Lecture 24

Aakash Jog

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1 Vector Fields

1.1 Curl

Definition 1. Let

$$\overline{F}(x,y,z) = \big(P(x,y,z), Q(x,y,z), R(x,y,z)\big)$$

be a vector field in $\mathbb{R}^3$ and let there exist first order partial derivatives of P, Q, R. Then $\operatorname{curl} \overline{F}$ is a vector field in $\mathbb{R}^3$ which is defined as

$$\operatorname{curl} \overline{F} = (R_y - Q_z, P_z - R_x, Q_x - P_y) = \nabla \times \overline{F}$$

Theorem 1. *If $\overline{F} = (P,Q,R)$ is a vector field defined of $\mathbb{R}$ with continuous first order partial derivatives of P, Q, R and $\operatorname{curl} \overline{F} = \overline{0}$, then $\overline{F}$ is a conservative vector field.*

1.2 Divergence

Definition 2. Let $\overline{F}(x,y,z) = \big(P(x,y,z), Q(x,y,z), R(x,y,z)\big)$ be a vector field in $\mathbb{R}^3$ and let there exist P_x, Q_y, R_z. Then the divergence of $\overline{F}$ is

$$\operatorname{div} \overline{F} = P_x + Q_y + R_z = \nabla \cdot \overline{F}$$

2 Stoke's and Gauss' Theorem

Definition 3 (Curve with positive orientation)**.** Let S be a surface with normal $\hat{n}$ and let C be a curve which bounds S. Then C is a curve with positive orientation with respect to S if, as we walk on C in this direction and with our head in the direction of $\hat{n}$, the surface S is always on our left.

Theorem 2 (Stoke's Theorem)**.** *Let S be a piecewise smooth surface with normal $\hat{n}$ and let S be bounded by a curve C which is piecewise smooth, simple, closed and with positive orientation with respect to S. Let $\overline{F}(x,y,z) = \big(P(x,y,z), Q(x,y,z), R(x,y,z)\big)$ be a vector field such that there exist continuous first order partial derivatives of P, Q, R in an open domain of $\mathbb{R}^3$ which contains S. Then*

$$\int\limits_C \overline{F} \cdot \hat{T} \, \mathrm{d}s = \iint\limits_S \operatorname{curl} \overline{F} \cdot \hat{n} \, \mathrm{d}S$$

Remark 1. Stoke's Theorem is a generalization of Green's Theorem.