- Reflexive Closure:

→ Reflexive
→ Contains R
→ Minimum (add only whats required)

$A = \{1, 2, 3\}$
$R = \{(1,1), (2,2), (1,2), (2,1)\}$ R is not reflexive
$R_R^* = \{(2,1), (2,2), (1,2), (3,3), (3,1) (1,3)\}$
This isn't minimum

$\therefore$, $R_R^* = \{(1,1), (2,2), (1,2), (2,1), (3,3)\}$

- Transitive Closure

(adding all elements which will make it transitive)

→ Transitive
→ Contains R
→ Minimum

$A = \{1, 2, 3, 4\}$
$R = \{(1,2), (2,3)\}$ (not transitive)

$R_T^* = \{(1,2), (2,3), (1,3)\}$

$A = \{1, 2, 3, 4\}$
$R = \{(1,2), (2,3), (3,4) (4,1)\}$

$R_T^* = \{(1,2), (2,3), (1,3), (3,4), (4,1), (2,4), (3,1), (4,3), (1,4), (2,1), (3,3), (1,1), (4,2) (4,4), (3,2), (2,2)\}$

Date : / /

- Consider the following binary relation

$S = \{x, y \mid y = x+1 \text{ and } x, y \in \{0,1,2\}\}$

A) $\{(x,y) \mid y > x \ \& \ x, y \in \{0,1,2\}\}$
B) $\{(x,y) \mid y \geq x \ \&$ " $\}$
C) $\{x, y \mid y < x \ \&$ " $\}$
D) $\{x, y \mid y \leq x \ \&$ " $\}$

$R = \{(0,1), (1,2), (2,3)\}$
$R_R^* = \{(0,1), (1,2), (2,3), (1,1), (2,2), (3,3)\}$

Note: $C \mid (A \cap B) = (C \mid A) \cap (C \mid B)$. prove.

$C \mid (A \cap B) = C \cap \overline{(A \cap B)}$

$\Rightarrow$

$C \cap (\bar{A} \cap \bar{B}) = (C \cap \bar{A}) \cup (C \cap \bar{B})$
$= (C \mid A) \cup (C \mid B)$

$\therefore$, Not provable

Self Study :

→ Warshall Algorithm — find transitive closure using this algorithm

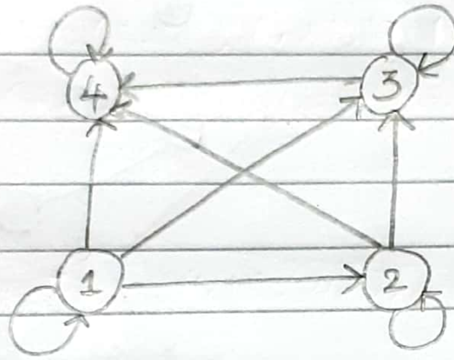
→ Equivalence relation
→ Equivalence classes
→ Partial order relation — Reflexive, Anti symmetric, Transitive } conditions

Hasse Diagram / Poset Representation

$A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

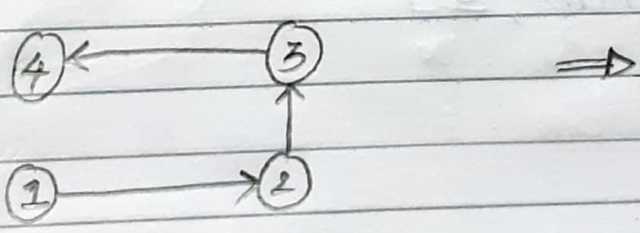
1. Reflexive
2. Antisymmetric
3. Transitive



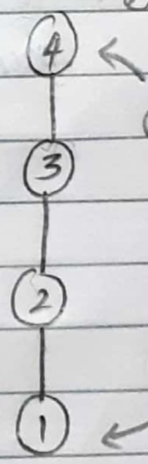
To reduce:

1. Remove all self loops as partial order will always have self loops
2. Remove transitive edges.

Remove $(1,3), (2,4), (1,4)$



Hasse Diagram



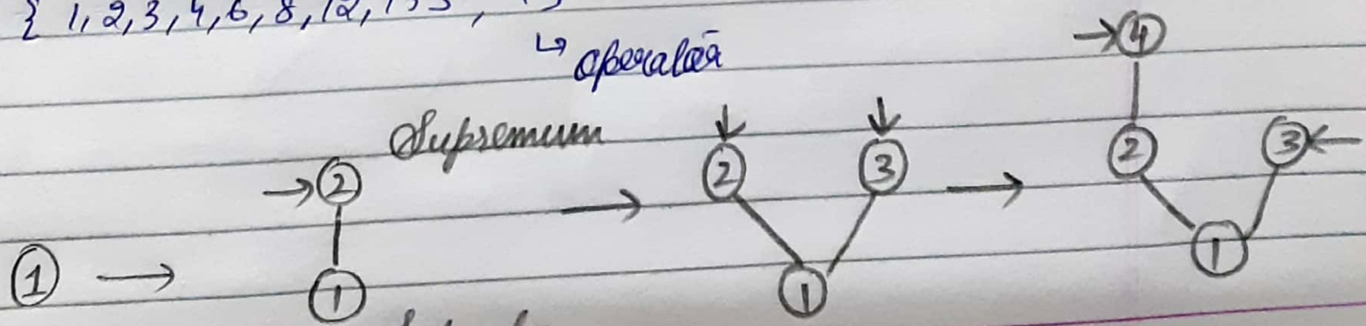
Supremum

Suborder

eg2. $[\{1, 2, 3, 4, 6, 8, 12, 15\}, \backslash]$

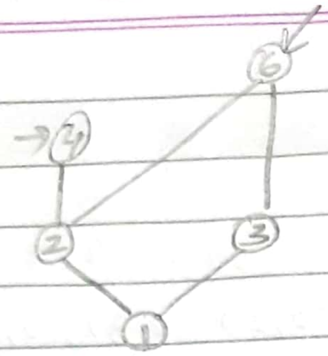
↳ operator

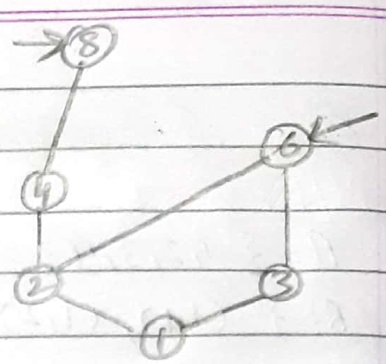
Supremum

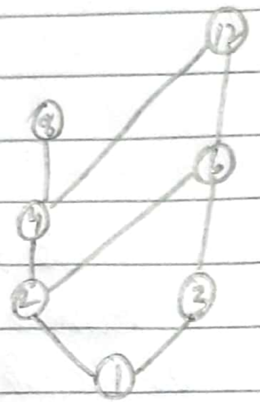


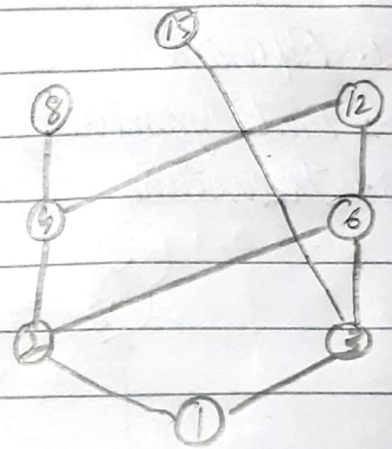
Suborder

as 4 doesn't divide 6 i.e, look at 4's suborder









Hasse Diagram

eg: $A = \{1, 2, 3\}$

make Hasse Diagram for $P(A)$

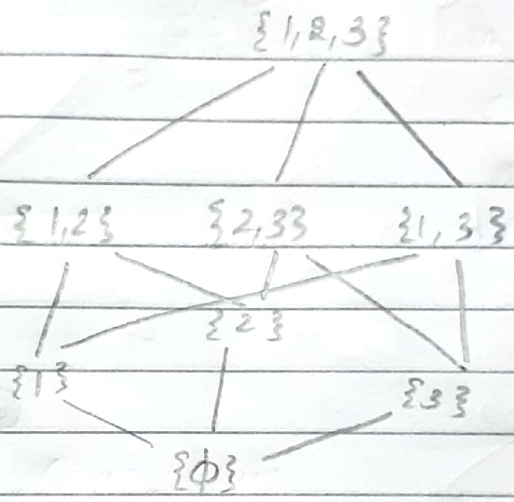
$[P(A), \subseteq]$

Hasse Diagram

$A = \{1, 2, 3\}$

$\boxed{P(A), \subseteq}$

$P(A) = \{ \{\phi\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$

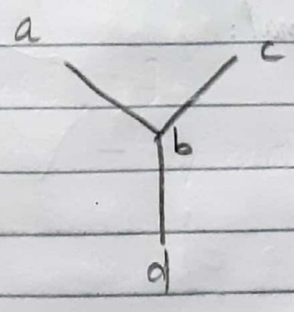


everyone relates to it, but it doesn't relate to anyone

Hash Diagram

- **Maximal and Minimal**

noone relates to it, but it relates to everyone.

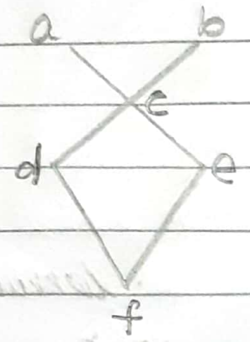


Maximal : a, c

Minimal : d

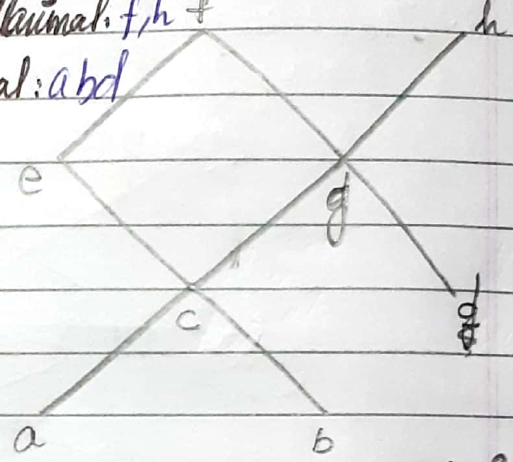
- **Maximal:**
  An element is not related to any other element in a poset.

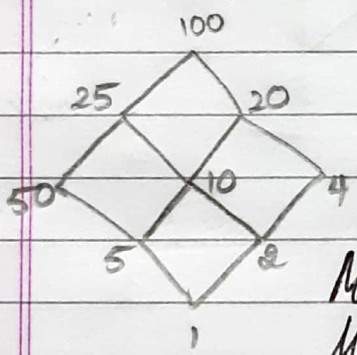
- **Minimal:**
  No element is related to an element in a poset.



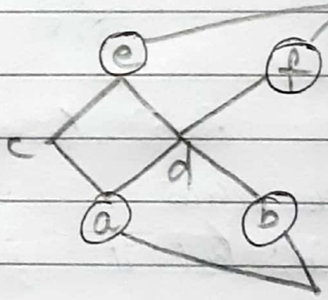
Maximal: a, b
Minimal: f

Maximal: f, h
Minimal: a b d



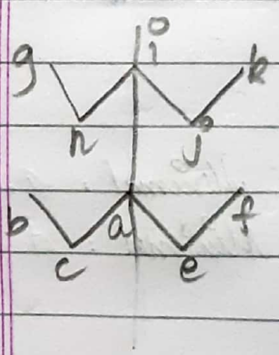


Maximal: 100
Minimal: 1

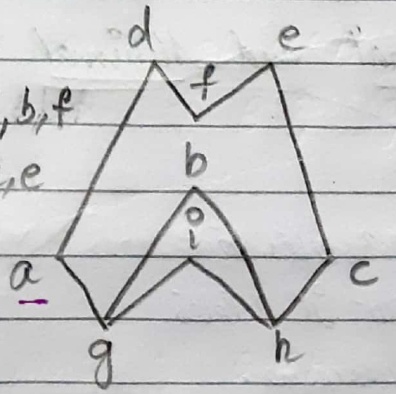


Maximal

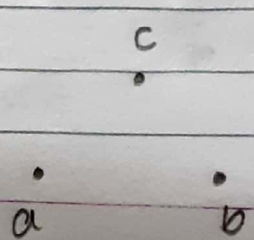
Minimal



Maximal: g, k, i, b, f
Minimal: h, j, c, e



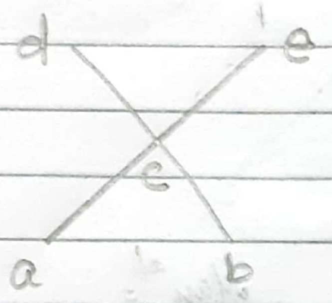
Maximal: d, e, i, b
Minimal: g, h, f



Maximal: a, b, c
Minimal: a, b, c

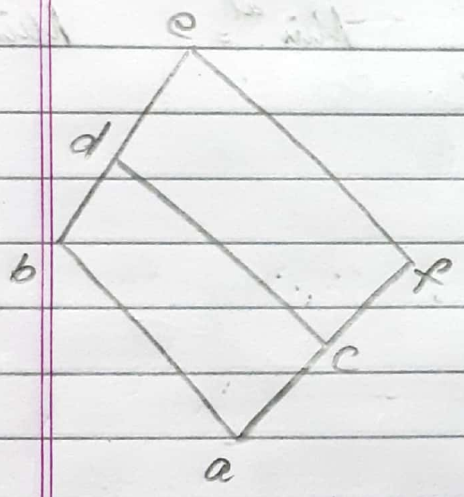
Maximum and Minimum :

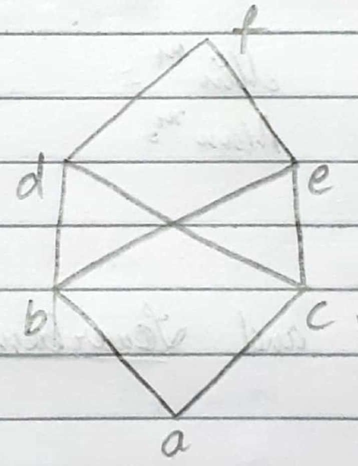
- Maximum: If it is a maximal and every element is related to it in a poset
- Minimum: If it is a minimal and it is related to every element in a poset.

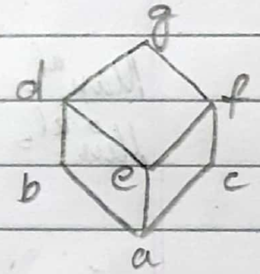


Maximal : d, e
Minimal : a, b

Maximum : ϕ
Minimum : ϕ
(because here we can't determine)







Maximal :
Minimal :

Minimum :
Maximum :

Maximal :
Minimal :

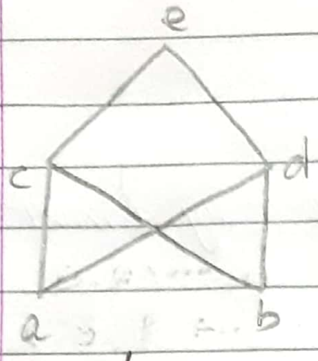
Maximum :
Minimum :

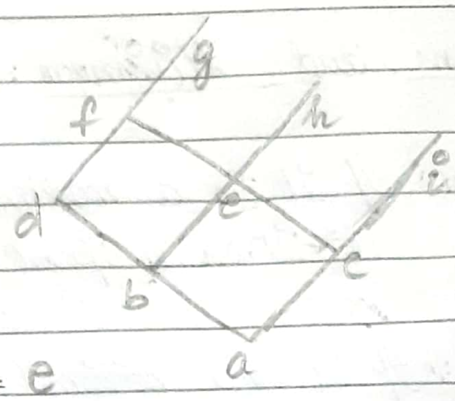
Maximal :
Minimal :

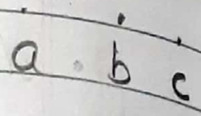
Maximum :
Minimum :

$Max^{al} = g, h, i$ $\quad$ $Max^{m} = \phi$

$Min^{al} = a$ $\quad$ $Min^{m} = a$





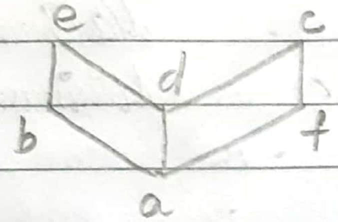


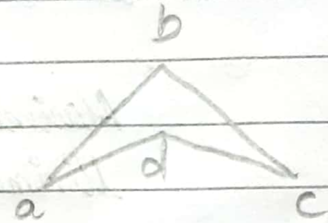
$Max^{al} = e$ $\quad$ $Max^{m} = e$

$Min^{al} = a, b$ $\quad$ $Min^{m} = \phi$

$Max^{al} =$ $\quad$ $Min^{m} =$

$Min^{al} =$ $\quad$ $Min^{m} =$





$Max^{al} =$ $\quad$ $Max^{m} =$

$Min^{al} =$ $\quad$ $Min^{m} =$

$Max^{al} =$ $\quad$ $Min^{m} =$

$Min^{al} =$ $\quad$ $Max^{m} =$

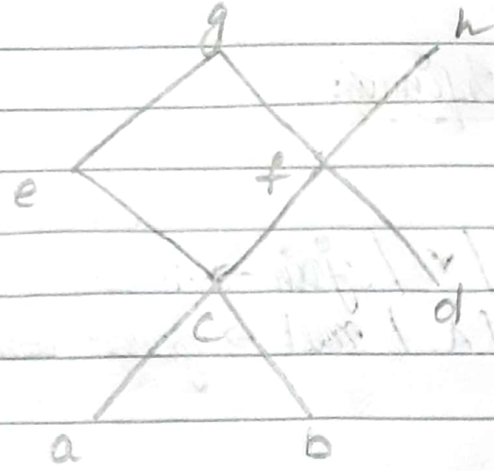
- **Upperbound** and **Lowerbound**

- **Upperbound**

  Let B be a subset of $a \in A$, an element $x \in A$ is in upperbound of B if $(y, x) \in$ poset $\forall\ y \in B$

- **Lowerbound**

  Let B be a subset of $a \in A$, an element $x \in A$ is in lowerbound of B if $(x, y) \in$ poset $\forall\ y \in B$



$B = \{e, c\}$

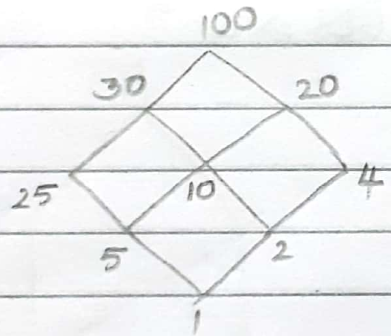
- UB = g, e

e → g, e
c → f, e, h, g, c

- LB = a, b, c

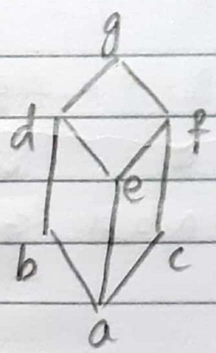
e	c
c → e	a → c
a → e	b → c
b → e	c → c

= x =



B = {50, 10}
UB = 100, 50
LB = 1, 2, 10, 5

B = {5, 10, 2, 4}
UB = 20, 100
LB = 1



B = {d, g}
UB = g
LB = a, b, e, d

B = {e, f}
UB = f, g
LB = a, e

UB
d → g, d
g → g

LB

d	g
b → d	a → g
e → d	b → g
a → d	d → g
d → d	e → g

UB
e → f, d, g, e
f → f, g

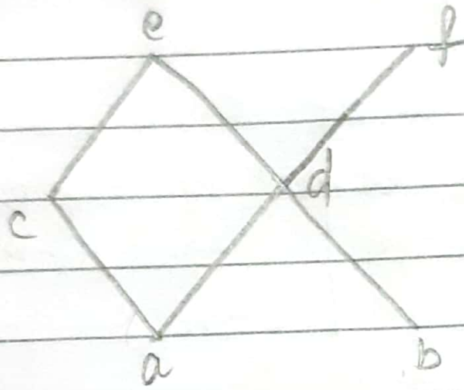
LB

e	f
a → e	e → f
e → e	a → f
	c → f

Date: / /

- Supremum and Infimum

⇒ Least UB / Supremum / $\vee$ / join → least element in UB
⇒ Greatest LB / Infimum / $\wedge$ / meet → greatest element in LB



$B = \{c, d\}$

UB = e
LB = a

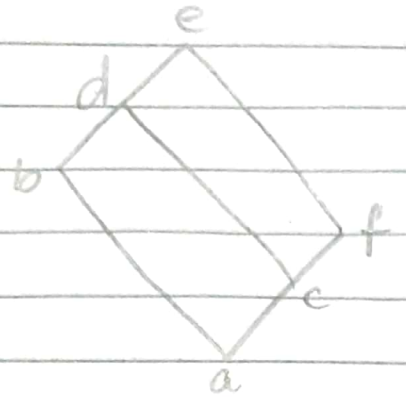
LUB ⇒ e
GLB ⇒ a

$B = \{a, b\}$

UB = d, e, f   LUB = d
LB = ϕ GLB = ϕ

$B = \{e, f\}$

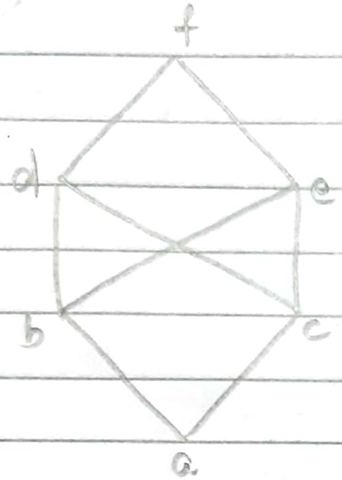
UB = ϕ LUB = ϕ
LB = a, b, d   GLB = d



$B = \{a, c, f\}$

UB = LUB =

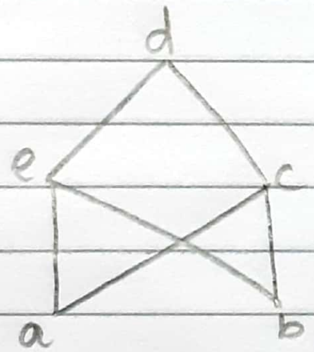
LB = GLB =



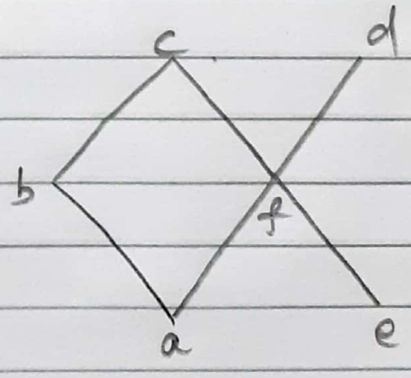
$B = \{d, e\}$ $B = \{b, c\}$

UB = LUB = UB = LUB =

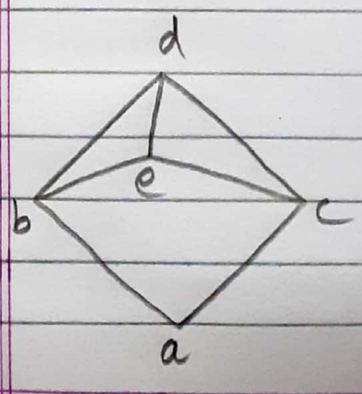
LB = GLB = LB = GLB =



$B = \{a, b\}$



$B = \{d, c\}$



$B = \{e, c\}$