Lecture 2

Thursday 30$^{\text{th}}$ October, 2014

Contents

1 Functions

1.1 Continuity

If $f : D(f) \to I(f)$ is one-to-one and onto. Then, we can define $g : I(f) \to D(f)$, which is one-to-one and onto, by $g(y) = x$, where $y = f(x)$. Therefore, $g(f(x)) = x$. g is called the inverse function of f.

The inverse function is denoted as $g = f^{-1}$(Note: $f^{-1} \neq \frac{1}{f}$)

$$D(f) = I(f^{-1}) \tag{1}$$

$$I(f) = D(f^{-1}) \tag{2}$$

The graphs of a f and f^{-1} are symmeteric about $y = x$.

1.2 Elementary Operations between Functions

1.2.1 $h = f \pm g$

$$h(x) = f(x) \pm g(x) \tag{3}$$

$$D(h) = D(f) \cap D(g) \tag{4}$$

1.2.2 $h = kf$

$$h(x) = kf(x) \tag{5}$$

$$D(h) = D(f) \tag{6}$$

1.2.3 $h = fg$

$$h(x) = f(x)g(x) \tag{7}$$

$$D(h) = D(f) \cap D(g) \tag{8}$$

1.2.4 $h = \frac{f}{g}$

$$h(x) = \frac{f(x)}{g(x)} \tag{9}$$

$$D(h) = \{x \in D(f) \cap D(g) : g(x) \neq 0\} \tag{10}$$

1.3 Composite Functions

Let $f : D(f) \to E$ and $g : D(g) \to F$ be two functions. A composition of f with g is a function $h : D(h) \to F$ where $h(x) = g(f(x))$. It is denoted as $g \circ f$

$$D(h) = \{x \in D(f) : f(x) \in D(g)\} \tag{11}$$

1.4 Elementary Functions

1.4.1 Polynomial

$$y = f(x) = a_0 + a_1 x + \cdots + a_n x^n; a_0, \ldots, a_n \in \mathbb{R} \tag{12}$$
$$D(f) = \mathbb{R} \tag{13}$$

1. If $n = 0, y = f(x) = a_0$ represents a constant function.
2. If $n = 1, y = f(x) = a_0 + a_1 x$ represents a straight line in the $X - Y$ plane.
3. If $n = 2, y = f(x) = a_0 + a_1 x + a_2 x^2$ represents a parabola in the $X - Y$ plane.

1.4.2 Power Function

$$y = f(x) = x^a; a \in R \tag{14}$$
$$D(f) \text{ depends on } a \tag{15}$$

1.4.3 Exponential Function

$$y = f(x) = a^x; a > 0, a \neq 1 \tag{16}$$
$$D(f) = \mathbb{R} \tag{17}$$
$$I(f) = (0, \infty) \tag{18}$$

1.4.4 Logarithmic Function

$$y = f(x) = \log_a x \tag{19}$$
$$D(f) = (0, \infty) \tag{20}$$
$$I(f) = \mathbb{R} \tag{21}$$

1.4.5 Trigonometeric Functions

$$y = f(x) = \sin x \tag{22}$$
$$y = f(x) = \cos x \tag{23}$$
$$y = f(x) = \tan x = \frac{\sin x}{\cos x} \tag{24}$$
$$y = f(x) = \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x} \tag{25}$$
$$y = f(x) = \csc x = \frac{1}{\sin x} \tag{26}$$
$$y = f(x) = \sec x = \frac{1}{\cos x} \tag{27}$$

1.4.6 Inverse Trigonometeric Functions

$$y = f^{-1}(x) = \sin^{-1} x = \arcsin x \tag{28}$$
$$D(\arcsin x) = [-1, 1] \tag{29}$$
$$I(\arcsin x) = [-\frac{\pi}{2}, \frac{\pi}{2}] \tag{30}$$

$$y = f^{-1}(x) = \cos^{-1} x = \arccos x \tag{31}$$
$$D(\arccos x) = [-1, 1] \tag{32}$$
$$I(\arccos x) = [0, \pi] \tag{33}$$

$$y = f^{-1}(x) = \tan^{-1} x = \arctan x \tag{34}$$
$$D(\arctan x) = \mathbb{R} \tag{35}$$
$$I(\arctan x) = [-\frac{\pi}{2}, \frac{\pi}{2}] \tag{36}$$

1.4.7 Hyperbolic Functions

$$\sinh x \doteq \frac{e^x - e^{-x}}{2} \tag{37}$$
$$D(\sinh x) = \mathbb{R} \tag{38}$$
$$I(\sinh x) = \mathbb{R} \tag{39}$$

$$\cosh x \doteq \frac{e^x + e^{-x}}{2} \tag{40}$$
$$D(\cosh x) = \mathbb{R} \tag{41}$$
$$I(\cosh x) = [1, \infty) \tag{42}$$

$$\tanh x \doteq \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{43}$$
$$D(\tanh x) = \mathbb{R} \tag{44}$$
$$I(\tanh x) = (-1, 1) \tag{45}$$

1.4.7.1 Identities of Hyperbolic Functions

$$\sinh(2x) = 2\sinh x \cosh x \tag{46}$$

$$\cosh^2 x + \sinh^2 x = \cosh(2x) \tag{47}$$

$$\cosh^2 x - \sinh^2 x = 1 \tag{48}$$

$$\frac{\cosh(2x) - 1}{2} = \sinh^2 x \tag{49}$$

$$\frac{\cosh(2x) + 1}{2} = \cosh^2 x \tag{50}$$

1.4.8 Absolute Value

$$y = f(x) = \begin{cases} x; x > 0 \\ 0; x = 0 \\ -x; x < 0 \end{cases} \tag{51}$$

1.4.9 Floor Function

$$y = f(x) = \lfloor x \rfloor = \text{the largest integer less than or equal to } x \tag{52}$$