$  are distinct primes.
10. Determine the subgroup lattice for  $Z_{p^n}$ , where  $p$  is a prime and  $n$  is some positive integer.
11. If  $|x| = 40$ , list all elements of  $\langle x \rangle$  that have order 10.
12. Determine the orders of the elements of  $D_{33}$  and how many there are of each.
13. If  $|a^5| = 12$ , what are the possibilities for  $|a|$ ? What if  $|a^5| = 15$ ?

# Answers

1. For  $Z_6$ , generators are 1 and 5; for  $Z_8$ , generators are 1, 3, 5, and 7; for  $Z_{20}$ , generators are 1, 3, 7, 9, 11, 13, 17, and 19.

2. 0, 10, 20, 30, 40, 50, 60, 70

3. Since  $\langle a^{24} \rangle = \langle a^{12} \rangle$  is a subgroup of order 5, the element  $a^{24}$  must have order 5 as well.

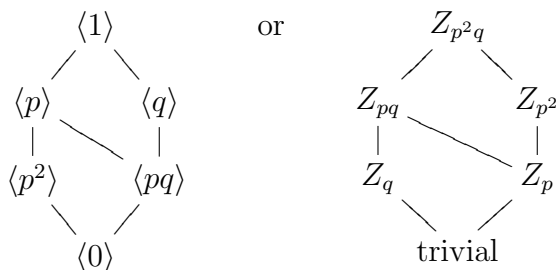
4. Six subgroups:  $Z_{20}$  (generated by 1, 3, 7, 9, 11, 13, 17, or 19), the subgroup of even numbers (generated by 2, 6, 14, or 18), the subgroup of multiples of 4 (generated by 4, 8, 12, or 16), the subgroup of multiples of 5 (generated by 5 or 15), the subgroup of multiples of 10 (generated by 10), and the trivial subgroup (generated by 0).

5.  $\langle a^{21} \rangle \cap \langle a^{10} \rangle = \langle a^6 \rangle$ . In the general case  $\langle a^m \rangle \cap \langle a^n \rangle = \langle a^k \rangle$ , where  $k = \text{lcm}(m, n) \bmod 24$ .

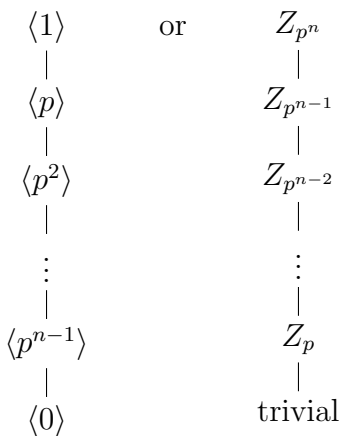
6. 5, 25, 35, and 55.      7. a.  $|a|$  divides 12. b.  $|a|$  divides  $m$ .

8. 1000000, 3000000, 5000000, 7000000. By Theorem 4.3,  $\langle 1000000 \rangle$  is the unique subgroup of order 8, and only those on the list are generators

9.



10.



11.  $x^4, x^{12}, x^{28}, x^{36}$ .

12. 33 of order 2, 20 of order 33, 10 of order 11, 2 of order 3, one of order 1.

13.  $|a| = 12$  or  $|a| = 60$ ;  $|a| = 75$ .