Recitation 7

Wednesday 10th December, 2014

Contents

1 Function Analysis

Example 1. Analyse

$$f(x) = \frac{x^3}{2(x+1)^2}$$

Solution. Domain of definition:

$$D(f) : x \neq -1$$

$(0, 0)$ is the only point of intersection with the axes.

The function is neither even nor odd. It is also non-periodic.

$$\begin{aligned} f'(x) &= \frac{3x^2 \cdot 2(x+1)^2 - 4(x+1)x^3}{4(x+1)^3} \\ &= \frac{x^2(x+3)}{2(x+1)^3} \\ \therefore f'(x) = 0 &\iff x = 0 \qquad \text{or } x = -3 \end{aligned}$$

Therefore, f is monotonically increasing in $(-\infty, -3) \cup (-1, \infty)$. f is monotonically decreasing in $(-3, -1)$.

$$f(-3) = -\frac{27}{8}$$

Therefore, $\left(-3, -\frac{27}{8}\right)$ is a local maximum point.

$$\begin{aligned} f''(x) &= \frac{3x}{(1+x)^4} \\ \therefore x < 0 &\implies f''(x) < 0 \\ \therefore x > 0 &\implies f''(x) > 0 \end{aligned}$$

Therefore, f is convex upwards in $(-\infty,-1)\cup(-1,0)$ and convex downwards in $(0,\infty)$. $(0,0)$ is a point of inflection.

$$\lim_{x\to -1}\frac{x^3}{2(x+1)^2} = -\infty$$

Therefore, $x=-1$ is a vertical asymptote.

$$\lim_{\pm\infty}\frac{f(x)}{x} = \frac{x^2}{2(x+1)^2}$$

$$\lim_{x\to\pm\infty} f(x) - ax = -1$$

Therefore, $\dfrac{x}{2}-1$ is an oblique asymptote at $\pm\infty$.

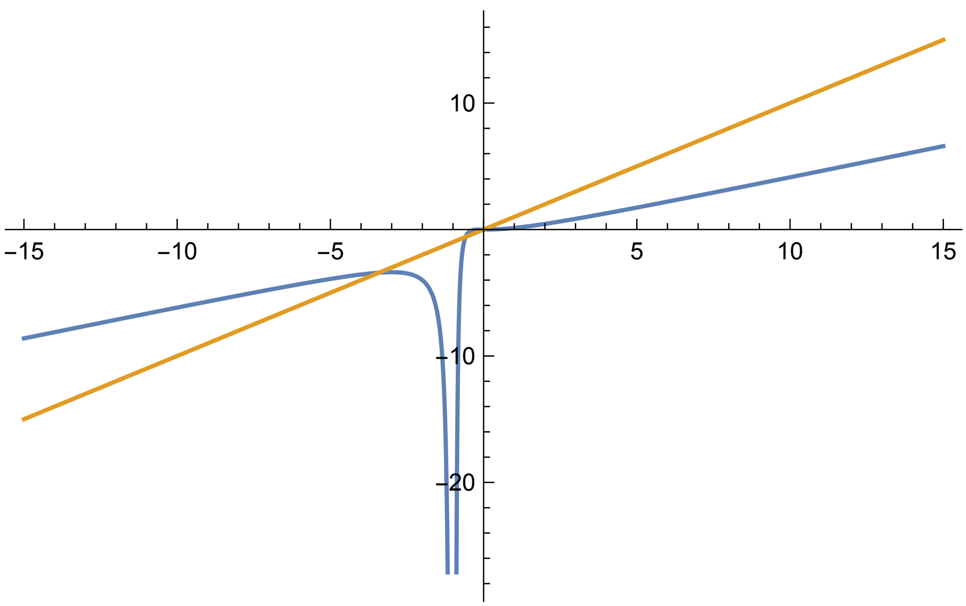


Figure 1: $f(x) = \dfrac{x^3}{2(x+1)^2}$

Example 2. Analyse

$$f(x) = \begin{cases} e^{-1/x^2} & ; \quad x\neq 0 \\ 0 & ; \quad x = 0 \end{cases}$$

Solution. Domain of definition:

$$D(f) = \mathbb{R}$$

The graph intersects the axes only at $(0,0)$.

The function is even and non-periodic.

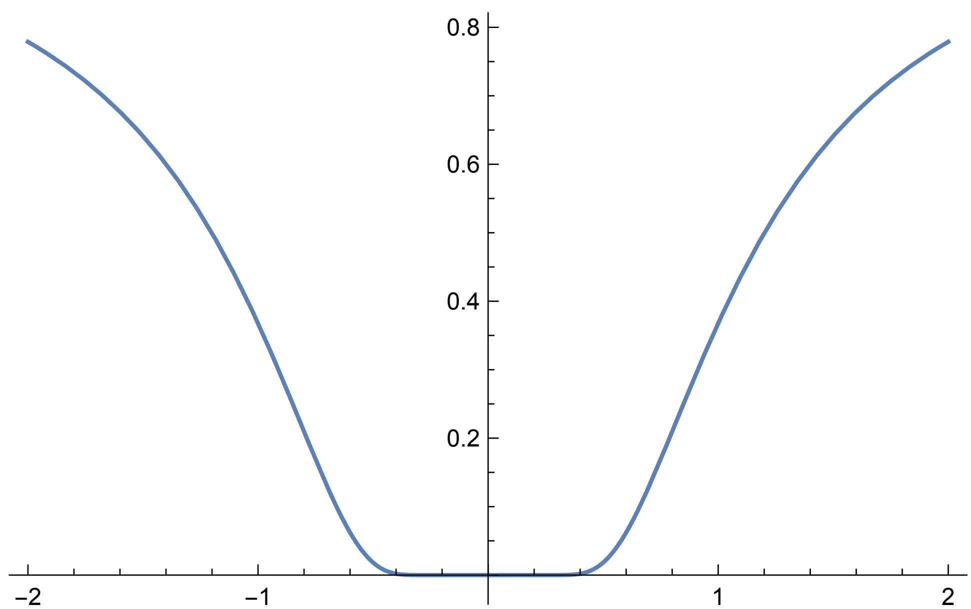
$$\begin{aligned} f'(0) &= \lim_{h \to 0} \frac{f(h) - f(0)}{h} \\ &= 0 x > 0 \implies f'(x) > 0 \end{aligned}$$

$$x < 0 \implies f'(x) < 0$$

Therefore, f is monotonically decreasing in $(-\infty, 0)$ and monotonically increasing in $(0, \infty)$.

f is convex downwards in $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$ and convex upwards in $\left(-\infty, -\sqrt{\frac{2}{3}}\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$.

$y = 1$ is a horizontal asymptote.


0.8
0.6
0.4
0.2
−2
−1
1
2