Practice Problem

1. Examine whether the following examples, if are groupoid (A's), Semigroup, monoid, group, or Abelian Group?

(a) $(\mathbb{Z}, *)$ where $a * b = (a+b)/2$

(b) $(\mathbb{Q}, *)$ where $a * b = (a+b)/2$

(c) $(\mathbb{R} \setminus \{0\}, /)$ i.e. non-zero Real Number with division

(2) Verify that $(\mathbb{Z}_5, +)$ is abelian group.

(3) Is $(\mathbb{Z}_5^*, \cdot)$ is cyclic

(4) find a non-trivial subgroup of $(\mathbb{Z}_{12}, \cdot)$

(5) Is $(\mathbb{Z}_8^*, \cdot)$ cyclic

(6) Show for any group $G$ $(ab)^{-1} = b^{-1}a^{-1}$.

(7) let $G$ be a group of the identity element ~~1 and a, b, c~~

(8) $1 \forall a, b, c \in G$ and (a) Given that $x \in G$ fulfills

$\{ ax^2 = b,\ x^3 = 1$, what is $x$?

(b)

(b) Given that $x \in G$ fulfills that

$\{ (xax)^3 = bx,\ x^2a = (xa)^{-1}$ what is $x$?

(9) Show that set $G$ of all numbers of the form $a + b\sqrt{2}$, $a, b \in$ Int forms a group under the operation

$(a + b\sqrt{2}) + (c + d\sqrt{2}) = (a+c) + (b+d)\sqrt{2}$.

(10) Show that the set of all real $(2 \times 2)$ matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a group under Matrix

$ad - bc \neq 0$

Multiplication as binary operation.

(11) Given $g = \{1, a, a^2, a^3, a^4, a^5\}$ such that $a^6 = 1$ is a group with multiplication. Suppose $H = \{1, a^3\}$, $a^6 = 1$ with multiplication is a sub group?

(12) find all generators of $Z_6$ $Z_8$ $Z_{20}$

(13) If $|a| = 60$ what is the order of $a^{24}$?

(14) ~~do~~ To do.

Let $G$ be a cyclic group of order 6. How many of its elements generate $G$.

(15) Define $f : \mathbb{Z} \to \mathbb{Z}$

$f(x) = 5x$

Prove or disprove $f$ is a group map (Homomorphism)

(16) Consider the set $G = \{-1, 1\}$ make $G$ into a group using multiplication. Show that $G$ is isomorphic to $\mathbb{Z}_2$

$f : \mathbb{Z}_2 \to G$

(17) To do: Show that the exponential map

$\exp : (\mathbb{R}, +) \to (\mathbb{R}^+, \cdot)$ given by

(18) $\exp(x) = e^x$ is a group isomorphism

(19)

(20) -

Scanned by CamScanner