Recitation 3

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Contents

1 Arithmetic of Limits

1.1 Theorem: If $\lim\limits_{x\to x_0} f(x) = a$ and $\lim\limits_{x\to x_0} g(x) = b$, then, $\lim\limits_{x\to x_0} (f(x) \pm g(x)) = a \pm b$, $\lim\limits_{x\to x_0} (f(x) \cdot g(x)) = a \cdot b$, $\lim\limits_{x\to x_0} \frac{f(x)}{g(x)} = \frac{a}{b}$

1.2 Examples

1.2.1 Example 1

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^2-4x+2}{x^2+4} &= \lim_{x\to\infty} \frac{\frac{x^2-4x+2}{x^2}}{\frac{x^2+4}{x^2}} \\
&= \lim_{x\to\infty} \frac{1-\frac{4}{x}+\frac{2}{x^2}}{1+\frac{4}{x^2}} \\
&= \frac{1}{1} \\
&= 1
\end{aligned}$$

1.2.2 Example 2

$$\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$$

Let $x = t^1 2$

$$\begin{aligned}
\therefore \lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1} &= \lim_{t\to 1} \frac{t^4-1}{t^3-1} \\
&= \lim_{t\to 1} \frac{(t-1)(t^3+t^2+t+1)}{(t-1)(t^2+t+1)} \\
&= \lim_{t\to 1} \frac{t^3+t^2+t+1}{t^2+t+1} \\
&= \frac{4}{3}
\end{aligned}$$

1.3 Infinite Arithmetic

$$\frac{\text{“}\infty\text{”}}{\text{“}a\text{”}} = \begin{cases} +\infty; a > 0 \\ -\infty; a < 0 \end{cases}$$

$$\frac{\text{“}a\text{”}}{\text{“}\infty\text{”}} = 0$$

2 Useful Limits

If $\lim_{x \to x_0} g(x) = 0$,

$$\lim_{x \to x_0} (1 + g(x))^{\frac{1}{g(x)}} = e$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$$

$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x = e^a$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

3 Sandwich Theorem

If, in a punctured neighbourhood of x_0 we have

$$g(x) \leq h(x) \leq f(x)$$

and

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x) = l$$

then

$$\lim_{x \to x_0} h(x) = l$$

4 Continuity

A function f is continuous at x_0 if $\lim_{x \to x_0} f(x) = f(x_0)$.

4.1 Types of Discontinuity

Let $f(x)$ be defined in a neighbourhood of x_0.

4.1.1 Removable Discontinuity

$$\lim_{x \to x_0} f(x) \neq f(x_0)$$

4.1.2 Discontinuity of First Kind

$$\lim_{x \to x_{0+}} f(x) \neq \lim_{x \to x_{0-}} f(x)$$

4.1.3 Discontinuity of Second Kind

Atleast one of the one-sided limits of $f(x)$ at x_0 does not exist. Note that the limits are defined as finite numbers only.