# Recitation 3

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## Contents

# 1 Arithmetic of Limits

## 1.1 Theorem: If $\lim_{x \to x_0} f(x) = a$ and $\lim_{x \to x_0} g(x) = b$ , then, $\lim_{x \to x_0} (f(x) \pm g(x)) = a \pm b$ , $\lim_{x \to x_0} (f(x) \cdot g(x)) = a \cdot b$ , $\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{a}{b}$

## 1.2 Examples

### 1.2.1 Example 1

$$
\begin{aligned}
\lim_{x \to \infty} \frac{x^2 - 4x + 2}{x^2 + 4} &= \lim_{x \to \infty} \frac{\frac{x^2 - 4x + 2}{x^2}}{\frac{x^2 + 4}{x^2}} \\
&= \lim_{x \to \infty} \frac{1 - \frac{4}{x} + \frac{2}{x^2}}{1 + \frac{4}{x^2}} \\
&= \frac{1}{1} \\
&= 1
\end{aligned}
$$

### 1.2.2 Example 2

$$
\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{\sqrt[4]{x} - 1}
$$

Let $x = t^1 2$

$$
\begin{aligned}
\therefore \lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{\sqrt[4]{x} - 1} &= \lim_{t \to 1} \frac{t^4 - 1}{t^3 - 1} \\
&= \lim_{t \to 1} \frac{(t-1)(t^3 + t^2 + t + 1)}{(t-1)(t^2 + t + 1)} \\
&= \lim_{t \to 1} \frac{t^3 + t^2 + t + 1}{t^2 + t + 1} \\
&= \frac{4}{3}
\end{aligned}
$$

### 1.3 Infinite Arithmetic

$$\frac{\text{“}\infty\text{”}}{\text{“}a\text{”}} = \begin{cases} +\infty; a > 0 \\ -\infty; a < 0 \end{cases}$$

$$\frac{\text{“}a\text{”}}{\text{“}\infty\text{”}} = 0$$

## 2 Useful Limits

If $\lim_{x \to x_0} g(x) = 0$,

$$\lim_{x \to x_0} (1 + g(x))^{\frac{1}{g(x)}} = e$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$$

$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x = e^a$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

## 3 Sandwich Theorem

If, in a punctured neighbourhood of $x_0$ we have

$$g(x) \leq h(x) \leq f(x)$$

and

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x) = l$$

then

$$\lim_{x \to x_0} h(x) = l$$

## 4 Continuity

A function $f$ is continuous at $x_0$ if $\lim_{x \to x_0} f(x) = f(x_0)$.

### 4.1 Types of Discontinuity

Let $f(x)$ be defined in a neighbourhood of $x_0$.

### 4.1.1 Removable Discontinuity

$$\lim_{x \to x_0} f(x) \neq f(x_0)$$

### 4.1.2 Discontinuity of First Kind

$$\lim_{x \to x_{0+}} f(x) \neq \lim_{x \to x_{0-}} f(x)$$

### 4.1.3 Discontinuity of Second Kind

Atleast one of the one-sided limits of $f(x)$ at $x_0$ does not exist. Note that the limits are defined as finite numbers only.