Recitation 14

Aakash Jog

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Contents

1 Double Integrals

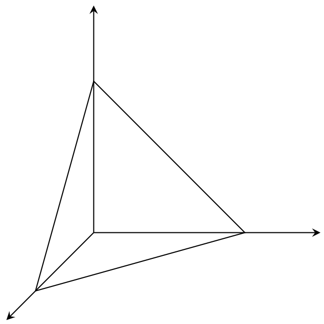
Example 1. Calculate $\iint\limits_D y \,\mathrm{d}x \,\mathrm{d}y$ where D is bounded by $x = y^2, y \geq 0$ and $y = x^2$.

Solution.

$$\begin{aligned}
\iint\limits_D y \,\mathrm{d}x \,\mathrm{d}y &= \int\limits_0^1 \int\limits_{x^2}^{\sqrt{x}} y \,\mathrm{d}y \,\mathrm{d}x \\
&= \int\limits_0^1 \frac{y^2}{2} \Bigg|_{y=x^2}^{y=\sqrt{x}} \mathrm{d}x \\
&= \int\limits_0^1 \left(\frac{x}{2} - \frac{x^4}{2} \right) \mathrm{d}x \\
&= \frac{x^2}{4} - \frac{x^5}{10} \Bigg|_0^1 \\
&= \frac{1}{4} - \frac{1}{10} \\
&= \frac{3}{20}
\end{aligned}$$

2 Triple Integrals

Example 2. Find $\iiint\limits_E x^2 + y^2 + z^2 \,\mathrm{d}V$ where E is bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a, a > 0$.



Solution. Therefore,

$$\iiint_E x^2+y^2+z^2\,\mathrm{d}V = \int_0^a \int_0^{a-x} \int_0^{a-x-y} x^2+y^2+z^2\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

Example 3. Calculate $\iiint_E xe^z\,\mathrm{d}V$ where E is bounded by $z = x^2+y^2$ and $z = 8-x^2-y^2$.

Solution. The two boundaries intersect at $x^2+y^2 = 4$. Therefore the projection of the volume is the circle.
Therefore,

$$\iiint_E xe^z\,\mathrm{d}V = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{8-x^2-y^2} xe^z\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

3 Line Integrals of Scalar Functions

Example 4. Calculate $\int_C x^2+y^2\,\mathrm{d}s$ where C is a circle of radius 2.

Solution.

$$\begin{aligned}\int_C x^2+y^2\,\mathrm{d}s &= \int_0^{2\pi} \left((2\cos t)^2+(2\sin t)^2\right)\cdot 2\,\mathrm{d}t \\ &= 16\pi\end{aligned}$$

4 Line Integrals of Vector Functions

Example 5. Calculate $\int\limits_C \frac{x}{y}\,\mathrm{d}x + \frac{y-x}{x}\,\mathrm{d}y$ where C is the path over the parabola $y = x^2$ from $(2,4)$ to $(1,1)$.

Solution.

$$\begin{aligned}
\int\limits_C \left(\frac{x}{y}, \frac{y-x}{x}\right) \mathrm{d}r &= \int\limits_2^1 \left(\frac{t}{t^2} + \frac{t^2-t}{t}\right) \cdot (1, 2t)\,\mathrm{d}t \\
&= \int\limits_2^1 \left(\frac{1}{t} + (t-1)\cdot 2t\right) \mathrm{d}t \\
&= \ln t + \frac{2t^3}{3} - t^2 \Bigg|_2^1 \\
&= \ln\frac{1}{2} + \frac{2}{3} - \frac{16}{3} - 1 + 4 \\
&= 3 - \frac{14}{3} - \ln 2 \\
&= \frac{5}{3} - \ln 2
\end{aligned}$$