

Recitation 9

Wednesday 24th December, 2014

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1 Definite Integrals

Example 1. Calculate

$$\int_1^e \frac{dx}{x\sqrt{1+\ln x}}$$

Solution. Let

$$\begin{aligned} y &= \ln x \\ \therefore dy &= \frac{dx}{x} \end{aligned}$$

Therefore,

$$\begin{aligned} \int_1^e \frac{dx}{x\sqrt{1+\ln x}} &= \int_0^1 \frac{dy}{\sqrt{1+y}} \\ &= 2\sqrt{1+y} \Big|_0^1 \\ &= 2\sqrt{2} - 2 \end{aligned}$$

Example 2. Find the area bounded by $x = 2$, $y = x$, $y = \frac{1}{\sqrt{x}}$.

Solution.

$$\begin{aligned} A &= \left(\int_1^2 x - \frac{1}{\sqrt{x}} \right) dx \\ &= \frac{x^2}{2} - 2\sqrt{x} \Big|_1^2 \\ &= 2 - 2\sqrt{2} + 2 + \frac{1}{2} \\ &= \frac{7}{2} - 2\sqrt{2} \end{aligned}$$

Example 3. Find the length of the graph of $f(x) = \ln(-\cos x)$ in $[2, 4]$.

Solution.

$$\begin{aligned}l &= \int_a^b \sqrt{1 + (f'(x))^2} \, dx \\&= \int_2^4 \sqrt{1 + \frac{\sin^2(x)}{\cos^2(x)}} \, dx \\&= \int_2^4 \sqrt{\sec^2 x} \, dx \\&= - \int_2^4 \sec x \, dx \\&= - \ln |\sec x + \tan x| \Big|_2^4 \\&= - \ln |\sec 4 + \tan 4| + \ln |\sec 2 + \tan 4|\end{aligned}$$

Example 4. Find the volume of the body obtained from rotation of $y = \sqrt{x}$ in $[0, 1]$ around the x -axis, and the y -axis.

Solution. Rotating the graph around the x -axis,

$$\begin{aligned}V &= \pi \int_0^1 (\sqrt{x})^2 \, dx \\&= \pi \left. \frac{x^2}{2} \right|_0^1 \\&= \frac{\pi}{2}\end{aligned}$$

$$\begin{aligned}V &= \pi \int_0^1 (y^2)^2 \, dy \\&= \pi \left. \frac{y^5}{5} \right|_0^1 \\&= \frac{\pi}{5}\end{aligned}$$