# DIFFERENTIAL AND INTEGRAL METHODS - EXERCISE 11

(1) Check whether the following functions are continuous at $(0,0)$:

    (a) $f(x,y) = \begin{cases} \frac{x}{3x+5y}, & (x,y) \neq (0,0) \\ 5, & (x,y) = (0,0) \end{cases}$

    (b) $f(x,y) = \begin{cases} \frac{x^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

(2) Find the domain of definition of the following functions:

    (a) $f(x,y) = \sqrt{\frac{1+x+y}{1-x-2y}}$

    (b) $f(x,y) = \frac{1}{\cos(x^2+y^2)}$

(3) Find the following limits:

    (a) $\lim\limits_{(x,y)\to(\frac{\pi}{2},0)} [\sin x + (\ln \frac{x+y}{x-y}) \cdot \frac{x}{\sqrt{x^2+y^2}}$

    (b) $\lim\limits_{(x,y,z)\to(0,0,0)} \frac{\sqrt{x^2+y^2+z^2+1}-1}{\sqrt{x^2+y^2+z^2}}$

    (c) $\lim\limits_{\substack{x \to \infty \\ y \to 4}} \left(1+\frac{1}{x}\right)^{\frac{x^2}{x+y}}$

    (d) $\lim\limits_{(x,y)\to(0,0)} \frac{x-y}{x+y}$

    (e) $\lim\limits_{(x,y)\to(2,1)} \frac{y\sin(xy-2)}{3xy-6}$

    (f) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2y^2+(x^2-y^2)^2}$

(4) Find the following limits or show that they do not exist:

    (a) $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$.

    (b) $\lim\limits_{(x,y)\to(0,0)} \frac{3xy^2-5y^4}{x^2+2y^2}$

    (c) $\lim\limits_{(x,y)\to(0,0)} \frac{e^{xy}-1}{x^2+y^2}$ (Hint: Calculate $\lim_{a\to 0} \frac{e^a-1}{a}$ and use it)

    (d) $\lim\limits_{\substack{x \to \infty \\ y \to \infty}} \frac{2x+3y}{x^2-xy+y^2}$

    (e) $\lim\limits_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$

    (f) $\lim\limits_{(x,y)\to(0,0)} \frac{3y^4+x^2y^2+3x^2}{x^2+y^4}$