Complex Functions

Fall Semester

LECTURER **Zahi Hazan**

Office: room 003 in the small building behind Exact Sicences & Engineering Library
E-mail: zahihaza@post.tau.ac.il

PREREQUISITES
Calculus 1, Calculus 2, Linear Algebra

COURSE DESCRIPTION
This course is an introduction to the theory of analytic functions of one complex variable. Main topics include Cauchy's theorem, series representation of analytic functions, i.e. Taylor and Laurent series, residue theorem, evaluation of improper real integrals using the residue theorem.

COURSE TOPICS

Week 1: The Field of complex numbers: The algebra and geometry of complex numbers. Polar representation. Complex conjugate. Absolute value. Euler identity and De-Moivre's formula: Powers, roots and geometric interpretation.
Week 2: Series of Complex numbers and convergence. Topology: Regions on the complex plane, e.g. disk, annulus, limits in the complex plane
Week 3-4: Functions of a complex variable. Image, limits, continuity and derivatives of complex functions, differentiation rules, Cauchy-Riemann equations and consequences.
Week 5-6: Elementary functions, i.e. exponential function, logarithmic function, trigonometric functions, hyperbolic functions, inverse functions. The logarithmic and exponential functions. Powers, roots and their geometrical interpretations. Branches of multi-valued functions and analytic branches.
Week 7-8: Path integration in the complex plane. Evaluation Theorem. Connected and simply connected regions. Cauchy's theorem. Morera's Theorem.
Week 9: Cauchy's integral and its use to evaluate derivatives. Any order derivatives of analytic function.
Liouville's theorem for entire functions. The fundamental theorem of algebra. Maximum and minimum principals.
Week 10-11: Power series. Radius of convergence. Cauchy-Hadamard's formula for radius of convergence. (Local) Uniform convergence. Weierstrass M-test for uniform

convergence of power series. Term by term differentiation \ integration.
Uniqueness Theorems.
Week 12: Laurent and Taylor series and isolated singular points of analytic functions. Casorati Weierstrass Theorem.
Week 13: Residue Theorem and its applications. Calculation of improper integrals of real valued functions using the residue theorem
(If time permits) The argument principle. Rouche's theorem

ASSIGNMENTS
75% of all homework assignments submission is obligatory.

MIDTERM COURSE POLICY
A midterm exam will be scheduled in the beginning of the semester. During an examination, student shall not use books, papers, or other materials not authorized by the instructor. The midterm will count for 15% of the total course grade using the better grade principle: the midterm grade counts if and only if it is better than or equal to your exam grade. If your midterm grade is lower than your exam grade, the exam grade will be used for that portion of your assessment instead.

FINAL COURSE POLICY
The final exam will cover the entire course material. If the final exam grade is lower than 60, this means failure. Otherwise, the better grade principle will be applied. The duration will be 3 hours. During an examination, student shall not use books, papers, or other materials not authorized by the instructor.
Students will have a first exam, Moed A. If the student does not pass, they can retake the exam, Moed B. The last exam taken will be the student's final grade for the exam.

RECOMENDED READING
James Ward Brown & Ruel V. Churchill, "Complex Variables and Applications", McGraw-Hill, Inc. 1996.
D. Zill, P. Shanahan, "Complex Variables with Applications", Jones and Bartlett Publishers.

ADDITIONAL READING
Saff, Edward B., and Arthur David Snider. *Fundamentals of Complex Analysis with Applications to Engineering, Science, and Mathematics.* 3rd ed. Upper Saddle River, NJ: Prentice Hall, 2002. ISBN: 0139078746.
Sarason, Donald. *Complex Function Theory.* American Mathematical Society. ISBN: 0821886223
Alfhors, Lars. Complex Analysis: *An Introduction to the Theory of Analytic Functions of One Complex Variable.* McGraw-Hill Education, 1979. ISBN: 0070006571.