Recitation 8

Wednesday 17$^{\text{th}}$ December, 2014

Contents

1 Indefinite Integrals

Example 1.

$$\int \frac{x+2}{x(x+1)^2}\,\mathrm{d}x$$

Solution. Let

$$\begin{aligned}
\frac{x+2}{x(x+1)^2} &= \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \\
&= \frac{A(x+1)^2 + B(x)(x+1) + C(x)}{x(x+1)^2} \\
\therefore x+2 &= A(x+1)^2 + B(x)(x+1) + C(x) \\
\therefore x+2 &= (A+B)x^2 + (2A+B+C)x + A
\end{aligned}$$

Therefore,

$$\begin{aligned}
A &= 2 \\
B &= -2 \\
C &= -1
\end{aligned}$$

Therefore,

$$\begin{aligned}
\int \frac{x+2}{x(x+1)^2}\,\mathrm{d}x &= \int \left(\frac{2}{x} - \frac{2}{x+1} - \frac{1}{(x+1)^2}\right)\mathrm{d}x \\
&= 2\ln|x| - 2\ln|x+1| + \frac{1}{x+1} + d
\end{aligned}$$

Example 2.

$$\int \frac{\cos x}{10+\sin x}\,\mathrm{d}x$$

Solution. Let

$$\begin{aligned}
y &= \sin x \\
\therefore \mathrm{d}y &= \cos x\,\mathrm{d}x
\end{aligned}$$

Therefore,

$$\begin{aligned}
\int \frac{\cos x}{10+\sin x}\,\mathrm{d}x &= \int \frac{\mathrm{d}y}{10+y} \\
&= \ln|10+y| + c \\
&= \ln|10+\sin x| + c
\end{aligned}$$

Example 3.

$$\int \frac{\mathrm{d}x}{\sqrt{x}+\sqrt[3]{x}}\,\mathrm{d}x$$

Solution. Let

$$\begin{aligned}
x &= t^6 \\
\therefore \mathrm{d}x &= 6t^5\,\mathrm{d}t
\end{aligned}$$

Therefore,

$$\begin{aligned}
\int \frac{\mathrm{d}x}{\sqrt{x}+\sqrt[3]{x}}\,\mathrm{d}x &= \int \frac{6t^5}{t^3+t^2}\,\mathrm{d}t \\
&= \int \frac{(t^3+t^2)(6t^2-6t+6)-6t^2}{t^3+t^2}\,\mathrm{d}t \\
&= \int 6t^2-6t+6-\frac{6}{t+1}\,\mathrm{d}t \\
&= \frac{6t^3}{3}-\frac{6t^2}{2}+6t-6\ln|t+1|+c \\
&= \frac{6\sqrt{x}}{3}-\frac{6\sqrt[3]{x}}{2}+6\sqrt[6]{x}-6\ln|\sqrt[6]{x}+1|+c
\end{aligned}$$

Example 4.

$$\int \sin^3 x\,\mathrm{d}x$$

Solution.

$$\begin{aligned}
\int \sin^3 x\,\mathrm{d}x &= \int \sin^2 x \sin x\,\mathrm{d}x \\
&= \int (1-\cos^2 x)\sin x\,\mathrm{d}x
\end{aligned}$$

Let

$$\begin{aligned} y &= \cos x \\ \therefore dify &= -\sin x \,\mathrm{d}x \end{aligned}$$

Therefore,

$$\begin{aligned} \int (1 - \cos^2 x) \sin x \,\mathrm{d}x &= -\int (1 - y^2) \,\mathrm{d}y \\ &= -y + \frac{y^3}{3} + c \\ &= -\cos x + \frac{\cos^3 x}{3} + c \end{aligned}$$

Example 5.

$$\int \frac{\mathrm{d}x}{\sqrt{28 - 12x - x^2}}$$

Solution.

$$\int \frac{\mathrm{d}x}{\sqrt{28 - 12x - x^2}} = \int \frac{\mathrm{d}x}{\sqrt{-(x+6)^2 + 36 + 28}}$$

Let

$$\begin{aligned} t &= x + 6 \\ \therefore \mathrm{d}t &= \mathrm{d}x \end{aligned}$$

Therefore,

$$\begin{aligned} \int \frac{\mathrm{d}x}{\sqrt{-(x+6)^2 + 36 + 28}} &= \int \frac{\mathrm{d}t}{\sqrt{-t^2 + 64}} \\ &= \int \frac{\mathrm{d}t}{8\sqrt{-\left(\frac{t}{8}\right)^2 + 1}} \end{aligned}$$

Let

$$\begin{aligned} y &= \frac{t}{8} \\ \therefore \mathrm{d}y &= \frac{1}{8} \,\mathrm{d}t \end{aligned}$$

Therefore,

$$
\begin{aligned}
\int \frac{\mathrm{d}t}{8\sqrt{-\left(\frac{t}{8}\right)^2 + 1}} &= \int \frac{\mathrm{d}y}{\sqrt{1-y^2}} \\
&= \arcsin y + c \\
&= \arcsin \frac{x+6}{8} + c
\end{aligned}
$$