Tutorial Sheet 1: String Language & Grammar Introduction

1. Given the language $L=\{ab,aa,baa\}$, which of the following strings are in L^*?

a) abaabaaabaa b) aaaabaaaa c) baaaaabaaaab d) baaaaabaa

explain your logic behind answer.

2. Prove that $(w^R)^R = w$ for all $w \in \Sigma^*$.

Give a simple description of the language generated by the grammars with productions

4. $S \rightarrow aA \mid \lambda,\ A \rightarrow bS$
5. $S \rightarrow Aa,\ A \rightarrow B,\ B \rightarrow Aa$
6. $S \rightarrow aS \mid bS \mid \lambda$
7. $S \rightarrow aS \mid bS \mid a \mid b$
8. $S \rightarrow aS \mid bS \mid a$
9. $S \rightarrow aSb \mid \lambda$
10. $S \rightarrow aSb \mid ab$
11. $S \rightarrow aSbS \mid bSaS \mid \lambda$
12. $S \rightarrow aS \mid B,\ B \rightarrow b \mid bB$
13. $S \rightarrow aA \mid bB,\ A \rightarrow aA \mid B \mid \lambda,\ B \rightarrow bB \mid \lambda$
14. $S \rightarrow abScT \mid abcT,\ T \rightarrow bT \mid b$
15. $S \rightarrow aSa \mid aa$
16. $S \rightarrow a \mid c \mid aS \mid cS \mid bB,\ B \rightarrow b \mid c \mid bB \mid cB$
17. $S \rightarrow b \mid c \mid bS \mid cS \mid aB,\ B \rightarrow b \mid bS$
18. $S \rightarrow aS \mid bS \mid \lambda$

19. Show that the grammars $G = (\{S\}, \{a, b\}, S, P)$, with productions $S \rightarrow aSb|bSa|SS|a$ and $S \rightarrow aSb|bSa|a$ are not equivalent.

20. Show that for every language L, $LL^* = L^*$ if and only if $\lambda \in L$

21. Show that $(L^*)^* = L^*$ for all languages.

22. Let $\Sigma=\{a,b\}$ and $L=\{aa,bb\}$, find out the complement of L?

23. Let L1 and L2 be subsets of $\{a, b\}^*$. Show that if $L1 \subseteq L2$, then $L1^* \subseteq L2^*$.

24. Let L1 and L2 be subsets of $\{a, b\}^*$. Show that $L1^* \cup L2^* \subseteq (L1 \cup L2)^*$.

25. Give an example of two languages L1 and L2 such that $L1^* \cup L2^*$ =/= $(L1 \cup L2)^*$.

26. One way for the two languages $L1^* \cup L2^*$ and $(L1 \cup L2)^*$ to be equal is for one of the two languages L1 and L2 to be a subset of the other, or more generally, for one of the two languages $L1^*$ and $L2^*$ to be a subset of the other. Find an example of languages L1 and L2 for which neither of $L1^*$, $L2^*$ is a subset of the other, but $L1^* \cup L2^* = (L1 \cup L2)^*$.

27. Let L1, L2, and L3 be languages over some alphabet $\sum$. In each case below, two languages are given. Say what the relationship is between them. (Are they always equal? If not, is one always a subset of the other?) Give reasons for your answers, including counterexamples if appropriate.

a. $L1(L2 \cap L3)$, $L1L2 \cap L1L3$

b. $L1^* \cap L2^*$, $(L1 \cap L2)^*$

c. $L1^*L2^*$, $(L1L2)^*$

28. Consider the language L of all strings of a's and b's that do not end with b and do not contain the substring bb. Find a finite language S such that $L = S^*$.

29. Show that there is no language S such that S* is the language of all strings of a's and b's that do not contain the substring bb.

30. Consider the language $L = \{yy \mid y \in \{a, b\}^*\}$. We know that $L = L\{\lambda\} = \{\lambda\}L$, because every language L has this property. Is there any other way to express L as the concatenation of two languages? Prove your answer.

Find grammars for the following languages:

31. $L= \{ a^n b^m c^m \mid n, m \geq 0\}$

32. $L= \{ a^n b^m c^m \mid n, m \geq 1\}$

33. $L= \{ a^n b^m c^m \mid n \geq 0, m \geq 1\}$

34. $L= \{a^n b^n c^p d^p \mid n \geq 0, p \geq 1\}$

35. $L= \{a^n b^{n+m} c^m \mid n,m \geq 1\}$

36. $L= \{a^n b^{2n} \mid n, m \geq 1\}$

37. L= {w|w contains at least three a's}.

38. L= {w|w starts and ends with same symbol}.

39. L= { }

40. $L=\{a^{n+2} b^n \mid n \geq 1\}$

41. $L= \{a^n b^{n-3} \mid n \geq 3\}$

42. $L=\{a^n b^m \mid (n+m) \text{ is even}\}$

43. $L= \{ 0^{2n} \mid n >= 1\}$

44. L= { $0^m 0^n 0^{(m+n)} 1$ | m >= 1 and n >= 2}

45. L = {$a^n b^m c^n$ | m, n >= 1}

46. L = {$a^n b^n c^{2n}$ | n >= 1}

47. L = {a^n b^m | n,m ≥ 0}

48. L = {w ∈ {a,b}* | w has equal number of a's and b's}

49. L = {an | n ≥ 0} ∪ {bn | n ≥ 0} ∪ {an bn | n ≥ 0}

50. L = {a,b}*

51. The set of all strings with an even number of 0's

52. The set of all strings of even length (length multiple of k)

53. The set of all strings that begin with 110

54. The set of all strings containing exactly three 1's

55. The set of all strings divisible by 2

56. The set of strings where third last symbol is 1

57. Let L = x ∈ {a, b} ∗ : x does not contain two consecutive b's .

58. Find the grammar over ∑={a,b} that generates all strings with exactly one a.