Lecture 10

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1 Full Investigation of Functions

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Example 1. Investigate

$$y = f(x) = \frac{(x-1)^3}{(x+1)^2}$$

Solution.

$$D(f) = \mathbb{R} - \{-1\}$$

$$\begin{aligned} y &= 0 && \implies x = 1 \\ x &= 0 && \implies y = -1 \end{aligned}$$

The function is not periodic.

$$\begin{aligned} f(-x) &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

Therefore, the function is not symmetric.

$$f'(x) = \frac{(x-1)^2(x+5)}{(x+1)^3}$$

Therefore, $x = -5$ is a local maximum point.
The function is monotonically increasing in $(-\infty, -5) \cup (-1, +\infty)$ and is monotonically decreasing in $(-5, -1)$.

$$f''(x) = \frac{24(x-1)}{(x+1)^4}$$

Therefore, the function is convex upwards in $(-\infty,-1)\cup(-1,1)$ and convex downwards in $(1,\infty)$.

$$\begin{aligned}
\lim_{x\to-1^-}\frac{(x-1)^3}{(x+1)^2} &= \frac{-8}{+0}\\
&= -\infty\\
\lim_{x\to-1^+}\frac{(x-1)^3}{(x+1)^2} &= \frac{-8}{+0}\\
&= -\infty
\end{aligned}$$

Therefore, $x=-1$ is a vertical asymptote of $f(x)$.

$$\begin{aligned}
a_1 &= \lim_{x\to+\infty}\frac{f(x)}{x} && = 1\\
b_1 &= \lim_{x\to+\infty}\big(f(x)-a_1x\big) && = -5\\
a_2 &= \lim_{x\to-\infty}\frac{f(x)}{x} && = 1\\
b_2 &= \lim_{x\to-\infty}\big(f(x)-a_1x\big) && = -5
\end{aligned}$$

Therefore, $y=x-5$ is an oblique asymptote of the function, at $+\infty$ and $-\infty$.

Example 2. Investigate

$$f(x)=\begin{cases} x^2\sin x & ;x\neq 0\\ 0 & ;x=0\end{cases}$$

Solution.

$$\begin{aligned}
f'(0) &= \lim_{\Delta x\to 0}\frac{f(0+\Delta x)-f(0)}{\Delta x}\\
&= \lim_{\Delta x\to 0}\frac{(\Delta x)^2\sin\dfrac{1}{\Delta x}}{\Delta x}\\
&= \lim_{\Delta x\to 0}\Delta x\sin\frac{1}{\Delta x}\\
&= 0
\end{aligned}$$

Therefore $x=0$ is a critical point of $f(x)$, but it is not a local extremum point.